ANSWER TO SECOND PROBLEM

following Laffont the solution is obtained by:

$$\psi'(e^*) = 1 - \frac{\lambda F(\beta)\psi''(e^*)}{(1+\lambda) f(\beta)} \qquad (*)$$

and

$$U(\beta) = \int_{\underline{\beta}}^{\bar{\beta}} \psi'(e(\beta))d\beta$$

For the highest type $[\underline{\beta}]$, we assume $F(\underline{\beta}) = 0$ and thus $\psi'(e(\beta)) = 1 - 0 = 1$ as in the perfect information case. The utility will also be positive as $\int_{\underline{\beta}}^{\bar{\beta}} \psi'(e(\beta))d\beta > 0$ for $\beta \neq \overline{\beta}$.

For the lowest type $[\overline{\beta}]$, we assume $F(\overline{\beta}) = 1$ and thus $\psi'(e(\beta)) = 1 - \frac{\lambda}{1+\lambda}\frac{1}{f(\beta)}\psi''(e(\beta)) < 1$. The utility will also be 0 as $U(\overline{\beta}) = \int_{\underline{\beta}}^{\bar{\beta}} \psi'(e(\beta))d\beta = 0$ for $\beta = \overline{\beta}$.

The result will move towards the perfect information solution for $\beta \Rightarrow \underline{\beta}$ and likewise the inforamtional rent (utility) will increase.

Mathematical Appendix:

1. How to get to the optimal solution:

$$H\{S - (1+\lambda)(\beta - e + \psi(e)) - \lambda U(\beta)\} f(\beta)d\beta + \mu[-\psi(e(\beta))]$$

$$\frac{\partial H}{\partial e} = 0 \Rightarrow (1+\lambda)[1 - \psi'(e)] f(\beta) = \mu\psi''(e(\beta))$$

Following the Hamiltonion approach, the following has to hold (see Kamien and Schwartz, if you want to):

$$\frac{\partial H}{\partial \mu} = -\psi(e(\beta)) = \dot{U}(\beta)$$

$$-\frac{\partial H}{\partial \mu} = \dot{\mu} \Rightarrow \dot{\mu} = \lambda f(\beta) \Rightarrow \mu = \lambda F(\beta)$$

Consequently:

$$(1+\lambda)[1 - \psi'(e)] f(\beta) = \lambda F(\beta)\psi''(e(\beta)$$

$$1 - \psi'(e) = \frac{\lambda}{1+\lambda}\frac{F(\beta)}{f(\beta)}\psi''(e(\beta)$$

$$\Rightarrow \psi'(e) = 1 - \frac{\lambda}{1+\lambda}\frac{F(\beta)}{f(\beta)}\psi''(e(\beta))$$

2. How to transform the first oder constraint from t to U (based on Schmidt 1995):

given:
$$C'(\beta) \;\geq\; 0 \;, \qquad (4.38)$$
$$\Psi'(\beta - C(\beta))C'(\beta) + t'(\beta) \;=\; 0 \;. \qquad (4.39)$$

if follows:

Beachte, daß $e'(\beta) = 1 - C'(\beta)$. Also ist (4.38) äquivalent zu

$$e'(\beta) \leq 1 \;. \qquad (4.40)$$

Ferner, sei $U(\beta) = U(\beta, \beta) = t(\beta) - \Psi(\beta - C(\beta))$. Dann ist

$$U'(\beta) \;=\; t'(\beta) - (1 - C'(\beta))\Psi'(\beta - C(\beta)) \qquad (4.41)$$
$$\;=\; \underbrace{t'(\beta) + \Psi'(\beta - C(\beta))C'(\beta)}_{= 0 \text{ wegen } (4.39)} - \Psi'(\beta - C(\beta)) \qquad (4.42)$$

Also ist (4.39) äquivalent zu

$$U'(\beta) \;=\; -\Psi'(\beta - C(\beta)) \;. \qquad (4.43)$$

3. How to get to the utility integral (based on Schmidt 1995):

Wenn wir (4.43) von β bis $\overline{\beta}$ aufintegrieren, erhalten wir:

$$\int_{\beta}^{\overline{\beta}} U'(\tilde{\beta})d\tilde{\beta} \;=\; U(\overline{\beta}) - U(\beta) = -\int_{\beta}^{\overline{\beta}} \Psi'(\tilde{\beta} - C(\tilde{\beta}))d\tilde{\beta} \;, \qquad (4.44)$$

bzw.:

$$U(\beta) \;=\; U(\overline{\beta}) + \int_{\beta}^{\overline{\beta}} \Psi'(e(\tilde{\beta}))d\tilde{\beta} \;. \qquad (4.45)$$

We just assume that the utility for the least efficient type $U(\overline{\beta})$ is 0 (we could also assume 1, 2 or some other positive number), in order to guarantee that the participation constraint U >= 0.