Strategic Information Acquisition in Networked Groups with "Informational Spillovers"*

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Abstract

This paper develops a model of costly information acquisition by agents who are connected through a network. For a exogenously given network, each agent decides first on information acquisition from his neighbors and then, after processing the information acquired, takes an action. Each agent is concerned about the extent to which other agents align their actions with the underlying state. A new equilibrium notion, which is in the spirit of perfect Bayesian equilibrium, is proposed to analyze information acquisition decisions within networked groups. This equilibrium notion allows each agent to compute, when deciding about information acquisition, the extent to which changes in his information acquisition decision will affect his own perception of future expected payoffs. Agents anticipate and incorporate such changes in their information acquisition decisions. Both the efficient and the equilibrium information acquisition profiles are characterized and the compatibility between them is related to the density of the network.

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1 Introduction

In many economic and social settings, agents acquire information from others in order to improve their knowledge of the underlying fundamentals. For example, a researcher acquires information from colleagues in order to improve his knowledge of a certain scientific problem and of the possible alternatives to address it. Also, it is common that investors in a new sector acquire information from other investors to obtain more accurate predictions of the economic variables affecting the profitability of the sector. Most of these information acquisition activities often take place through networks.

Despite the widespread use of information acquisition within networked groups, little is known about this phenomenon. How do agents interact with respect to their information acquisition decisions when they are connected through a network? How is the compatibility between efficient and equilibrium information acquisition related to the network architecture? To address these questions, this paper provides a game theoretical framework that treats the transmission of information as a result of a Bayesian belief revision process.

In this model, the architecture of the network is exogenously given and common knowledge, and agents are engaged in a two-stage game. In the first stage, each agent chooses at a cost the amount of information that he acquires from his neighbors. In the second stage, each agent chooses a payoff-relevant action. Agents are able to receive information only from their direct neighbors, so that I do not consider the *network effect* which forms an essential part of most of the analyses of communication networks.

This model is built on the assumption that, when the agents choose the amount of information that they acquire, they correctly anticipate and compute the extent to which the newly acquired information will change their perceptions of their own future expected payoffs. This assumption constitutes the crucial sequential rationality requirement of the equilibrium concept proposed in this paper, *information acquisition equilibrium* (IAE). The IAE concept requires that each agent be sequentially rational in both stages of the underlying game and that posterior beliefs be consistent, according to Bayes' rule, with the strategies over messages chosen in the first stage. Thus, IAE requirements seem analogous to those of perfect Bayesian equilibrium. In fact, IAE departs from perfect Bayesian equilibrium only in the way in which the agents compute their expected payoffs in the first stage. In an IAE, an agent's expected payoff in the first stage is specified by discounting his expected payoffs at the various information sets in the second stage according to the combination of strategies over messages chosen by the agents. Given this specification, when an agent changes his information acquisition choice at the first stage, he is able to compute the extent to which his own perception of his payoff in the second stage will change.

The motivation for this key sequential rationality requirement of IAE has a behavioral nature and clearly contrasts that of perfect Bayesian equilibrium, the equilibrium concept usually proposed to analyze information revelation decisions. In signaling[1] and cheap talk[2] models, an agent who decides about information revelation cares about the action that he induces the receiver to take rather than about any changes on his own posterior beliefs.[3] However, when an agent decides about acquiring new information, it seems reasonable to assume that he anticipates the self-induced changes on his posterior beliefs and the extent to which such changes will affect his perception of own future payoffs. Then, it seems appropriate to consider that, at the date when the agent decides about information acquisition, he cares about both the induced optimal actions and the anticipated perception of his own future payoffs. The nature of the problem of information acquisition seems different from that of information revelation. This paper proposes an equilibrium concept suitable to incorporate into the agents' rationality the fact that they anticipate the role of the acquired information in shaping their own posterior beliefs and, accordingly, their own perceptions of future expected payoffs. For the two-agent version of the underlying game, we can simplify an agent's expected payoff in the first stage as it is specified in an IAE so as to obtain the expected payoff used in a perfect Bayesian equilibrium. This shows that both concepts of equilibrium coincide for the two-agent case. However, they turn out to be different equilibrium concepts for the case with more than two agents.

Regarding preferences, I adopt a particular choice which seems reasonable to study information acquisition within groups. In this model, a sender cannot decide about the amount of information that he discloses to his neighbors. Therefore, strategic interactions over actions are ruled out. Each agent's payoff depends on the appropriateness of his own action to the underlying state. In addition, an agent's payoff decreases with the distance between the others' actions and the state—this is the way in which positive "informational spillovers" are formalized. Also, to render the analysis tractable, I assume that payoffs are quadratic.

The main motivation for the assumed preferences comes from organizations or groups where their members face similar problems which they must solve independently, and where each of them wishes to solve his problem but also values that the other agents solve theirs too. Clearly, in this framework no agent has incentives to refuse to transmit his

[1] See, e.g., Spence [20], Rothschild and Stiglitz [19], and Wilson [23].

[2] The seminal work on cheap talk is due to Crawford and Sobel [11].

[3] In fact, the study of posterior beliefs and perceptions of the agent who decides about information transmission is not the purpose of these sender-receiver models since they assume that the receiver is completely informed about the underlying state.

information to others so that the analysis of information revelation decisions is irrelevant. Then, we can aptly restrict attention to information acquisition decisions. Examples of such groups or organizations are those of a research department, where its members pursuit independently similar innovations, or a group of investors in a new sector, where the profitability of the sector increases as more investors choose investment strategies appropriate to the underlying state.

To study the efficiency properties of information acquisition through a network, I consider that the planner seeks to maximize the sum of the ex ante payoffs of the agents. Proposition 1 provides the following necessary and sufficient condition for an information acquisition profile to be efficient: it is efficient to acquire full information from a given neighbor if and only if the cost of information acquisition does not exceed the variance of the neighbor's type. Otherwise, it is efficient to acquire no information at all from that neighbor. Not surprisingly, this result gives us an efficiency criterion in terms of the marginal cost and the marginal benefit derived from information acquisition.

The second result of this paper characterizes an agent's best response information acquisition strategy with respect to a given neighbor. Proposition 2 shows that the incentives of an agent to acquire information from a neighbor increase with the amount of information that the rest of neighbors of that neighbor acquire from him.

Both the sequential rationality requirement at the first stage of the underlying game imposed by the IAE notion and the presence of positive informational spillovers are crucial to obtain the result that agents wish to coordinate their information acquisition decisions. The following example illustrates the forces behind this result. Consider three agents such that agent 1 is linked to agent 2 and agent 2 is linked to agent 3. We are then encouraged to ask what forces cause agent 1's decision about information acquisition from agent 2 to depend on agent 3's choice about information acquisition from agent 2. Agent 1 knows the strategy over messages that agent 2 adopts with respect to agent 3 (which is indeed chosen by agent 3). However, so long as he does not acquire full information from agent 2, he is still uncertain about agent 2's type. As a consequence, he is also uncertain about the particular message that agent 2 sends to agent 3. In other words, the information that agent 1 acquires from agent 2 improves his knowledge about 2's private information but also about the extent to which agent 3 acquires information from agent 2. Therefore, this information also changes his perception of the extent to which agent 3 is able to solve his problem. Then, by changing his information acquisition decision with respect to agent 2, agent 3 changes the relation between agent 1's information acquisition choice with respect to agent 2 and agent 1's own (anticipated) perception of agent 3's most preferred action. This affects agent 1's information acquisition choice with respect to agent 2 given that (i) information acquisition is costly, (ii) agent 1 cares about agent 3's action, and (iii) agent 1 is risk averse with respect to agent 3's action.

Regarding its welfare implications, this paper provides conditions in terms of a precise measure of the network density—the minimum degree of the network—under which efficient information acquisition can be either reached in an IAE or not. These results, provided by Corollaries 1 and 2, suggest that it is more likely that the IAE be efficient when the least connected agent is highly connected relative to the size of the entire group.

To the best of my knowledge this paper is the first to conduct an analysis of strategic information acquisition decisions and their welfare implications for networked groups. For networks that allow for communication among connected agents, Jackson and Wolinsky [15], and Bala and Goyal [4] pioneered the study of the compatibility between efficient and equilibrium networks.[4] For tractability reasons, most of this literature do not consider the information transmission problem in terms of a Bayesian belief revision process. Instead, certain given relations are assumed between an agent's payoff and the number of agents whose information he can access. By doing so, the effects of information on payoffs are exogenously modeled and the role of information in shaping beliefs is ignored.

Recently, some papers have analyzed communication networks using Bayesian belief revision processes to model information transmission. Calvó-Armengol and de Martí [8] consider a framework where agents communicate through a given network as a result of a Bayesian belief revision process that takes place in successive rounds. The main differences between their approach and that followed in this paper are: (i) they do not consider endogenous information transmission decisions, and (ii) the class of preferences that they assume include a second-guessing coordination motive. Hagenbach and Koessler [13] consider a model where each agent decides whether or not to reveal his private information to the others before choosing his own action. The choices on information revelation determine endogenously a communication network. The main difference with this paper is in the fact that they study information revelation decisions. Consequently, they use perfect Bayesian equilibrium as solution concept. Perfect Bayesian equilibrium is arguably an appropriate equilibrium concept for that problem. As a result, they do not obtain strategic interactions over information transmission decisions at equilibrium. This marks a sharp contrast with the results of this paper.

The rest of the paper is structured as follows. The model and the notions of equilibrium and efficiency are introduced in Section 2. Section 3 characterizes both the set of efficient and the set of equilibrium information acquisition profiles, and presents the results that

[4]The line of research on communication networks has been pursued further in different contexts, among others, by Suk-Young Chwe [21], Calvó-Armengol [7], Bloch and Dutta [5], Calvó-Armengol and de Martí [8], and Calvó-Armengol and Jackson [9].

relate the compatibility between equilibrium and efficiency to the network density. In Section 4, I discuss the robustness of the model. Formal justifications are provided by considering two perturbations of the model: one where types are drawn according to a Normal distribution and each agent receives a signal consisting of the true type plus some noise; the other with non-linear information acquisition costs. Section 5 concludes with a discussion of the results. The proofs of all the propositions are grouped together in the Appendix.

2 The Model

2.1 Network Notation

There is a finite set of agents $N := \{1, \ldots, n\}$, with $n \geq 2$. The shorthand notation[5] ij denotes the subset of N, of size two, containing agents i and j, which is referred to as the *link* ij. A *communication network* g is a collection of links where $ij \in g$ means that i and j are directly linked and able to acquire information from each other under network g. Let G denote the set of all possible networks on N. For a network $g \in G$, the set of agent i's *neighbors* is $N_i(g) := \{j \in N : ij \in g\}$ and the number of his neighbors is $n_i(g) := |N_i(g)|$. Finally, let $\delta(g) := \min_{i \in N} n_i(g)$ and $\rho(g) := \max_{i \in N} n_i(g)$ denote, respectively, the minimum and the maximum degree of network g. Both $\delta(g)$ and $\rho(g)$ can be understood as measures of the extent to which agents are connected in network g.

The architecture of the network itself is exogenously given and common knowledge.

2.2 Information Structure, Actions, and Payoffs

Given a network $g \in G$, agents are engaged in a game that is played in two consecutive stages numbered 1 and 2. In stage 1, each agent $i \in N$ decides the amount of information that he acquires from each agent in his neighborhood $N_i(g)$. In stage 2, each agent chooses an action using the information that he has acquired from his neighbors in stage 1.

The *initial private information* of each agent $i \in N$ is described by his type t_i, an element of $T_i := [0, 1]$. For each variable, set, or function, denote its *profile* over all agents by the corresponding bold letter and its profile over all agents except that of agent i with the corresponding letter with subscript $-i$.[6] A *state of the world* is denoted $\boldsymbol{t} := (t_i)_{i \in N}$

[5] The network notation presented here was developed by Jackson and Wolinsky [15].

[6] This notation is standard. Specifically, for each set Y_i with generic element $y_i \in Y_i$, for some agent $i \in N$, write $\boldsymbol{Y}$ to denote the Cartesian product $\times_{i \in N} Y_i$, and, accordingly, write $\boldsymbol{y} := (y_i)_{i \in N} \in \boldsymbol{Y}$ and $y_{-i} := (y_1, \ldots, y_{i-1}, y_{i+1}, \ldots, y_n) \in Y_{-i}$. Likewise, for each family of functions $h_i : Y \to Z$, write $\boldsymbol{h}(\boldsymbol{y}) := (h_i(y_i))_{i \in N}$ and $h_{-i}(y_{-i}) := (h_j(y_j))_{j \neq i}$.

and the *state space* is[7] $\boldsymbol{T} := \times_{i\in N} T_i = [0,1]^n$.[8] Thus, agent i's type is the respective coordinate t_i of the actual state $\boldsymbol{t}$. All aspects of this game, except $\boldsymbol{t}$, are common knowledge. Clearly, this information structure exhibits complementarities in the sense that two distinct agents improve their knowledge about the underlying state by sharing their pieces of private information. In particular, the true state is always obtained by combining the pieces of private information of all the agents.[9]

Although the proposed information structure relates generally to situations with informational complementarities, the main motivation of this model comes from situations where agents face independently a common (or similar) decision problem with several independent "aspects" so that solving the problem requires to solve the various aspects. Each agent is an "expert" in one aspect so that the knowledge about how to solve the problem is improved by information sharing.

In stage 2, each agent chooses a payoff-relevant action. An action for agent i is an n-coordinate vector $a_i \in A_i := [0,1]^n$. Thus, the action space available to each agent $i \in N$ coincides with the state space, $A_i := \boldsymbol{T} = [0,1]^n$. The idea here is to think of an action as a collection of all the independent steps that an agent must take in order to solve his decision problem (one step for each aspect of the problem). Let $a_{ik} \in [0,1]$ denote the k-th coordinate of the action vector a_i taken by agent i, i.e., $a_i := (a_{ik})_{k\in N}$. Intuitively, a_{ik} summarizes the action taken by agent i with respect to the k-th aspect of the decision problem.

Under the chosen preferences, strategic interactions over actions are ruled out. Each agent wishes, on the one hand, to match his own action with the true state and, on the other hand, is concerned about the extent to which the other agents align their actions with the true state. I call this second motive the *team concern* and interpret it as a positive "informational spillover" or externality affecting the organization/group. I am assuming that the organization receives higher benefits, either monetary or in terms of prestige, as more of its members perform "well" in their independent tasks. Thus, contingent on the performance of the entire organization, each member is rewarded in terms of reputation or monetary payments. For example, consider a set of investors choosing their investment strategies in a new sector, where the profitability of the sector increases with the number of investors that choose a "good" investment strategy. Consequently, each investor cares about the extent to which the rest of investors align their actions with the true state.

[7] The proposed state space is similar to those used in models on multidimensional cheap talk. See, e.g., Chakraborty and Harbaugh [10], and Levy and Razin [17].

[8] For a set B and an integer l, write B^l to denote the l-fold Cartesian product of B.

[9] Jiménez-Martínez [16] proposes an analogous information structure to study a two-agent information sharing problem.

Of course, a broad class of applications can be covered by this model when one thinks of the team concern in terms of benefits derived to each agent from the prestige of the organization.

With regards to the team concern, let $r \in [0,1]$ be a scalar parameter that measures the extent to which each agent cares about the alignment of the other agents' actions with the true state. Let $\|\cdot\|$ denote the Euclidean norm. The payoff to agent i is given by the function $U_i : \boldsymbol{A} \times \boldsymbol{T} \times [0,1] \to \mathbb{R}$ defined by

$$U_i(\boldsymbol{a}, \boldsymbol{t}; r) := -(1-r)\|\boldsymbol{t} - a_i\|^2 - \frac{r}{n-1} \sum_{j \neq i} \|\boldsymbol{t} - a_j\|^2. \tag{1}$$

The first term in equation (1) above is the quadratic loss in the distance between agent i's own action and the true state. The second term is the team concern, i.e, the payoff loss derived from the discrepancy between the other agents' actions and the true state. Parameter r gives us the weight of such a team behavior motive. Notice that the payoff of each agent is strictly decreasing with respect to the (Euclidean) distance between the action that he chooses and the true state. Thus, each agent has incentives to acquire information since more information allows for actions better suited to the underlying state. Of course, for each $r \in (0,1]$, the specified preferences represent common interests for all agents. Finally, I am assuming that the team concern has the form of a positive informational spillover in the sense that, for each $i \in N$, $U_i(\boldsymbol{a}, \boldsymbol{t}; r)$ strictly decreases with $\|\boldsymbol{t} - a_j\|$, for each $j \neq i$.

Although the proposed payoffs are very specific, they can be viewed as a second-order approximation of a more general class of convex preferences. The assumptions imposed on preferences make the analysis tractable. More importantly, this class of preferences allows us to work with all the relevant ingredients that describe an environment without strategic interactions over actions and with external positive effects. The fact that strategic interactions over actions are absent will enable us to focus on the analysis of how the agents interact strategically *only* over their information acquisition decisions.

2.3 The Information Transmission Process

There is a set $M := [0,1]$ of feasible messages available to each agent for information transmission purposes. Thus, the message space coincides with the type space of each agent, and a message $m \in M$ sent by agent i may be interpreted as a statement that his type is $t_i = m$.

At the beginning of stage 1, each agent $i \in N$ chooses the message that each of his neighbors $j \in N_i(g)$ sends to him.[10] All messages are sent simultaneously. Write

[10]Formally, an agent chooses the message strategy that each of his neighbors adopts with respect to

$m_{ji} \in M$ to denote a generic message sent from agent j to agent i, $m_i := (m_{ji})_{j \neq i}$ to denote a combination of messages received by agent i, and $\boldsymbol{m} := (m_i)_{i \in N} \in M^{n(n-1)}$ to denote a message profile.

In terms of strategies, each agent chooses the degree of informativeness of the message strategy that each of his neighbors adopts with respect to him. As it will be specified below, a scalar parameter $x_{ij} \in [0, 1]$ is used to summarize the degree of informativeness of agent j's message strategy with respect to agent i. Thus, for each $j \in N_i(g)$, agent i must choose an information acquisition parameter x_{ij} and we interpret this choice as agent i acquiring an amount of information x_{ij} from agent j.

After each agent has chosen the information acquisition parameter for each of his neighbors, a state $\boldsymbol{t}$ is randomly drawn from $\boldsymbol{T}$ according to a continuous joint density $q(\cdot)$ and each agent learns the corresponding type.[11] Each type t_i is drawn from T_i according to a (common) probability distribution, with continuous marginal density $f(\cdot)$, supported on $[0, 1]$. I assume that the agents' types are independent so that a state $\boldsymbol{t}$ is drawn from $\boldsymbol{T}$ according to density $q(\boldsymbol{t}) := \prod_{i \in N} f(t_i)$. Let us denote the mean and the variance of each agent i's type, respectively, by $\mu := \int_0^1 f(t_i) t_i dt_i$ and by $\sigma^2 := \int_0^1 f(t_i)(t_i - \mu)^2 dt_i$.

I can now be more specific about the information acquisition parameter and its interpretation. For $i \in N$ and $j \in N_i(g)$, x_{ij} is the weight parameter of a linear combination between a totally non-informative (pooling) and a totally informative (completely separating) message strategy. Formally, given the information acquisition parameter x_{ij}, agent j's type t_j sends message $m_{ji} \in M$ to agent i according to the function $\beta_{ji} : M \times T_j \times [0, 1] \to [0, 1]$ defined as

$$\beta_{ji}(m_{ji}|t_j; x_{ij}) := (1 - x_{ij}) f(m_{ji}) + x_{ij} \mathbb{I}(m_{ji}|t_j), \tag{2}$$

where $\mathbb{I} : M \times T_j \to [0, 1]$ is the indicator function defined, for each $(m_{ji}, t_j) \in M \times T_j$, $i \neq j$, by (i) $\mathbb{I}(m_{ji}|t_j) = 1$ if $m_{ji} = t_j$, and (ii) $\mathbb{I}(m_{ji}|t_j) = 0$ if $m_{ji} \neq t_j$.

For $x_{ij} \in [0, 1]$, $i, j \in N$, $i \neq j$, $\beta_{ji}(\cdot; x_{ij})$ specifies a *message strategy* for agent j with respect to agent i, parameterized by x_{ij}. Therefore, $\beta_{ji}(m_{ji}|t_j; x_{ij})$ is the density associated to type t_j sending message m_{ji} to agent i, given that agent i chooses information acquisition parameter x_{ij}. Thus, x_{ij} can be interpreted as agent i choosing quality x_{ij} for the message service through which he receives information from agent j about his type (or simply as agent i acquiring amount x_{ij} of information from agent j).

Since the message space coincides with the type space of each agent, density f can be evaluated meaningfully at each message $m \in M$. Therefore, expression (2) above specifies

him. In this sense, this choice may be interpreted as a decision about the quality of a message service.

[11] In the present context, it would be equivalent to assume that agents learn their private information *before* they decide about information acquisition.

an appropriate class of message strategies for information transmission purposes.

Each agent $i \in N$ incurs a cost $c > 0$ (in terms of time, effort, or money) for each unit of information that he acquires from each of his neighbors. Thus, the cost function is assumed to be linear. The described two-stage game typically has multiple equilibria. Under the assumptions imposed on payoffs, the objective problem of an agent with respect to information acquisition is not concave. These assumptions also imply that agents make corner choices at equilibrium.

The class of message strategies allowed for is admittedly very specific. Three points should be made in defense of this choice. The first is that it captures quite conveniently, and without loss of generality for our purposes, the extent to which an agent j transmits his information to another agent i: (i) if $x_{ij} = 0$, then agent j reveals no information at all (i.e., he *pools*), (ii) if $x_{ij} = 1$, then agent j fully reveals (i.e., he *completely separates*), and (iii) if $x_{ij} \in (0, 1)$, then agent j reveals partially (i.e., he *semi-separates*). Furthermore, the relation between the degree of informativeness of agent j's message strategy with respect to agent i and x_{ij} is continuous and strictly increasing on the interval $[0, 1]$.[12]

The second point is that, as it will be discussed in Subsection 4.1, the assumed message strategies induce posterior beliefs whose (conditional) mean and variance behave in a way totally analogous to those obtained by assuming that the information transmission process is described by a Normal signal consisting of the true type plus some noise. This way of modeling information transmission is standard in the recent literature on the social value of information and on communication networks.[13] Thus, an interesting class of information transmission processes falls qualitatively within this model.

The third point is that the underlying game where the agents decide about information acquisition has typically multiple equilibria. This makes problematic any analysis of welfare implications. The proposed message strategies have a linear structure which, together with the linear structure assumed for preferences and for the cost function, mitigates crucially this problem. This makes tractable the analysis of social efficiency. This paper aims at studying the compatibility between equilibrium and efficient information acquisition in networks. The chosen message strategies, together with the assumptions on preferences over actions and the linearity assumption on the cost function, allows us to concentrate on this question.

Let $x_i := (x_{ij})_{j \neq i} \in X_i := [0, 1]^{n-1}$ denote an *information acquisition strategy* for

[12]The fact that the amount of information transmitted by an agent j to his neighbor i is completely described by parameter $x_{ij} \in [0, 1]$ enables us to model the information possessed by each agent as a perfectly divisible good. Thus, using the proposed class of message strategies, we avoid the complicated problem that results when information is modeled as an indivisible good, as successfully studied by Allen [1], [2].

[13]See, e.g., Angeletos and Pavan [3], and Calvó-Armengol and de Martí [8].

agent i and let $\boldsymbol{X} := \times_{i \in N} X_i$ be the set of all information acquisition profiles. For a given network $g \in G$, each agent $i \in N$ is able to acquire information *only* from his neighbors. So, I shall set $x_{ij} = 0$ for $j \notin N_i(g) \cup \{i\}$ throughout the paper.

I turn now to describe how the posterior beliefs of the agents are formed. For two agents $i, j \in N$, $i \neq j$, let $\lambda_{ij} : T_j \times M \times [0,1] \to [0,1]$ denote the density corresponding to agent i's posterior beliefs over agent j's type, given the information acquisition parameter x_{ij}. Agents use Bayes' rule to update their priors.[14] Bayes' rule imposes

$$\lambda_{ij}(t_j | m_{ji}; x_{ij}) = \beta_{ji}(m_{ji} | t_j; x_{ij}) f(t_j) \Big/ \int_0^1 \beta_{ji}(m_{ji} | \tau; x_{ij}) f(\tau) d\tau. \tag{3}$$

Since types are independent, an agent can update his beliefs over states by doing separately the corresponding Bayesian belief revision over each of the other agents' types. Thus, agent i's posterior beliefs over $\boldsymbol{T}$ can be described by the function $\lambda_i : \boldsymbol{T} \times M^{n-1} \times X_i \to [0,1]$, defined as

$$\lambda_i(\boldsymbol{t} | m_i; x_i) := \prod_{j \neq i} \lambda_{ij}(t_j | m_{ji}; x_{ij}).$$

Let Q be the set of all densities on $\boldsymbol{T}$ so that $\lambda_i \in Q$ for each agent i.

2.4 Information Acquisition Equilibrium and Efficient Information Acquisition

Let us now introduce the notions of equilibrium and efficiency.

It is useful first to specify action strategies. An action strategy for agent i with respect to coordinate $k \neq i$ of the action space is a function $\alpha_{ik} : M \to [0,1]$ that associates his choice of action over coordinate k, $\alpha_{ik}(m_{ki}) \in [0,1]$, to the message that he receives from agent k, $m_{ki} \in M$. Since types are independent and all messages are sent simultaneously, an agent's choice of action over a particular coordinate depends *only* on the message that he receives from the expert in that coordinate, as specified. Likewise, an action strategy for agent i with respect to coordinate i is a function $\alpha_{ii} : T_i \to [0,1]$. Clearly, an agent's choice of action over the coordinate in which he is the expert depends only on his own initial private information. An action strategy for agent i is then a function $\alpha_i : T_i \times M^{n-1} \to A_i$ defined as $\alpha_i(t_i, m_i) := (\alpha_{ii}(t_i), (\alpha_{ik}(m_{ki}))_{k \neq i})$ for each $(t_i, m_i) \in T_i \times M^{n-1}$. Let Δ_i be the set of all action strategies for agent i.

The *expected payoff* of agent i in stage 2 is given by a function $V_{i,2} : A_i \times \Delta_{-i} \times Q \times T_i \times M^{n(n-1)} \to \mathbb{R}$ defined, given his own type t_i, a message profile $\boldsymbol{m} = (m_i, m_{-i})$, his

[14] As mentioned earlier, $x_{ij} = 0$ for $j \notin N_i(g) \cup \{i\}$, $i \in N$, so that each agent can indeed use *only* his neighbors' message strategies to update his beliefs.

own action $a_i = \alpha_i(t_i, m_i)$, a combination of action strategies followed by the other agents α_{-i}, and his own posterior beliefs about the true state λ_i, by[15]

$$V_{i,2}(a_i, \alpha_{-i}, \lambda_i; t_i, \boldsymbol{m}) := \int_{t_{-i}\in T_{-i}} \lambda_i(\boldsymbol{t}|m_i; x_i) U_i\big(\alpha_i(t_i, m_i), \alpha_{-i}(t_{-i}, m_{-i}), \boldsymbol{t}; r\big) dt_{-i}, \tag{4}$$

where $\alpha_{-i}(t_{-i}, m_{-i}) = (\alpha_j(t_j, m_j))_{j\neq i}$.

For $i \in N$ and $\lambda_i \in Q$, let the function $\widehat{\alpha}_i(\cdot; \lambda_i) : T_i \times M^{n-1} \to A_i$ defined by $\widehat{\alpha}_i(t_i, m_i; \lambda_i) := \arg\max_{a_i\in A_i} V_{i,2}(a_i, \alpha_{-i}, \lambda_i; t_i, m_i, m_{-i})$ for each $(t_i, m_i) \in T_i \times M^{n-1}$ be agent i's *optimal action strategy* given his posterior beliefs λ_i.[16] For the assumed preferences, the optimal action strategy of an agent i depends on the information that he acquires (which endows him with beliefs λ_i) but not on the action strategies followed by the rest of agents, α_{-i}. As discussed earlier, strategic interactions over actions are absent in this model.

The expected payoff of agent i in stage 1 is given by a function $V_{i,1} : \boldsymbol{\Delta} \times Q \to \mathbb{R}$ defined, for each given action strategy profile $\boldsymbol{\alpha}$ and posterior beliefs λ_i, by

$$\begin{aligned} V_{i,1}(\boldsymbol{\alpha}, \lambda_i) := \int_{\boldsymbol{t}\in \boldsymbol{T}} q(\boldsymbol{t}) \int_0^1 \cdots \int_0^1 \prod_{k\in N} \prod_{j\neq k} \beta_{kj}(m_{kj}|t_k; x_{jk}) \times \\ \times V_{i,2}(\alpha_i(t_i, m_i), \alpha_{-i}, \lambda_i; t_i, \boldsymbol{m}) dm_{kj} d\boldsymbol{t} - c \sum_{k\in N_i(g)} x_{ik}. \end{aligned} \tag{5}$$

Equation (5) above gives us agent i's objective function corresponding to the sequential rationality requirement in stage 1 for the equilibrium concept proposed in this paper, IAE. With this specification agent i's posterior beliefs are taken into account in his expected utility at stage 1 through each $V_{i,2}(\alpha_i(t_i, m_i), \alpha_{-i}, \lambda_i; t_i, \boldsymbol{m})$, for the various information sets $(t_i, \boldsymbol{m}) \in T_i \times M^{n(n-1)}$ at stage 2. The idea here is to recognize the role of acquired information in shaping agent i's perception of his own future payoffs, and to incorporate such role into his optimal decision in the stage where he decides about information acquisition. Using the specification in (5), we see that changes in agent i's information acquisition choice in stage 1 will change his own perception of his payoff in stage 2. Agent i is then able to anticipate and compute the extent to which such perception changes. The IAE concept departs from the perfect Bayesian equilibrium of the underlying game only

[15]Let $d\boldsymbol{t} := \prod_{k\in N} dt_k$ and $dt_{-i} := \prod_{k\neq i} dt_k$ for $i \in N$.

[16]An agent's optimal action strategy is nothing but an action strategy that satisfies an additional requirement (it maximizes the agent's expected utility in stage 2 given certain posterior beliefs). Therefore, as specified for each action strategy, for $i \in N$ and $\lambda_i \in Q$, let $\widehat{\alpha}_i(t_i, m_i; \lambda_i) := (\widehat{\alpha}_{ii}(t_i), (\widehat{\alpha}_{ik}(m_{ki}; \lambda_{ik}))_{k\neq i})$ for each $(t_i, m_i) \in T_i \times M^{n-1}$. Since types are independent and messages are sent simultaneously, agent i's optimal action strategy over the k-th coordinate of the action space depends only on the message m_{ki} that he receives from agent k, given his posterior beliefs λ_{ki}.

in that specification given in equation (5) of expected payoffs in stage 1.[17] In a perfect Bayesian equilibrium an agent considers in stage 1 only prior beliefs and cares about the messages and the actions chosen by everyone rather than about the effects induced on his own posterior beliefs. This makes it an equilibrium notion suitable for signaling and cheap talk games, where the relevant decisions are about information revelation. However, if we wish to recognize the role of newly acquired information in shaping posterior beliefs and induced perceptions of future payoffs (and to consider that agents are ex-ante aware of such effects), then the payoff specification in (5) seems more suitable to analyze information acquisition decisions.

In the definition of IAE below, condition (i) requires that each agent's type choose an expected payoff maximizing action in stage 2, taking as given the action strategies followed by the others and the information acquisition strategies chosen by everyone. Condition (ii) imposes each agent to choose optimally his information acquisition strategy in stage 1, which gives him his own posterior beliefs, taking as given the information acquisition strategies chosen by the rest of agents. Condition (iii) simply requires that each agent use Bayes' rule to update his priors over states.

Definition 1. Given a network $g \in G$, an *Information Acquisition equilibrium* (IAE) is a triple $(\boldsymbol{\alpha}^*, \boldsymbol{\lambda}^*, \boldsymbol{x}^*)$ such that, for each $i \in N$:

(i) $\alpha_i^* = \widehat{\alpha}_i(\cdot; \lambda_i^*)$. (SR2)

(ii) For each $\lambda_i \in Q$,

$$V_{i,1}(\boldsymbol{\alpha}^*, \lambda_i^*) \geq V_{i,1}(\widehat{\alpha}_i(\cdot; \lambda_i), \alpha_{-i}^*, \lambda_i). \quad \text{(SR1)}$$

(iii) For each $(t_j, m_{ji}) \in T_j \times M$ such that $j \neq i$,

$$\lambda_{ij}^*(t_j|m_{ji}; x_{ij}^*) = (1 - x_{ij}^*) f(t_j) + x_{ij}^* \mathbb{I}(m_{ji}|t_j) \text{ with } x_{ij}^* = 0 \text{ for } j \notin N_i(g) \cup \{i\}. \quad \text{(BU)}$$

For an agent $i \in N$, say that the information acquisition strategy $x_i \in X_i$ induces beliefs $\lambda_i \in Q$ if λ_i is obtained from x_i using condition (BU) in Definition 1 above. It formalizes the way in which information is transmitted between two agents connected

[17]For the two-agent version of the underlying game, the expression for agent i's expected utility in stage 1 in (5) can be simplified, upon substitution of the expression in (4), so as to obtain the equivalent expression

$$V_{i,1}(\boldsymbol{\alpha}, \lambda_i) = \int_{\boldsymbol{t} \in \boldsymbol{T}} q(\boldsymbol{t}) \int_0^1 \int_0^1 \beta_{ij}(m_{ij}|t_i; x_{ji}) \beta_{ji}(m_{ji}|t_j; x_{ij}) U_i(\alpha_i(t_i, \boldsymbol{m}), \alpha_j(t_j, \boldsymbol{m}), \boldsymbol{t}; r) dm_{ji} dm_{ij} d\boldsymbol{t} - c x_{ij}.$$

This is the expected payoff specification in stage 1 that one uses in the perfect Bayesian equilibrium concept for the underlying two-stage game. Therefore, IAE coincides with perfect Bayesian equilibrium for the two-agent version of the game. However, such a simplification cannot be obtained for the case with more than two agents.

through a link by assuming that posterior beliefs are consistent with Bayesian updating. To state Bayes' rule as expressed in condition (BU) above, one must combine equations (2) and (3). Say that the information acquisition profile $\boldsymbol{x} \in \boldsymbol{X}$ induces the belief profile $\boldsymbol{\lambda} \in Q^n$ if each x_i, $i \in N$, induces the corresponding λ_i. So, if $(\boldsymbol{\alpha}^*, \boldsymbol{\lambda}^*, \boldsymbol{x}^*)$ is an IAE, then $\boldsymbol{x}^*$ induces $\boldsymbol{\lambda}^*$.

One may ask whether the fact that agents communicate through a network may lead to the result that an agent's optimal information acquisition strategy, as described by conditions (SR2) and (SR1) in Definition 1 above, depends on other agents' information acquisition strategies. Proposition 2 gives an affirmative answer to that question so that this model enables us to analyze strategic interactions *only* over information acquisition decisions.

I now describe the efficiency benchmark that we shall use to gauge the efficiency properties of IAE. The welfare measure proposed in this paper is the sum of the expected payoff of all the agents in stage 1. Here we require that agents choose optimally their action strategies in stage 2 and then compare information acquisition profiles. Hence, we consider the possibility that the planner changes the information acquired by the agents, who will then pay the corresponding *new* cost of information acquisition and use such information optimally to choose their actions. As indicated earlier, the expected utility of an agent in stage 1 incorporates his own perception of his expected utility in stage 2 using the posterior beliefs resulting from his information acquisition decisions. In contrast, the ex-ante welfare function is evaluated from the planner's perspective. Therefore, the agents' posterior beliefs are not considered in the proposed welfare function in stage 1. Formally,

Definition 2. Given a network $g \in G$ and an information acquisition profile $\boldsymbol{x} \in \boldsymbol{X}$ that induces a belief profile $\boldsymbol{\lambda} \in Q^n$, the welfare function evaluated in stage 2 is the function $W_2 : \boldsymbol{T} \times M^{n(n-1)} \times Q^n \to \mathbb{R}$ defined by

$$W_2(\boldsymbol{t}, \boldsymbol{m}; \boldsymbol{\lambda}) := \sum_{i \in N} U_i\big(\widehat{\alpha}_i(t_i, m_i; \lambda_i), \widehat{\alpha}_{-i}(t_{-i}, m_{-i}; \lambda_{-i}), \boldsymbol{t}; r\big),$$

where $\widehat{\alpha}_{-i}(t_{-i}, m_{-i}; \lambda_{-i}) := (\widehat{\alpha}_j(t_j, m_j; \lambda_j))_{j \neq i}$.

Definition 3. Given a network $g \in G$, $\boldsymbol{x} \in \boldsymbol{X}$ is an efficient information acquisition profile if it induces a belief profile $\boldsymbol{\lambda}$ that maximizes the welfare function evaluated in stage 1, $W_1 : \boldsymbol{X} \to \mathbb{R}$, defined by

$$\begin{aligned} W_1(\boldsymbol{x}) := \int_{\boldsymbol{t}\in\boldsymbol{T}} q(\boldsymbol{t}) \int_0^1 \cdots \int_0^1 \prod_{k\in N} \prod_{j\neq k} \beta_{kj}(m_{kj}|t_k; x_{jk}) W_2(\boldsymbol{t}, \boldsymbol{m}; \boldsymbol{\lambda}) dm_{kj}\, d\boldsymbol{t} \\ - c \sum_{i\in N} \sum_{k\in N_i(g)} x_{ik}. \end{aligned} \tag{6}$$

2.5 A Two-Agent Example

As an antidote to the complexity of the ingredients of the previous subsections, I now work out an example for the particular case where $n = 2$ to illustrate the model. Consider $N = \{1, 2\}$ and the network $g = \{12\}$ so that each agent is able to acquire information from the other. Following the development of the previous subsections, the type space of the agents is $T_1 = T_2 = [0, 1]$, and the state space is $\boldsymbol{T} = [0, 1] \times [0, 1]$ with typical element $\boldsymbol{t} = (t_1, t_2)$, where $t_1 \in T_1$ and $t_2 \in T_2$. The action space of the agents is $A_1 = A_2 = \boldsymbol{T} = [0, 1] \times [0, 1]$ and an action for agent $i = 1, 2$ is $a_i = (a_{i1}, a_{i2})$. Thus, an action profile is $\boldsymbol{a} = (a_{11}, a_{12}, a_{21}, a_{22}) \in \boldsymbol{A} = A_1 \times A_2 = [0, 1]^4$. The payoff to agent $i = 1, 2$ is given by the expression

$$U_i(\boldsymbol{a}, \boldsymbol{t}; r) = -(1 - r)\big[(t_1 - a_{i1})^2 + (t_2 - a_{i2})^2\big] - r\big[(t_1 - a_{(3-i)1})^2 + (t_2 - a_{(3-i)2})^2\big].$$

As for the information transmission process, each agent $i = 1, 2$ has a set of messages $M = [0, 1]$ available to transmit information about his own type to the other agent. Using the notation introduced in subsection 2.3, $m_1 = m_{21}$ and $m_2 = m_{12}$ denote, respectively, the message received by agent 1 (from agent 2) and the message received by agent 2 (from agent 1). Also, $x_1 = x_{12}$ and $x_2 = x_{21}$ denote, respectively, the information acquisition strategy for agent 1 (to acquire information from agent 2) and the information acquisition strategy for agent 2 (to acquire information from agent 1).

In this example, a message strategy for agent $i = 1, 2$, given the information acquisition strategy x_{3-i} chosen by the other agent, is simply

$$\beta_i(m_{3-i}|t_i; x_{3-i}) = (1 - x_{3-i}) f(m_{3-i}) + x_{3-i}\mathbb{I}(m_{3-i}|t_i).$$

Accordingly, the induced beliefs for agent $3 - i$ $(i = 1, 2)$ over agent i's type are given by

$$\lambda_{3-i}(t_i|m_{3-i}; x_{3-i}) = (1 - x_{3-i}) f(t_i) + x_{3-i}\mathbb{I}(m_{3-i}|t_i).$$

The action choice of agent $i = 1, 2$, given his own type $t_i \in [0, 1]$ and the message $m_i \in [0, 1]$ that he receives from agent $3 - i$, is given by his action strategy α_i:

$$a_i = (a_{i1}, a_{i2}) = \alpha_i(t_i, m_i) = (\alpha_{ii}(t_i), \alpha_{i(3-i)}(m_i)).$$

I proceed by computing the optimal action strategy and the optimal information acquisition strategy for the agents. For the sake of clarity, I will write down the arguments only for a given agent, say agent $i = 1$, but will also derive the analogous implications for agent 2.

Consider a given information acquisition profile $\boldsymbol{x} = (x_1, x_2) \in [0,1] \times [0,1]$ that induces a belief profile $\boldsymbol{\lambda} = (\lambda_1, \lambda_2)$. From the expression above for the payoff to agent $i = 1$ together with (4) agent 1's expected payoff in stage 2, given type $t_1 \in [0,1]$ and message profile $\boldsymbol{m} = (m_1, m_2) \in [0,1] \times [0,1]$, specializes to

$$\begin{aligned} V_{1,2}(a_1, \alpha_2, \lambda_1; t_1, \boldsymbol{m}) &= \int_0^1 \big[(1-x_1)f(t_2) + x_1\mathbb{I}(m_1|t_2)\big] U_1\big(a_1, \alpha_2(t_2, m_2), t_1, t_2; r\big) dt_2 \\ &= -(1-r)\int_0^1 \big[(1-x_1)f(t_2) + x_1\mathbb{I}(m_1|t_2)\big]\big[(t_1 - a_{11})^2 + (t_2 - a_{12})^2\big] dt_2 \\ &\quad - r\int_0^1 \big[(1-x_1)f(t_2) + x_1\mathbb{I}(m_1|t_2)\big]\big[(t_1 - \alpha_{21}(m_2))^2 + (t_2 - \alpha_{22}(t_2))^2\big] dt_2, \end{aligned}$$

where $a_1 = (a_{11}, a_{12}) = \alpha_1(t_1, m_1) = (\alpha_{11}(t_1), \alpha_{12}(m_1))$. From the expression above it follows that $a^*_{11} = t_1$ and (using the expression analog to the one above for agent 2) $\alpha^*_{22}(t_2) = t_2$ correspond to the optimal action strategy of the agents. Using this, we can rewrite the expression above for agent 1's expected payoff in stage 2, when both agents's choose their optimal actions for the aspect of the problem in which they are the experts, as

$$\begin{aligned} V_{1,2}(a^*_1, \alpha^*_2, \lambda_1; t_1, \boldsymbol{m}) = &-(1-r)\left[(1-x_1)\int_0^1 (t_2 - a^*_{12})^2 f(t_2) dt_2 + x_1(m_1 - a^*_{12})^2\right] \\ &- r(t_1 - \alpha^*_{21}(m_2))^2. \end{aligned}$$

From the expression above (and from the analog one for agent 2), we obtain that $a^*_{12} = (1-x_1)\mu + x_1 m_1$ and $\alpha^*_{21}(m_2) = (1-x_2)\mu + x_2 m_2$ correspond to the optimal action strategy of the agents. Using this and doing the algebra, yields

$$\begin{aligned} V_{1,2}(a^*_1, \alpha^*_2, \lambda_1; t_1, \boldsymbol{m}) = &-(1-r)(1-x_1)\big[\sigma^2 + x_1(m_1 - \mu)^2\big] \\ &- r\big(t_1 - (1-x_2)\mu - x_2 m_2\big)^2. \end{aligned}$$

Now, using (5) together with the expressions obtained above for both agents' message strategies, the expression for agent 1's expected payoff in stage 1, when both agents's choose their optimal action strategies, specializes to

$$\begin{aligned} V_{1,1}(\boldsymbol{\alpha}^*, \lambda_1) = &\int_0^1 \int_0^1 f(t_1)f(t_2) \int_0^1 \int_0^1 \big[(1-x_2)f(m_2) + x_2\mathbb{I}(m_2|t_1)\big] \times \\ &\times \big[(1-x_1)f(m_1) + x_1\mathbb{I}(m_1|t_2)\big] V_{1,2}(\alpha^*_1(t_1, m_1), \alpha^*_2, \lambda_1; t_1, \boldsymbol{m}) dm_2 dm_1 dt_2 dt_1 - cx_1. \end{aligned}$$

By substituting the expression of agent 1's expected payoff in stage 2, $V_{1,2}(\alpha_1^*(t_1,m_1),\alpha_2^*,\lambda_1;t_1,\boldsymbol{m})$, obtained earlier, into the expression above and by doing the algebra, we finally obtain

$$V_{1,1}(\boldsymbol{\alpha}^*,\lambda_1)=-(1-r)(1-x_1^2)\sigma^2-r(1-x_2^2)\sigma^2-cx_1.$$

Therefore, the optimal information acquisition strategy of agent 1 is given by (i) $x_1^*=0\Leftrightarrow c\geq(1-r)\sigma^2$, (ii) $x_1^*=1\Leftrightarrow c\leq(1-r)\sigma^2$, and (iii) $x_1^*\in\{0,1\}\Leftrightarrow c=(1-r)\sigma^2$, regardless of the information acquisition strategy chosen by agent 2. Of course, for agent 2 one obtains an analogous optimal information acquisition strategy.

I turn now to study efficient information acquisition in this example. Addition of the payoffs of the two agents, when both of them choose their optimal action strategies, together with the expressions above for such optimal strategies, gives us the following expression for the welfare function evaluated in stage 2:

$$\begin{aligned}W_2(t_1,t_2,m_1,m_2;\lambda_1,\lambda_2)&=\big(t_2-\alpha_{12}^*(m_1)\big)^2+\big(t_1-\alpha_{21}^*(m_2)\big)^2\\&=\big(t_2-(1-x_1)\mu-x_1m_1\big)^2+\big(t_1-(1-x_2)\mu-x_2m_2\big)^2.\end{aligned}$$

Using the expression in equation (6), the welfare function evaluated in stage 1 specializes to

$$\begin{aligned}W_1(\boldsymbol{x})=&\int_0^1\int_0^1 f(t_1)f(t_2)\int_0^1\int_0^1\big[(1-x_2)f(m_2)+x_2\mathbb{I}(m_2|t_1)\big]\times\\&\times\big[(1-x_1)f(m_1)+x_1\mathbb{I}(m_1|t_2)\big]W_2(t_1,t_2,m_1,m_2;\lambda_1,\lambda_2)dm_2dm_1dt_2dt_2-c[x_1+x_2].\end{aligned}$$

Then, by substituting the expression for the welfare function evaluated in stage 2 obtained earlier into the expression above and by doing the algebra, it can be checked that the expression for the welfare function evaluated in stage 1 in equation (6) becomes

$$W_1(\boldsymbol{x})=-2\sigma^2+x_1[x_1\sigma^2-c]+x_2[x_2\sigma^2-c].$$

Therefore, the efficient information acquisition profile (x_1,x_2) must satisfy, for each $i=1,2$, (i) $x_i=0\Leftrightarrow c\geq\sigma^2$, (ii) $x_i=1\Leftrightarrow c\leq\sigma^2$, and (iii) $x_i\in\{0,1\}\Leftrightarrow c=\sigma^2$.

In this example we observe that the (possible) discrepancy between the efficient and the equilibrium information acquisition profiles is due to the team concern. This example illustrates the main ingredients of the model but it does not allow us to obtain insights for the case where the agents are indeed connected through a network. In particular, under the requirement imposed by the IAE notion that agents correctly anticipate the role of information in shaping their posterior beliefs (and incorporate it in their information acquisition decisions), interesting strategic interactions over information acquisition decisions arise when more than two agents are connected through a network. Thus, the fact that the agents acquire information through a network plays an essential role in this model. The rest of the paper is devoted to that analysis.

3 Efficiency and Equilibrium

This section characterizes both the set of efficient information acquisition profiles and the set of IAE, and relates the compatibility between them to the network density.

I start by studying the optimal action strategies followed by the agents. For $i, k \in N$, $i \neq k$, let

$$\mathbb{E}[t_k|m_{ki}; x_{ik}] := \int_0^1 \lambda_{ik}(t_k|m_{ki}; x_{ik}) t_k dt_k$$

and

$$\text{Var}[t_k|m_{ki}; x_{ik}] := \int_0^1 \lambda_{ik}(t_k|m_{ki}; x_{ik}) \big(t_k - \mathbb{E}[t_k|m_{ki}; x_{ik}]\big)^2 dt_k$$

denote, respectively, the expected value and the variance of type t_k for the received message m_{ki}, given the information acquisition parameter x_{ik}. Thus, by applying the belief revision rule specified in (BU) to agent i, with respect to agent k's type, one obtains

$$\mathbb{E}[t_k|m_{ki}; x_{ik}] = (1 - x_{ik})\mu + x_{ik} m_{ki}, \tag{7}$$

and

$$\text{Var}[t_k|m_{ki}; x_{ik}] = (1 - x_{ik})\big[\sigma^2 + x_{ik}(m_{ki} - \mu)^2\big]. \tag{8}$$

Since the expected payoff of each agent in stage 2 is concave with respect to his own action, agent i's optimal action strategy $\widehat{\alpha}_i(\cdot; \lambda_i)$, $\lambda_i \in Q$, is given by the first order conditions

$$\widehat{\alpha}_{ii}(t_i) = t_i \quad \text{and} \quad \widehat{\alpha}_{ik}(m_{ki}; \lambda_{ik}) = \mathbb{E}[t_k|m_{ki}; x_{ik}] \text{ for each } k \neq i, \tag{9}$$

where x_i induces λ_i. Thus, each agent chooses optimally his expectation of the underlying state $\boldsymbol{t}$ according to the posteriors that he obtains with the information acquired from his neighbors.

I turn now to characterize the efficient information acquisition profiles.

3.1 Efficient Information Acquisition

Using the payoff specification given by equation (1) and the specification of the welfare function evaluated in stage 2 in Definition 2, one obtains

$$W_2(\boldsymbol{t}, \boldsymbol{m}; \boldsymbol{\lambda}) = -\sum_{i \in N} \sum_{k \neq i} \big(t_k - \widehat{\alpha}_{ik}(m_{ki}; \lambda_{ik})\big)^2.$$

Hence, a social planner who faces the problem of maximizing the welfare function evaluated in stage 2 seeks to keep the action of each agent close to the underlying state and ignores the team concern of each agent. This is due to the fact that agents are ex-ante identical so that the influence of each agent's action on any other agent's payoff is homogenous across agents. Therefore, the efficient information acquisition profile is characterized by the condition that ensures the optimal behavior of each agent with respect to information acquisition in the limit case where the team concern is absent, i.e., when $r = 0$, as provided by Proposition 1 below.

Proposition 1. *Let $g \in G$ and let $\boldsymbol{x}$ be an efficient information acquisition profile. Then, for each agent $i \in N$ and each neighbor $k \in N_i(g)$, either*
(i) $x_{ik} = 0$ if and only if $c \geq \sigma^2$,
(ii) $x_{ik} = 1$ if and only if $c \leq \sigma^2$, or
(iii) $x_{ik} \in \{0, 1\}$ if and only if $c = \sigma^2$.

Consider an efficient information acquisition profile $\boldsymbol{x} \in X$. From the assumed homogeneity with respect to the variance of the agents' types, together with the fact that the information acquisition cost is identical for all agents, it follows that[18]

$$\boldsymbol{x} = \underline{0} \Leftrightarrow c \geq \sigma^2 \tag{10}$$

and

$$\boldsymbol{x} = \underline{1} \Leftrightarrow c \leq \sigma^2. \tag{11}$$

That is, for $c \neq \sigma^2$, in an efficient information acquisition profile either all the agents acquire full information from their neighbors or acquire no information at all.

Proposition 2 in Subsection 3.3 characterizes the best response information acquisition strategy of an agent—as a function of the information acquisition strategies taken by the rest of agents. It shows that, for a given network $g \in G$, the incentives of each agent $i \in N$ to acquire full information in an IAE from a neighbor $k \in N_i(g)$ increase with the amount of information that the rest of neighbors of agent k acquire from him. Thus, under the sequential rationality requirement in stage 1 imposed by the IAE concept, positive informational spillovers over actions induce a certain degree of coordination (in the same direction) over the information acquisition strategies followed by the agents at equilibrium. Before stating the formal result, I provide an example in the next subsection, for a network involving three agents, that illustrates the forces behind that coordination effect.[19]

[18]The notation $\underline{0}$ and $\underline{1}$ denotes, respectively, the vector $(0, 0, \cdots, 0)$ and the vector $(1, 1, \cdots, 1)$ in a space of conformal dimension.

[19]I am grateful to Dragan Filipovich for suggesting me to provide an example along these lines.

3.2 A Three-Agent Example

Consider $N = \{1, 2, 3\}$ and the network $g = \{12, 23\}$. In this example we ask ourselves: why should, in an IAE, the amount of information that agent 1 acquires from 2 depend on the amount of information that 3 acquires from 2?

To address this question, it suffices to account for that part of agent 1's expected payoff due to the team concern. Using the payoff specification in (1), we see that agent 1 cares about $-\|\boldsymbol{t} - a_3\|^2$. In particular, he wishes that the difference $(t_2 - a_{32})^2$ be minimized, i.e., he cares about the extent to which agent 3 solves aspect 2 (for which agent 2 is the expert) of his problem. Notice that agent 1 is risk averse with respect to agent 3's choice over the second coordinate of the state of world. From (7), we know that, given a message m_{23} received from agent 2, agent 3's optimal action choice over the second aspect of the problem is given by his expectation of that coordinate according to the induced posteriors λ_{32} (that he obtains from his information acquisition decision x_{32} regarding agent 2). That is,

$$\widehat{\alpha}_{32}(m_{23}; \lambda_{32}) = \mathbb{E}[t_2 | m_{23}; x_{32}] = (1 - x_{32})\mu + x_{32} m_{23}.$$

First, suppose that agent 3 acquires no information at all from agent 2. Then, for each message $m_{23} \in [0, 1]$ received by agent 3 from agent 2, agent 1 knows that agent 3 optimally chooses $\widehat{\alpha}_{32}(m_{23}; \lambda_{32}) = \mu$, so that agent 1 cares about $-(t_2 - \mu)^2$. Thus, the only source of uncertainty affecting agent 1 is with respect to t_2.

If agent 1 decides to acquire no information at all from agent 2, then at stage 1 he knows that at stage 2 he would compute $-(t_2 - \mu)^2$ according to his priors $f(t_2)$, obtaining expected payoff $-\sigma^2$ in stage 2. Consequently, he knows that at stage 1 he would compute $-\sigma^2$ again according to his priors $f(t_2)$. Thus the component of his expected payoff in stage 1 due to concern $-(t_2 - a_{32})^2$ and to his information acquisition decision amounts to $-\sigma^2$.

If, on the other hand, agent 1 decides to acquire full information from agent 2, then at stage 1 he knows that at stage 2 he would know the true value of $-(t_2 - \mu)^2$. However, at stage 1 he does not know the way in which μ relates to t_2. In particular, he does not know whether t_2 coincides with μ or not, and, consequently, he continues to compute $-(t_2 - \mu)^2$ according to his priors $f(t_2)$. Then, the component of his expected payoff in stage 1 due to concern $-(t_2 - a_{32})^2$ and to his information acquisition decision amounts to $-\sigma^2 - c$. So long as $c > 0$ agent 1 prefers, regarding component $-(t_2 - a_{32})^2$ of his payoff (and other things being equal), to acquire no information from agent 2 when agent 3 acquires no information from agent 2.

Second, suppose that agent 3 acquires full information from agent 2. Then, for a given message $m_{23} \in [0,1]$ received by agent 3 from agent 2, agent 1 knows that agent 3 optimally chooses $\widehat{\alpha}_{32}(m_{23};\lambda_{32}) = m_{23}$, so that agent 1 cares now about $-(t_2 - m_{23})^2$. So, there are now two sources of uncertainty affecting agent 1 (over this component of his payoffs), one due to t_2, the other corresponding to m_{23}. Agent 1 can use the information that he acquires from agent 2 to improve his knowledge about the way in which m_{23} relates to t_2. Notice that, even though agent 1 knows the value of x_{32}, his information about the particular message m_{23} depends on the amount of information about t_2 that he acquires.

If agent 1 decides to acquire no information at all from agent 2, then at stage 1 he knows that at stage 2 he would compute $-(t_2 - m_{23})^2$ using his priors $f(m_{23})$. Consequently, he knows that at stage 1 he would compute $-\int_0^1 (t_2 - m_{23})^2 f(m_{23}) dm_{23}$ using his priors $f(t_2)$. Thus, the component of his expected payoff in stage 1 due to concern $-(t_2 - a_{32})^2$ and to his information acquisition decision amounts to $-2\sigma^2$ in stage 1.

If, on the other hand, agent 1 decides to acquire full information from agent 2, then at stage 1 he knows that at stage 2 he would compute $-(t_2 - m_{23})^2$ knowing exactly the way in which m_{23} relates to t_2 for agent 2's message strategy with respect to agent 3. Therefore, agent 1 knows at stage 1 that at stage 2 he would know (i) that m_{23} coincides with t_2 and (ii) the exact value of m_{23}. Consequently, agent 1 knows that at stage 2 he would compute a zero expected payoff. Then, at stage 1 he would compute a zero payoff according to his priors $f(t_2)$, so that the component of his expected payoff in stage 1 due to concern $-(t_2 - a_{32})^2$ and to his information acquisition decision amounts to $-c$.

We see that agent 1 is more inclined to acquire full information from agent 2 when agent 3 acquires full information from agent 2 than in the case where agent 3 acquires no information from agent 2.

In this example, agent 3's information acquisition decisions from agent 2 affect the relation between agent 1's information acquisition decisions from agent 2 and his own (anticipated) posterior perception of the expected value of $-(t_2 - a_{32})^2$. Note that the IAE concept imposes that the posterior beliefs over t_2 of agent 1 and of agent 3 enter agent 1's expected payoff in stage 1 in a multiplicative way. This implies that the information a acquisition parameters x_{12} and x_{32} enter also agent 1's expected payoff in stage 1 in a multiplicative way, which leads to the result that the information acquisition decisions of agents 1 and 3 are interdependent at equilibrium when information acquisition is costly.

3.3 Information Acquisition Equilibrium

The next proposition characterizes the best response information acquisition strategies of the agents.

Proposition 2. *Let $g \in G$ and let $(\boldsymbol{\alpha}^*, \boldsymbol{\lambda}^*, \boldsymbol{x}^*)$ be an IAE. Then, for each agent $i \in N$ and each neighbor $k \in N_i(g)$, either*
(i) $x^*_{ik} = 0$ *if and only if* $c \geq \sigma^2\left[(1-r) + 2r\frac{1}{n-1}\sum_{j\in N_k(g)\setminus\{i\}}(x^*_{jk})^2\right]$,
(ii) $x^*_{ik} = 1$ *if and only if* $c \leq \sigma^2\left[(1-r) + 2r\frac{1}{n-1}\sum_{j\in N_k(g)\setminus\{i\}}(x^*_{jk})^2\right]$, *or*
(iii) $x^*_{ik} \in \{0,1\}$ *if and only if* $c = \sigma^2\left[(1-r) + 2r\frac{1}{n-1}\sum_{j\in N_k(g)\setminus\{i\}}(x^*_{jk})^2\right]$.

Since we are considering an externality, with the form of the team concern, one might expect that the conditions that characterize the set of equilibria (provided by Proposition 2 above) do not coincide with those characterizing the set of efficient information acquisition profiles (provided by Proposition 1). The forces behind this discrepancy for this set-up are, however, more subtle than those involved in traditional inefficiency results in the presence of externalities. The fact that the agents' expected payoffs in stage 1 incorporate their expected payoffs in stage 2 using their own posterior beliefs is crucial to explain the differences between efficient and equilibrium information acquisition.

In this model, the information that an agent $i \in N$ acquires from a neighbor $k \in N_i(g)$ shapes his own beliefs about the underlying state as well as about the extent to which the optimal action of any other neighbor of agent k, $j \in N_k(g)\setminus\{i\}$ approaches the true state. In other words, by changing his information acquisition decision, agent i changes the way in which he anticipates his perception of the extent to which agent $j \in N_k(g)\setminus\{i\}$ solves aspect k of his own problem. This is mathematically expressed by the fact that, under the sequential rationality condition in stage 1 imposed by the IAE concept, the posterior beliefs of agent i and of agent j about t_k enter agent i's expected utility at stage 1 in a multiplicative way. Finally, note that, to obtain the result stated in Proposition 2, is crucial that (i) information acquisition be costly, that (ii) agent i cares about agent j's action choice over coordinate k of the action space, and that (iii) agent i be risk averse with respect to the difference $t_k - a_{jk}$.

An obvious consequence of Proposition 2 is that, in equilibrium, it is less likely that a hub in a network acquires full information from the agents in his periphery than each of the agents in the periphery acquire full information from that particular hub. Also, the likelihood with which an agent in the periphery of a hub acquires full information from that hub increases with the number of agents in the periphery of the hub. To see this, consider the star network $g = \{12, 13, \ldots, 1n\}$. It follows from Proposition 2 (ii) that there exists an IAE where each agent in the periphery of agent 1 acquires full information from that hub if $0 \leq c \leq \sigma^2\left[(1-r) + 2r\frac{n-2}{n-1}\right]$. If we consider a large group so that $n \to \infty$, then there exists an IAE where each agent in the periphery of agent 1 acquires full information from that hub if $0 \leq c \leq \sigma^2(1+r)$. In contrast, in equilibrium, agent 1 acquires full information from an agent in the periphery only if $0 \leq c \leq \sigma^2(1-r)$.

Another consequence of Proposition 2 is that the incentives of the agents to acquire full information from their neighbors in a network increase with the minimum and maximum degrees of that network. To see this, consider the complete circle network $g = \{12, 23, \ldots, (n-1)n\}$ so that $\delta(g) = \rho(g) = 2$. From Proposition 2 (ii), it follows that there is an IAE where each agent acquires full information from his neighbors if and only if $0 \leq c \leq \sigma^2\Big[(1-r) + 2r\frac{1}{n-1}\Big]$. If we consider a large group so that $n \to \infty$, that condition becomes $0 \leq c \leq \sigma^2(1-r)$. This gives us an upper bound on the cost lower than that ensuring that each agent in the periphery of the hub acquires full information from that hub in a star network, as shown above.

3.4 Equilibrium, Efficiency, and the Network Density

Despite the linearity assumptions used in this paper, the best response information acquisition strategies characterized by Proposition 2 still allow for the existence of multiple IAE for a broad class of networks. To compare efficient and equilibrium information acquisition profiles, it is useful to use Proposition 2 to characterize IAE where either all the agents acquire full information from their neighbors or acquire no information at all.

Let us define $\kappa(\mathrm{a}(g), n) := \max\{2[\mathrm{a}(g)-1]/(n-1), 0\}$ where $\mathrm{a}(g) \in \{\delta(g), \rho(g)\}$. That is, $\kappa(\mathrm{a}(g), n)$ is a function strictly increasing with $\mathrm{a}(g)$, $\mathrm{a}(g) \in \{\delta(g), \rho(g)\}$, i.e., with the minimum and the maximum degrees of network g, and strictly decreasing with the number of agents in N. Thus, $\kappa(\mathrm{a}(g), n)$, $\mathrm{a}(g) \in \{\delta(g), \rho(g)\}$, can be understood as measures of the degree of density of network g relative to the size of the organization/group.

Consider an information acquisition profile $\boldsymbol{x}^* \in \boldsymbol{X}$ corresponding to an IAE. It follows from Proposition 2 (i) that

$$\boldsymbol{x}^* = \underline{0} \Leftrightarrow c \geq \sigma^2(1-r). \tag{12}$$

On the other hand, Proposition 2 (ii) implies that

$$\boldsymbol{x}^* = \underline{1} \Leftrightarrow c \leq \sigma^2[(1-r) + r\kappa(\delta(g), n)]. \tag{13}$$

Notice that an IAE equilibrium for each of the regions of the information acquisition cost delimited by equations (12) and (13) above is not necessarily unique. It needs not be so even when one restricts attention to IAE where either each agent acquires full information or each agent acquires no information at all. To see this, notice that $\sigma^2(1-r) \leq \sigma^2[(1-r) + r\kappa(\delta(g), n)]$ for each $g \in G$ and each $n \geq 3$ since $\kappa(\delta(g), n) \geq 0$ for each $g \in G$ and each $n \geq 3$. Therefore, for $\sigma^2(1-r) \leq c \leq \sigma^2[(1-r) + r\kappa(\delta(g), n)]$, both $\boldsymbol{x}^* = \underline{0}$ and $\boldsymbol{x}^{**} = \underline{1}$ correspond to IAE. It can be easily checked that this is the only case where multiplicity of equilibria arises when one restricts attention to IAE where either all the agents acquire full information from their neighbors or acquire no information at all.

Corollary 1. *Let $g \in G$ be a network such that $\delta(g) \geq \frac{n+1}{2}$. Then, for each $r \in [0,1]$ and each $c \in \mathbb{R}_+$, for each efficient information acquisition profile $\boldsymbol{x} \in \boldsymbol{X}$ there exists a belief profile $\boldsymbol{\lambda} \in Q^n$ induced by $\boldsymbol{x}$ such that $(\boldsymbol{\alpha}, \boldsymbol{\lambda}, \boldsymbol{x})$ is an IAE.*

Proof. First, suppose that $0 \leq c \leq \sigma^2$. Then, using (11), we know that $\boldsymbol{x} = \underline{1}$ is the efficient information acquisition profile. Since $\delta(g) \geq \frac{n+1}{2} \Leftrightarrow \kappa(\delta(g), n) \geq 1$ for each $n \geq 3$ and each $r \in [0,1]$, we know that $0 \leq c \leq \sigma^2$ implies necessarily $0 \leq c \leq \sigma^2[(1-r) + r\kappa(\delta(g), n)]$ for each $n \geq 3$. But then, using (13), one obtains that $\boldsymbol{x} = \underline{1}$ corresponds to an IAE.

Second, suppose that $c \geq \sigma^2$. Then, using (10), we know that $\boldsymbol{x} = \underline{0}$ is the efficient information acquisition profile. But then $c \geq \sigma^2(1-r)$ for each $r \in [0,1]$ so that, using (12), we obtain that $\boldsymbol{x} = \underline{0}$ corresponds to an IAE. □

The result in Corollary 1 allows us to relate the network density to the compatibility between equilibrium and efficient information acquisition. In particular, if the minimum degree of the network is high enough relative to the size of the organization/group, then each efficient information acquisition profile can be reached in an IAE.

However, one must consider the comparison between equilibrium and efficient information acquisition obtained from Corollary 1 with due care since, as mentioned earlier, there are multiple IAE for some regions of the cost. In particular, for $r \in (0,1]$, using (12), one obtains that, regardless of the network architecture, if $\sigma^2(1-r) < c < \sigma^2$, then there exists an IAE where all the agents acquire no information at all from his neighbors. However, it follows from (11) that the efficient information profile for cost in that interval requires that all the agents acquire full information, regardless of the network architecture.

Corollary 2. *Let $g \in G$ be a network such that $\delta(g) < \frac{n+1}{2}$. Then, for each $r \in (0,1]$ and each $\sigma^2[(1-r) + r\kappa(\delta(g), n)] < c < \sigma^2$, each agent acquires full information from each of his neighbors in the efficient information acquisition profile whereas at least some agent acquires no information at all in the information acquisition profile corresponding to each IAE.*

Proof. Since $\delta(g) < \frac{n+1}{2} \Leftrightarrow \kappa(\delta(g), n) < 1$ for each $n \geq 3$ and each $r \in (0,1]$, we know that $\sigma^2[(1-r) + r\kappa(\delta(g), n)] < \sigma^2$. So, suppose that $\sigma^2[(1-r) + r\kappa(\delta(g), n)] < c < \sigma^2$. Then, from (11), we know that $\boldsymbol{x} = \underline{1}$ is the efficient information acquisition profile. However, since $c > \sigma^2[(1-r) + r\kappa(\delta(g), n)]$, the result in Proposition (i) implies that at least some agent acquires no information at all in each IAE. $\square$

Obviously, the existence of multiple IAE does not impose any qualification to the message conveyed by Corollary 2 since it identifies a region of the information acquisition cost where *all* IAE are inefficient.

The intuition behind the results in Corollary 1 and Corollary 2 is as follows. By comparing the result in Proposition 1 with that in Proposition 2, we observe that the (possible) discrepancy between efficient and equilibrium information acquisition is driven by the coordination effect (in the same direction) identified in Proposition 2. For information acquisition cost relatively high, $c > \sigma^2$, this coordination effect has no influence on the agents' decisions for one of the possible equilibria (the one with no information acquisition). So, one obtains that an equilibrium information acquisition profile coincides with the efficient one. However, for lower values of the information acquisition cost, $c < \sigma^2$, such a coordination effect does influence the agents' decisions at each equilibrium. If $c < \sigma^2$, full information acquisition of each agent from each of his neighbors corresponds to the efficient profile. Then, given the coordination effect, an agent $i \in N$ will choose, at equilibrium, to acquire full information from a neighbor $k \in N_i(g)$ if the number of other neighbors of agent k that acquire full information from him is relatively high. For this to happen, agent k needs to be "minimally connected" in network g. Since we are looking at equilibria where all the agents either acquire full information or acquire no information at all, the fact that each agent $k \in N$ be minimally connected is a sufficient condition to guarantee efficiency of the equilibrium profile. The minimal connectivity condition that we obtain, $\delta(g) \geq \frac{n+1}{2}$, requires that the minimum degree of network g be larger than half of the size of the group.

Corollary 3. *Consider a network $g \in G$ and suppose that $r \in [0, 1]$. Then:*
(i) For each $0 < c < \sigma^2(1-r)$, the efficient information acquisition profile coincides with the information acquisition profile corresponding to the unique IAE. In this IAE, each agent acquires full information from each of his neighbors.
(ii) If $\rho(g) \geq \frac{n+1}{2}$, then, for each $c > \sigma^2[(1-r) + r\kappa(\rho(g), n)]$ the efficient information acquisition profile coincides with the information acquisition profile corresponding to the unique IAE. In this IAE, each agent acquires no information at all from each of his neighbors. Moreover, if $\rho(g) < \frac{n+1}{2}$, then the same conclusion holds for each $c > \sigma^2$.

Proof. (i) Clearly, $0 < c < \sigma^2(1-r)$ implies $0 < c < \sigma^2$ for each $r \in [0, 1]$. So, using (11), we know that, for each $0 < c < \sigma^2(1-r)$, the efficient information acquisition profile is

$\boldsymbol{x} = \underline{1}$. Also, since, for each $r \in [0,1]$ and each $i \in N$,

$$\sigma^2(1-r) \leq$$
$$\leq \inf \left\{ \sigma^2 \left[(1-r) + \frac{2r}{n-1} \sum_{j \in N_k(g) \setminus \{i\}} (x^*_{jk})^2 \right] \; : \; x^*_{jk} \in [0,1] \, , \, j \in N_k(g) \setminus \{i\} \, , \, k \in N_i(g) \right\},$$

Proposition 2 (ii) implies that $\boldsymbol{x}^* = \underline{1}$ is the information acquisition profile corresponding to the unique IAE when $c < \sigma^2(1-r)$.

(ii) First suppose that $\rho(g) \geq \frac{n+1}{2}$. Then, $\kappa(\rho(g), n) \geq 1$ and, therefore, $\sigma^2[(1-r) + r\kappa(\rho(g), n)] \geq \sigma^2$ for each $r \in [0,1]$. So, using (10), we know that, for each $c > \sigma^2[(1-r) + r\kappa(\rho(g), n)]$, the efficient information acquisition profile is $\boldsymbol{x} = \underline{0}$. Also, since for each $r \in [0,1]$ and each $i \in N$,

$$\sigma^2[(1-r) + r\kappa(\rho(g), n)] \geq$$
$$\geq \sup \left\{ \sigma^2 \left[(1-r) + \frac{2r}{n-1} \sum_{j \in N_k(g) \setminus \{i\}} (x^*_{jk})^2 \right] \; : \; x^*_{jk} \in [0,1] \, , \, j \in N_k(g) \setminus \{i\} \, , \, k \in N_i(g) \right\},$$

Proposition 2 (i) implies that $\boldsymbol{x}^* = \underline{0}$ is the information acquisition profile corresponding to the unique IAE when $c > \sigma^2[(1-r) + r\kappa(\rho(g), n)]$.

Finally, suppose that $\rho(g) < \frac{n+1}{2}$. Then, $\kappa(\rho(g), n) < 1$ and, therefore, $\sigma^2[(1-r) + r\kappa(\rho(g), n)] < \sigma^2$ for each $r \in [0,1]$. So, since $c > \sigma^2$ implies $c > \sigma^2[(1-r) + r\kappa(\rho(g), n)]$, we can use again the arguments above to obtain that, for $c > \sigma^2$, the efficient information acquisition profile is $\boldsymbol{x} = \underline{0}$ and $\boldsymbol{x}^* = \underline{0}$ is the information acquisition profile corresponding to the unique IAE. □

Corollary 3 gives us sufficient conditions for each efficient information acquisition profile to coincide with that at the *unique* equilibrium. In particular, the sufficient condition provided by Corollary 3 (ii) depends on whether the maximum degree of the network exceeds $\frac{n+1}{2}$ or not. It can be easily checked that the lower bound identified in Corollary 3 (ii) increases with the maximum degree of the network on the interval $[\sigma^2, \sigma^2(1+r)]$. Thus, Corollary 3 (ii) may seem to convey the message that, for values of the information acquisition cost high enough, $c \in [\sigma^2, \sigma^2(1+r)]$, the compatibility between efficient and equilibrium information acquisition is favored when the maximum degree of the network is relatively low. However, Corollary 3 (ii) gives us just a sufficient condition on the existence of a unique efficient IAE profile. As shown by Corollary 2, even for cost in the interval $[\sigma^2, \sigma^2(1+r)]$, there exists an efficient IAE profile when the minimum degree of the network is relatively high ($\delta(g) \geq \frac{n+1}{2}$).

The intuition behind the result in Corollary 3 (ii) is as follows. If $c \in [\sigma^2, \sigma^2(1+r)]$, agents acquire no information at all in the efficient information acquisition profile. Then, given the coordination effect identified in Proposition 2, an agent $i \in N$ will choose, at equilibrium, to acquire no information from a neighbor $k \in N_i(g)$ if the number of other neighbors of agent k that acquire no information from him is relatively high. A sufficient condition for this to happen is, clearly, that the number of neighbors of agent k be relatively low.

Furthermore, Corollary 3 implies that, for either sufficiently low or sufficiently high values of the information acquisition cost, the efficient information acquisition behavior coincides with that at equilibrium, regardless of the network architecture. This result is provided formally by Corollary 4 below.

Corollary 4. *Consider a network $g \in G$ and suppose that $r \in [0,1]$. If either $0 \leq c \leq \sigma^2(1-r)$ or $c \geq \sigma^2(1+r)$, then each efficient information acquisition profile coincides with the information acquisition profile corresponding to the unique IAE.*

The result in Corollary 4 is a straightforward consequence of Corollary 3 combined with the fact that, from the definition of $\kappa(\mathrm{a}(g), n)$, $\mathrm{a}(g) \in \{\delta(g), \rho(g)\}$, we have

$$0 \leq \kappa(\delta(g), n) \leq \kappa(\rho(g), n) \leq 2 \cdot \frac{n-2}{n-1} < 2 \ \text{ for each } \ g \in G \ \text{ and each } \ n \geq 3.$$

Therefore,

$$\sigma^2(1-r) \leq \sigma^2[(1-r) + r\kappa(\delta(g), n)] \leq \sigma^2[(1-r) + r\kappa(\rho(g), n)] < \sigma^2(1+r)$$

is satisfied for each $g \in G$ and each $n \geq 3$.

The intuition behind the result in Corollary 4 is simply that, for either very low or very high values of the cost, the coordination effect identified in Proposition 2 becomes of no importance for the agents' incentives to acquire information. This leads to the result that the IAE profile coincides with the efficient one.

4 Robustness and Justifications of the Model

This paper has studied both equilibrium and efficiency properties of the information acquisition phenomenon through networks by using a specific description of the information transmission process and by making specific assumptions on payoffs. In this section, I discuss the robustness of the model by analyzing the implications of changing some of the assumptions.

The first subsection below discusses the implications of assuming an alternative information transmission process where the agents receive a Normal signal consisting of the true type plus some noise. The second subsection studies whether the main implications of the model continue to hold under a payoff perturbation that introduces a convex cost function.

4.1 Normal Signals

Following the approach pursued by recent works on the social value of information and on communication networks,[20] let us consider an alternative description of the information transmission process. Assume that each type t_i is drawn according to a Normal distribution with mean μ and variance σ^2. As a consequence of his information acquisition decisions, each agent $i \in N$ receives from each of his neighbors $k \in N_i(g)$ a private signal $y_{ki} := t_k + \epsilon_{ki}$ where ϵ_{ki} is an idiosyncratic noise normaly distributed with mean zero and variance ς^2_{ki}. Furthermore, for each $i \in N$, assume that: (i) t_k and ϵ_{ki} are independent for each $k \neq i$, and (ii) ϵ_{ki} and ϵ_{ji} are independent for each $k, j \in N$ such that $k \neq i$, $j \neq i$, and $k \neq j$. The rest of the game is identical to the one described in Section 2.

Then, it can be checked that agent i's posterior beliefs about type t_k, $k \neq i$, conditional on receiving signal y_{ki}, are given by a Normal distribution with mean $\mathbb{E}[t_k|y_{ki}] = \gamma y_{ki} + (1-\gamma)\mu$ and variance $\text{Var}[t_k|y_{ki}]$ satisfying $\gamma = \text{Var}[t_k|y_{ki}]/(\sigma^2 + \varsigma^2_{ki})$, where $\gamma \in [0, 1]$.

Therefore, since an agent's expected payoffs are concave with respect to his own action, his optimal action strategy for a given coordinate $k \neq i$ consists of a linear combination between the mean of type t_k (using prior beliefs) and the signal y_{ki} that he receives from agent k. The class of message strategies chosen in this paper leads to the same conclusion, as implied by equation (7).

Furthermore, $\text{Var}[t_k|y_{ki}]/(\sigma^2 + \varsigma^2_{ki})$ increases with γ, and one obtains the limit cases: (a) if $\gamma \to 1$, then $\text{Var}[t_k|y_{ki}] \approx \sigma^2 + \varsigma^2_{ki}$ whereas (b) if $\gamma \to 0$, then $\varsigma^2_{ki} \to \infty$ for a bounded $\text{Var}[t_k|y_{ki}]$. This implication that the variance of the type (conditioned on the signal) increases with $\gamma \in [0, 1]$ is anologous to the one obtained in this paper, as derived from equation (8). The corresponding implication obtained in this paper is that the variance of an unknown agent's type, from the perspective of the agent that acquires information from him, decreases with the amount of information that he acquires.

Thus, given the assumed payoffs, this alternative benchmark for information transmission with normal signals permits us to obtain implications qualitatively similar to those derived from the message strategies considered in this paper. In particular, in both benchmarks, one obtains that each agent's optimal action strategy (for a given dimension of the action space) is linear with respect to the received signal, according to a certain weight parameter. In addition, the variance of an unknown agent's type, conditional upon the received signal, decreases with such a weight parameter. This paper has analyzed infor-

[20]See, e.g., Angeletos and Pavan [3], and Calvó-Armengol and de Martí [8].

mation acquisition by allowing the agents to choose endogenously the value of that weight parameter.

4.2 Non-linear Information Acquisition Cost

This paper has concentrated on the analysis of both efficient and equilibrium information acquisition strategies where agents either acquire full information or acquire no information at all. The result that the planner and the agents make corner choices in their respective decision problems (i.e., $x_{ik} \in \{0, 1\}$ for each $k \in N_i(g)$ and each $i \in N$) is driven by the assumed message strategies and by the assumption of linear information acquisition cost. This has made tractable the problem of comparing efficient and equilibrium information acquisition profiles.

One may wonder, however, whether the results obtained here would change under a slightly modified class of preferences that allows for the study of choices about information acquisition given by interior solutions (i.e., $x_{ik} \in (0, 1)$ for each $k \in N_i(g)$ and each $i \in N$).

To answer this, I consider a payoff perturbation by assuming that each agent $i \in N$ incurs a cost of information acquisition with respect to each neighbor $k \in N_i(g)$ given by a (twice differentiable) cost function $c : [0, 1] \to \mathbb{R}_+$ satisfying: (i) $c'(x) > 0$ for each $x \in (0, 1]$, (ii) $c'(0) = 0$ and $c'(1) > 2\sigma^2(1 + r)$, and (iii) $c''(x) > 2\sigma^2(1 + r)$ for each $x \in [0, 1]$. Condition (ii) above ensures that the planner and the agents make interior choices in their respective decision problems with respect to information acquisition. Condition (iii) guarantees that the respective information acquisition decision problems for the planner and the agents are concave. The rest of the game is identical to the one presented in Section 2.

Under the alternative assumption introduced above, one obtains the following results regarding efficient and equilibrium information acquisition behavior. They are analogs to those provided by Proposition 1 and Proposition 2.

Proposition 3. *Let $g \in G$ and let $\boldsymbol{x}$ be an efficient information acquisition profile. Then, for each agent $i \in N$ and each neighbor $k \in N_i(g)$, the information adquisition parameter x_{ik} satisfies $x_{ik} \in (0, 1)$ and*

$$\frac{c'(x_{ik})}{x_{ik}} = 2\sigma^2. \tag{14}$$

Proposition 4. *Let $g \in G$ and let $(\boldsymbol{\alpha}^*, \boldsymbol{\lambda}^*, \boldsymbol{x}^*)$ be an IAE. Then, for each agent $i \in N$ and each neighbor $k \in N_i(g)$, for each given $x^*_{-i} \in X_{-i}$, the information acquisition* *parameter* x_{ik}^* *satisfies* $x_{ik}^* \in (0,1)$ *and*

$$\frac{c'(x_{ik}^*)}{x_{ik}^*} = 2\sigma^2\Big[(1-r) + \frac{2r}{n-1}\sum_{j\in N_k(g)\setminus\{i\}}(x_{jk}^*)^2\Big]. \tag{15}$$

Let us see briefly whether the earlier result (obtained in Proposition 2) stating that the incentives of an agent to acquire information from a given neighbor increase with the amount of information that the rest of neighbors of that neighbor acquire from him continues to apply. Consider a network $g \in G$. For a given agent $i \in N$ and a given neighbor $k \in N_i(g)$, let us define the function

$$H(x_{ik};y) := \Big[(1-r) + \frac{2r}{n-1}y\Big]\sigma^2 x_{ik}^2 - c(x_{ik}) - \Big[(1-r) + \frac{r}{n-1}y\Big]\sigma^2,$$

where $y = \sum_{j\in N_k(g)\setminus\{i\}} x_{jk}^2$. The proof of Proposition 4 shows that agent i chooses optimally the amount of information that he acquires from his neighbor k if and only if he chooses $x_{ik}^* \in [0,1]$ so as to maximize $H(x_{ik};y)$. Therefore, the first order condition $\partial H(x_{ik}^*;y)/\partial x_{ik} = 0$ gives us equation (15) above. Furthermore, it can be checked that $\partial^2 H(x_{ik};y)/\partial x_{ik}\partial y \geq 0$ for each $x_{ik} \in [0,1]$ and each $y \geq 0$. Then, using Topkis' Monotonicity Theorem,[21] one obtains that x_{ik}^* is an increasing function with respect to y.

Hence, under this alternative cost specification, the result that the agents wish to coordinate their information acquisition decisions in the same direction continues to apply. This is important since the results of this paper relating the network density to the compatibility between efficient and equilibrium information acquisition rely strongly on that coordination effect.

It would be interesting to analyze the compatibility between efficient and equilibrium information acquisition profiles under this alternative cost specification. However, we observe that equation (15) characterizes a multiplicity of IAE where the agents make interior choices. Consequently, some selection criterion would be necessary in order to carry out that welfare analysis.

5 Concluding Comments

This paper considered a multi-agent information transmission model, in terms of Bayesian belief revision processes, to analyze information acquisition decisions by agents involved in a network. The environment investigated here is one with no conflict of interests over actions and with positive informational spillovers. The IAE concept, that incorporates the role of the newly acquired information in shaping own (anticipated) perception of future

[21] See, e.g., Topkis [22].

expected payoffs into the agent's sequential rationality requirements, has been proposed to analyze information acquisition decisions. The main contributions of this paper were (i) to propose an appealing game theoretical solution concept, IAN, to study information acquisition decisions within networked groups, (ii) to characterize both the efficient and the equilibrium behavior with respect to information acquisition, and (iii) to relate the compatibility between efficient and equilibrium information acquisition to the network density.

One may expect that the results of this paper hold in a wide class of environments where the information structure features complementarities, where there are no strategic interactions over actions, and where each agent cares about his own action and wishes the others to align theirs with the true state. Although the assumptions of the model are specific, they do not appear to be essential for its main results to follow. In this respect, quadratic payoffs can be considered as a second-order approximation of a more general class of convex preferences. The chosen payoffs over actions, together with the chosen message strategies and the linearity assumption on the cost function, are crucial to obtain that, in equilibrium, an agent acquires either full information or no information at all from a given neighbor. This alleviates the problem of multiplicity of equilibria, making tractable the welfare exercise of comparing efficient and equilibrium information acquisition profiles.

However, the results obtained here need not extend to environments with strategic complementarities over actions and/or a second-guessing coordination motive in payoffs, as it is the case under the class of preferences proposed, for example, by Morris and Shin [18], Angeletos and Pavan [3], Calvó-Armengol and de Martí [8], and Hagenbach and Koessler [13]. For these models, strategic interactions over actions are rich and constitute an important part of their analyses. In contrast, this work has concentrated only on the study of strategic interactions over information acquisition decisions.

Finally, this paper assumed that information cannot be transmitted through agents indirectly linked in a network. It would be interesting to investigate the information acquisition problem when such a network effect is allowed for.

Appendix

This appendix is devoted to the proofs of the various propositions.

A bit of notation will be useful. For $i, k \in N$, $i \neq k$, let $\theta(t_k|m_{ki}; x_{ik}) := \big(t_k - \mathbb{E}[t_k|m_{ki}; x_{ik}]\big)^2$ denote agent i's square forecast error about t_k, conditioned on receiving message m_{ki}, given the piece of information x_{ik} that he acquires from agent k. Using equation (7), one then easily obtains

$$\theta(t_k|m_{ki};x_{ik}) = (t_k-\mu)^2 + x_{ik}^2(m_{ki}-\mu)^2 - 2x_{ik}(t_k-\mu)(m_{ki}-\mu). \tag{16}$$

With this in hand, let us proceed to the proofs of the propositions.

Proof of Proposition 1. Consider a network $g \in G$. The optimal action strategy in equation (9), together with the expression for the square forecast error in equation (16), allows us to write the welfare function evaluated in stage 2 as

$$\begin{aligned} W_2(\boldsymbol{t},\boldsymbol{m};\boldsymbol{\lambda}) &= -\sum_{i\in N}\sum_{k\neq i}\theta(t_k|m_{ki};x_{ik}) \\ &= -(n-1)\sum_{i\in N}(t_i-\mu)^2 - \sum_{i\in N}\sum_{k\in N_i(g)} x_{ik}^2(m_{ki}-\mu)^2 \\ &\quad + 2\sum_{i\in N}\sum_{k\in N_i(g)} x_{ik}(t_k-\mu)(m_{ki}-\mu). \end{aligned}$$

By combining the expression above with equation (6), we can write the welfare function evaluated in stage 1 as

$$\begin{aligned} W_1(\boldsymbol{x}) = &-(n-1)n\sigma^2 - c\sum_{i\in N}\sum_{k\in N_i(g)} x_{ik} \\ &- \sum_{i\in N}\sum_{k\in N_i(g)} x_{ik}^2\int_0^1 f(t_k)\int_0^1 \beta_{ki}(m_{ki}|t_k;x_{ik})(m_{ki}-\mu)^2 dm_{ki}\,dt_k \\ &+ 2\sum_{i\in N}\sum_{k\in N_i(g)} x_{ik}\int_0^1 f(t_k)\int_0^1 \beta_{ki}(m_{ki}|t_k;x_{ik})(t_k-\mu)(m_{ki}-\mu) dm_{ki}\,dt_k. \end{aligned} \tag{17}$$

I proceed by expressing each of the terms in equation (17) as a function of the information acquired by the agents. Consider an agent $i \in N$ and an agent $k \in N_i(g)$. Applying the parameterization of message strategies in (2) to agent k with respect to agent i, we obtain

$$\begin{aligned} &\int_0^1 f(t_k)\int_0^1 \beta_{ki}(m_{ki}|t_k;x_{ik})(m_{ki}-\mu)^2 dm_{ki}\,dt_k \\ &\quad = \int_0^1 f(t_k)\int_0^1 [(1-x_{ik})f(m_{ki}) + x_{ik}\mathbb{I}(m_{ki}|t_k)](m_{ki}-\mu)^2 dm_{ki}\,dt_k \\ &\quad = \int_0^1 f(t_k)[(1-x_{ik})\sigma^2 + x_{ik}(t_k-\mu)^2]dt_k = \sigma^2. \end{aligned} \tag{18}$$

Similarly, we can compute

$$\begin{aligned} &\int_0^1 f(t_k)\int_0^1 \beta_{ki}(m_{ki}|t_k;x_{ik})(t_k-\mu)(m_{ki}-\mu) dm_{ki}\,dt_k \\ &\quad = \int_0^1 f(t_k)\int_0^1 [(1-x_{ik})f(m_{ki}) + x_{ik}\mathbb{I}(m_{ki}|t_k)](t_k-\mu)(m_{ki}-\mu) dm_{ki}\,dt_k \\ &\quad = x_{ik}\int_0^1 f(t_k)(t_k-\mu)^2 dt_k = x_{ik}\sigma^2. \end{aligned} \tag{19}$$

So, by substituting equations (18) and (19) into equation (17), we can express the welfare function evaluated in stage 1 as

$$W_1(\boldsymbol{x}) = -(n-1)n\sigma^2 + \sum_{i\in N}\sum_{k\in N_i(g)} x_{ik}[x_{ik}\sigma^2 - c]. \tag{20}$$

Let $\psi(x_{ik}) := x_{ik}[x_{ik}\sigma^2 - c]$. Given the parabolic shape of function ψ, we obtain that the welfare function evaluated in stage 1 is maximized according to: for each $i \in N$ and each $k \in N_i(g)$, (i) $x_{ik} = 1$ if and only if $c \leq \sigma^2$, (ii) $x_{ik} = 0$ if and only if $c \geq \sigma^2$, and (iii) $x_{ik} \in \{0, 1\}$ if and only if $c = \sigma^2$, as stated. □

Proof of Proposition 2. Consider a network $g \in G$. Consider an agent $i \in N$, a type $t_i \in T_i$, a message profile $\boldsymbol{m} = (m_i, m_{-i}) \in M^{n(n-1)}$, and an information acquisition profile $\boldsymbol{x} \in \boldsymbol{X}$ that induces a belief profile $\boldsymbol{\lambda} \in Q^n$. Substitution of equation (1) into equation (4), gives us the following expression for the expected payoff of agent i in stage 2:

$$\begin{aligned} V_{i,2}(\alpha_i(t_i, m_i), \alpha_{-i}, \lambda_i; t_i, \boldsymbol{m}) = &-(1-r)\big(t_i - \alpha_{ii}(t_i)\big)^2 \\ &-(1-r)\sum_{k\neq i}\int_0^1 \lambda_{ik}(t_k|m_{ki}; x_{ik})\big(t_k - \alpha_{ik}(m_{ki})\big)^2 dt_k \\ &-\frac{r}{n-1}\sum_{j\neq i}\big(t_i - \alpha_{ji}(m_{ij})\big)^2 \\ &-\frac{r}{n-1}\sum_{k\neq i}\sum_{j\neq i,k}\int_0^1 \lambda_{ik}(t_k|m_{ki}; x_{ik})\big(t_k - \alpha_{jk}(m_{kj})\big)^2 dt_k \\ &-\frac{r}{n-1}\sum_{k\neq i}\int_0^1 \lambda_{ik}(t_k|m_{ki}; x_{ik})\big(t_k - \alpha_{kk}(t_k)\big)^2 dt_k. \end{aligned} \tag{21}$$

By substituting the optimal action strategies in equation (9) into equation (21) above, we can write the expected payoff of agent $i \in N$ for $(t_i, \boldsymbol{m}) \in T_i \times M^{n(n-1)}$, when all the agents choose their optimal action strategies, as

$$\begin{aligned} V_{i,2}(\widehat{\alpha}_i(t_i, m_i; \lambda_i), \widehat{\alpha}_{-i}, \lambda_i; t_i, \boldsymbol{m}) = &-(1-r)\sum_{k\neq i}\operatorname{Var}[t_k|m_{ki}; x_{ik}] \\ -\frac{r}{n-1}\sum_{j\neq i}\theta(t_i|m_{ij}; x_{ji}) &- \frac{r}{n-1}\sum_{k\neq i}\sum_{j\neq i,k}\int_0^1 \lambda_{ik}(t_k|m_{ki}; x_{ik})\theta(t_k|m_{kj}; x_{jk}) dt_k. \end{aligned} \tag{22}$$

Now, using equations (5) and (22), we can write agent i's expected payoff in stage 1, when all the agents choose their optimal action strategies, as

$$
\begin{aligned}
V_{i,1}(\widehat{\boldsymbol{\alpha}}, \lambda_i) = & -(1-r)\sum_{k\neq i}\int_0^1 f(\tau)\int_0^1 \beta_{ki}(m_{ki}|\tau;x_{ik})\mathrm{Var}[t_k|m_{ki};x_{ik}]dm_{ki}\,d\tau \\
& -\frac{r}{n-1}\sum_{j\neq i}\int_0^1 f(\tau)\int_0^1 \beta_{ij}(m_{ij}|\tau;x_{ji})\theta(\tau|m_{ij};x_{ji})dm_{ij}\,d\tau \\
& -\frac{r}{n-1}\sum_{k\neq i}\sum_{j\neq i,k}\int_0^1 f(\tau)\int_0^1 \beta_{ki}(m_{ki}|\tau;x_{ik})\int_0^1 \beta_{kj}(m_{kj}|\tau;x_{jk})\times \\
& \qquad\qquad \times\int_0^1 \lambda_{ik}(t_k|m_{ki};x_{ik})\theta(t_k|m_{kj};x_{jk})dt_k\,dm_{kj}\,dm_{ki}\,d\tau \\
& -\sum_{k\in N_i(g)} cx_{ik}.
\end{aligned} \tag{23}
$$

I proceed by expressing each of the first three terms that appear in expression (23) above as a function of the agents' information acquisition parameters.

As regards the first term, consider an agent $k\neq i$. Then, applying the parameterization of message strategies in (2) to agent k and the expression for the variance of t_k for the information acquisition parameter x_{ik} in (8), we obtain

$$
\begin{aligned}
& \int_0^1 f(\tau)\int_0^1 \beta_{ki}(m_{ki}|\tau;x_{ik})\mathrm{Var}[t_k|m_{ki};x_{ik}]dm_{ki}\,d\tau \\
& \quad = \int_0^1 f(\tau)\int_0^1 \big[(1-x_{ik})f(m_{ki})+x_{ik}\mathbb{I}(m_{ki}|t_k)\big](1-x_{ik})\times \\
& \qquad\qquad \times\big[\sigma^2+x_{ik}(m_{ki}-\mu)^2\big]dm_{ki}\,d\tau \\
& \quad = \int_0^1 f(\tau)\big[(1-x_{ik})(1+x_{ik}-x_{ik}^2)\sigma^2+x_{ik}^2(1-x_{ik})(\tau-\mu)^2\big]d\tau \\
& \quad = [1-x_{ik}^2]\sigma^2.
\end{aligned} \tag{24}
$$

As for the second term in expression (23), consider an agent $j\neq i$. Applying the parameterization of message strategies in (2) and the expression of the square forecast error in (16) to agent j, we have

$$
\begin{aligned}
& \int_0^1 f(\tau)\int_0^1 \beta_{ij}(m_{ij}|\tau;x_{ji})\theta(\tau|m_{ij};x_{ji})dm_{ij}\,d\tau \\
& \quad = \int_0^1 f(\tau)\int_0^1 \big[(1-x_{ji})f(m_{ij})+x_{ji}\mathbb{I}(m_{ij}|t_i)\big]\times \\
& \qquad\qquad \times\big[(\tau-\mu)^2+x_{ji}^2(m_{ij}-\mu)^2-2x_{ji}(\tau-\mu)(m_{ij}-\mu)\big]dm_{ij}\,d\tau \\
& \quad = \int_0^1 f(\tau)\big[(1-x_{ji})x_{ji}^2\sigma^2+[(1-x_{ji})+x_{ji}+x_{ji}^3-2x_{ji}^2](\tau-\mu)^2\big]d\tau \\
& \quad = \big[x_{ji}^2-x_{ji}^3+1-x_{ji}+x_{ji}+x_{ji}^3-2x_{ji}^2\big]\sigma^2=\big[1-x_{ji}^2\big]\sigma^2.
\end{aligned} \tag{25}
$$

As for the third term in expression (23), consider two agents, $j \neq i$ and $k \neq i$, such that $j \neq k$. Application of the belief Bayesian updating rule specified in (BU) to agent i with respect to agent k's type, together with the expression of the square forecast error in (16), gives us

$$
\begin{aligned}
&\int_0^1 \lambda_{ik}(t_k|m_{ki};x_{ik})\theta(t_k|m_{kj};x_{jk})dt_k \\
&= \int_0^1 \big[(1-x_{ik})f(t_k) + x_{ik}\mathbb{I}(m_{ki}|t_k)\big]\big[(t_k-\mu)^2 + x_{jk}^2(m_{kj}-\mu)^2 - 2x_{jk}(t_k-\mu)(m_{kj}-\mu)\big]dt_k \\
&= (1-x_{ik})\sigma^2 + x_{jk}^2(m_{kj}-\mu)^2 + x_{ik}(m_{ki}-\mu)^2 - 2x_{ik}x_{jk}(m_{ki}-\mu)(m_{kj}-\mu).
\end{aligned}
$$

Next, application of the message strategy specified in (2) to agent k with respect to the message that he sends to agent j gives us

$$
\begin{aligned}
&\int_0^1 \beta_{kj}(m_{kj}|\tau;x_{jk}) \int_0^1 \lambda_{ik}(t_k|m_{ki};x_{ik})\theta(t_k|m_{kj};x_{jk})dt_k\, dm_{kj} \\
&= \int_0^1 \big[(1-x_{jk})f(m_{kj}) + x_{jk}\mathbb{I}(m_{kj}|\tau)\big]\times \\
&\qquad \times \big[(1-x_{ik})\sigma^2 + x_{jk}^2(m_{kj}-\mu)^2 + x_{ik}(m_{ki}-\mu)^2 - 2x_{ik}x_{jk}(m_{ki}-\mu)(m_{kj}-\mu)\big]dm_{kj} \\
&= \big[(1-x_{ik}) + (1-x_{jk})x_{jk}^2\big]\sigma^2 + x_{ik}(m_{ki}-\mu)^2 + x_{jk}^3(\tau-\mu)^2 - 2x_{ik}x_{jk}^2(m_{ki}-\mu)(\tau-\mu).
\end{aligned}
$$

Furthermore, application of the message strategy specified in (2) to agent k with respect to the message that he sends to agent i yields

$$
\begin{aligned}
&\int_0^1 \beta_{ki}(m_{ki}|\tau;x_{ik}) \int_0^1 \beta_{kj}(m_{kj}|\tau;x_{jk}) \int_0^1 \lambda_{ik}(t_k|m_{ki};x_{ik})\theta(t_k|m_{kj};x_{jk})dt_k\, dm_{kj}\, dm_{ki} \\
&= \int_0^1 \big[(1-x_{ik})f(m_{ki}) + x_{ik}\mathbb{I}(m_{ki}|\tau)\big]\times \\
&\times \big[(1-x_{ik}) + (1-x_{jk})x_{jk}^2\big]\sigma^2 + x_{ik}(m_{ki}-\mu)^2 + x_{jk}^3(\tau-\mu)^2 - 2x_{ik}x_{jk}^2(m_{ki}-\mu)(\tau-\mu)\big]dm_{ki} \\
&= \big[(1-x_{ik}) + (1-x_{jk})x_{jk}^2 + (1-x_{ik})x_{ik}\big]\sigma^2 + \Big[x_{jk}^3 + x_{ik}^2 - 2x_{ik}^2x_{jk}^2\Big](\tau-\mu)^2.
\end{aligned}
$$

Thus, one finally obtains

$$
\begin{aligned}
&\int_0^1 f(\tau) \int_0^1 \beta_{ki}(m_{ki}|\tau;x_{ik}) \int_0^1 \beta_{kj}(m_{kj}|\tau;x_{jk}) \int_0^1 \lambda_{ik}(t_k|m_{ki};x_{ik})\times \\
&\qquad \times \theta(t_k|m_{kj},x_{jk})dt_k dm_{kj} dm_{ki} d\tau = \big[1 + x_{jk}^2 - 2x_{jk}^2x_{ik}^2\big]\sigma^2.
\end{aligned} \tag{26}
$$

Then, by substituting equations (24)-(26) into equation (23), we can rewrite agent i's expected payoff in stage 1 as

$$
\begin{aligned}
V_{i,1}(\widehat{\boldsymbol{\alpha}},\lambda_i) = &-(1-r)\sum_{k\neq i}\big[1-x_{ik}^2\big]\sigma^2 - \frac{r}{n-1}\sum_{j\neq i}\big[1-x_{ji}^2\big]\sigma^2 \\
&- \frac{r}{n-1}\sum_{k\neq i}\sum_{j\neq i,k}\big[1 + x_{jk}^2 - 2x_{jk}^2x_{ik}^2\big]\sigma^2 - \sum_{k\in N_i(g)} c x_{ik}.
\end{aligned}
$$

For $k \in N_i(g)$, let $\phi_{ik} : [0,1] \to \mathbb{R}$ be the function defined as

$$\begin{aligned}\phi_{ik}(x_{ik}) :=& \Big[(1-r) + \frac{2r}{n-1} \sum_{j \in N_k(g)\setminus\{i\}} x_{jk}^2\Big]\sigma^2 x_{ik}^2 - c x_{ik} \\ &- \Big[(1-r) + \frac{r}{n-1} \sum_{j \in N_k(g)\setminus\{i\}} x_{jk}^2\Big]\sigma^2.\end{aligned} \tag{27}$$

Using this, taking into account the fact that, for each $i \in N$, $x_{ik} = 0$ for $k \notin N_i(g) \cup \{i\}$, and by rearranging terms, we can express agent i's expected payoff in stage 1 as

$$\begin{aligned}V_{i,1}(\widehat{\boldsymbol{\alpha}}, \lambda_i) =& -(1-r)(n - n_i(g) - 1)\sigma^2 - \frac{r(n-2)n_i(g)}{n-1}\sigma^2 - \frac{r}{n-1}\sum_{j \neq i}[1 - x_{ji}^2]\sigma^2 \\ &- \frac{r}{n-1} \sum_{k \notin N_i(g) \cup \{i\}} \sum_{j \neq i,k} [1 + x_{jk}^2]\sigma^2 + \sum_{k \in N_i(g)} \phi_{ik}(x_{ik}).\end{aligned}$$

It follows that the information acquisition strategy x_i^* and the corresponding induced beliefs μ_i^* satisfy conditions (SR2) and (SR1) in the definition of IAE, Definition 1, if and only if, for each $k \in N_i(g)$, x_{ik}^* solves the problem:

$$\max_{x_{ik} \in [0,1]} \ \phi_{ik}(x_{ik}).$$

Given the parabolic shape of the function ϕ_{ik}, we obtain that either (i) $x_{ik}^* = 0 \Leftrightarrow \phi_{ik}(0) \geq \phi_{ik}(1) \Leftrightarrow c \geq \sigma^2\Big[(1-r) + 2r\frac{1}{n-1}\sum_{j \in N_k(g)\setminus\{i\}}(x_{jk}^*)^2\Big]$, (ii) $x_{ik}^* = 1 \Leftrightarrow \phi_{ik}(1) \geq \phi_{ik}(0) \Leftrightarrow c \leq \sigma^2\Big[(1-r) + 2r\frac{1}{n-1}\sum_{j \in N_k(g)\setminus\{i\}}(x_{jk}^*)^2\Big]$, or (iii) $x_{ik}^* \in \{0,1\} \Leftrightarrow \phi_{ik}(1) = \phi_{ik}(0)$ $\Leftrightarrow c = \sigma^2\Big[(1-r) + 2r\frac{1}{n-1}\sum_{j \in N_k(g)\setminus\{i\}}(x_{jk}^*)^2\Big]$.

The result follows since we considered a generic agent $i \in N$. □

Proof of Proposition 3. Consider a network $g \in G$. Since the cost of information acquisition is given by function $c : [0,1] \to \mathbb{R}_+$, we can rewrite equation (20), which gives us the welfare function evaluated in stage 1, as

$$W_1(\boldsymbol{x}) = -(n-1)n\sigma^2 + \sum_{i \in N} \sum_{k \in N_i(g)} [x_{ik}^2 \sigma^2 - c(x_{ik})].$$

Therefore, as shown in the proof of Proposition 1, for each agent $i \in N$ and each $k \in N_i(g)$, the problem that the planner faces is:

$$\max_{x_{ik} \in [0,1]} \ \sigma^2 x_{ik}^2 - c(x_{ik}).$$

Corner solutions for this problem are ruled out by the assumptions $c'(0) = 0$ and $c'(1) > 2\sigma^2(1+r)$ on the shape of the cost function. It follows that $x_{ik} \in (0,1)$, for each agent

$i \in N$ and each neighbor $k \in N_i(g)$, must hold in each efficient information acquisition profile. Furthermore, it follows from the assumption $c''(x) > 2\sigma^2(1+r)$, for each $x \in [0,1]$, that the set of first order conditions

$$\frac{c'(x_{ik})}{x_{ik}} = \sigma^2 \quad \text{for each } i \in N \text{ and each } k \in N_i(g)$$

characterizes the solution of the planner's problem for each agent $i \in N$ and each neighbor $k \in N_i(g)$, as stated. □

Proof of Proposition 4. Consider a network $g \in G$ and an agent $i \in N$. Let x_i^* and μ_i^* be, respectively, an information acquisition strategy and the corresponding induced beliefs that satisfy conditions (SR2) and (SR1) in the definition of IAE, Definition 1. As shown in the proof of Proposition 2, for each $k \in N_i(g)$, x_{ik}^* must solve the problem:

$$\max_{x_{ik} \in [0,1]} \ \phi_{ik}(x_{ik}).$$

Now, since the cost of information acquisition function $c : [0,1] \to \mathbb{R}_+$, the definition of function ϕ_{ik}, for $k \in N_i(g)$ given in expression (27), becomes

$$\begin{aligned}\phi_{ik}(x_{ik}) :=& \Big[(1-r) + \frac{2r}{n-1} \sum_{j \in N_k(g) \setminus \{i\}} x_{jk}^2 \Big] \sigma^2 x_{ik}^2 - c(x_{ik}) \\ &- \Big[(1-r) + \frac{r}{n-1} \sum_{j \in N_k(g) \setminus \{i\}} x_{jk}^2 \Big] \sigma^2.\end{aligned}$$

From the assumptions $c'(0) = 0$ and $c'(1) > 2\sigma^2(1+r)$, it follows that $x_{ik}^* \in (0,1)$ for each $k \in N_i(g)$. Furthermore, from the assumption $c''(x) > 2\sigma^2(1+r)$ for each $x \in [0,1]$, one obtains that the optimal information acquisition choice of agent i with respect to each of his neighbors $k \in N_i(g)$ is characterized by the first order condition:

$$\frac{c'(x_{ik}^*)}{x_{ik}^*} = 2\sigma^2 \Big[(1-r) + \frac{2r}{n-1} \sum_{j \in N_k(g) \setminus \{i\}} (x_{jk}^*)^2 \Big].$$

The result follows since we considered a generic agent $i \in N$. □

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