A Note on Bargaining over Complementary Pieces of Information in Networks*

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Abstract

We consider two specific network structures, the star and the line, and study the set of bilateral alternating-offers bargaining procedures for the pairs of connected agents. Agents have complementary information and bargain over the relative price of their pieces of information. We characterize the unique subgame perfect equilibrium outcome for each of these network structures.

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1. Introduction

Consider a group of agents who are connected through an exogenously given network. Each agent has an endowment of information which is valuable to every other agent, so that pairs of agents have mutual interest in exchanging their pieces of information. Agents negotiate over the relative prices of their pieces of information. We study how the network structure influences these bilateral bargaining procedures. More precisely, we model the negotiation process for each pair of agents as an infinite-period alternating-offers bargaining model as in Rubinstein (1982). Each agent bargains simultaneously with each of her neighbors. We assume that each agent can benefit from the exchange of information with her neighbors only when the bargaining procedures with all of them finish. As a consequence, an agent's bargaining decisions, as well as her equilibrium payoffs, depend on her position in the network.

Situations of this sort arise naturally when an agent faces a decision problem and others have pieces of information which are useful to solve it. Consider an environment where taking the action most suitable to solve the problem depends crucially on the amount of information one has. Suppose that not using some relevant pieces of information decreases drastically the probability of picking the right action. In such cases, the agent will wish to gather the pieces of information possessed by others prior to solving the problem.

Calvó-Armengol (1999) analyzes a situation where one central agent bargains over a certain surplus with two peripheral agents. The central agent bargains first for a given number of periods with one of the peripheral agents and then switches to the other agent for another negotiation process for a given number of periods. He studies whether the central agent takes advantage in equilibrium of her position in the network. Corominas-Bosch (2004) explores bargaining situations between sellers and buyers who are connected through a network. Each seller can trade one unit of a good. In her model, an agent's bargaining power depends on her number of neighbors. Polanski (2007) investigates how the network structure influences the pricing of a unit of information which circulates along connected agents. In his model, the number of paths between a buyer and a seller determines the seller's relative bargaining power. Manea (2009) proposes a procedure to determine limit equilibrium payoffs in an infinite horizon game in which pairs of linked agents in a network are randomly chosen to bargain over a certain surplus. Agents who reach an agreement are replaced by new agents in the same positions of the network. A similar bargaining protocol, with the only difference that agents who reach an agreement are removed from the network without replacement, is considered by Abreu and Manea (2009). They study the efficiency properties of the Markov perfect equilibria of such a game. They obtain that for each network there is a subgame perfect equilibrium which is asymptotically efficient.

Section 2 presents the model and Section 3 states the main results. We restrict attention to two specific network structures: the star and the line. We obtain that, for such networks, the proposed game of pairwise negotiations has a unique subgame perfect equi-

librium outcome. Each agent's equilibrium payoff is a fraction of the total payoffs that the society can obtain given agents' relative positions, so that there is no efficiency loss in the negotiation processes through the network. Since each pair of agents trade with each other independently of their other trading partners, the gain associated with each link is pairwise. We abstract from situations where the gains of an agent from trading with a neighbor have externality on her surplus derived from trading with other neighbors. In the star, agents are favored when they propose first in their negotiation processes, regardless of their position in the network. However, an agent prefers to be a central agent when either the central agent or all the peripheral agents are restricted to propose first in each link while she prefers to be a peripheral agent when either the central agent or all the peripheral agents are restricted to propose second in each link. In the line, ex-ante bargaining power propagates through the network and each agent benefits from the position of agents who are indirectly connected. Section 4 discusses some extensions of the model and concludes. All the proofs are relegated to the Appendix.

2. The Model

2.1. Network Notation

We consider a finite set of agents $N = \{1, \ldots, n\}$, with $n \geq 3$. A *network* g on N is a collection of pairs from the set N and each pair $\{i, j\} \in g$ is a *link*. A network g restricts agents' bargaining possibilities: two agents $i, j \in N$ can bargain with each other only if $\{i, j\} \in g$. Let N_i^g denote the set of agent i's *neighbors* in network g. We will restrict our attention to two specific network structures, namely, the *star* and the *line*. Without loss of generality, we will specify the star network as $g_S := \{\{1, 2\}, \{1, 3\}, \ldots, \{1, n\}\}$ and the line network as $g_L := \{\{1, 2\}, \{2, 3\}, \ldots, \{n - 1, n\}\}$.

2.2. Bargaining in the Network

Each agent is initially endowed with one unit of information and receives payoffs from her use of information. In addition, each agent receives revenues from exchanging her endowment of information with her neighbors in the network. We assume that an agent's payoffs due to the use of information are strictly increasing in any other agent's piece of information. Thus, the pieces of information held by different agents are complementary in nature.[1] Since information is a non-depletable good, it follows that each agent finally receives the unit of information initially owned by each of her neighbors, provided that an agreement is reached with each of them. Then, an agent's payoffs due to the use of information depends on her number of neighbors. Let v_i^g be the payoff that accrues to agent i from the use of information in network g. Also, let $V^g = \sum_{i=1}^n v_i^g$ be the total

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[1]Information structures where agents held complementary information about the state of the world and each agent values the pieces of information possessed by others are considered, among others, by Hagenbach and Koessler (2010), and Jiménez-Martínez (2006).

payoffs due to the use of information in the society when agents are connected through network g.

Each pair of linked agents in a network bargain over the relative price of their initial endowments of information following the *bargaining game of alternating offers* proposed by Rubinstein (1982). The time period for the bargaining procedure within any link is discrete and labelled by $t \in T$, where T is the set of positive integers. In each date $t \in T$, one of the agents proposes an agreement price and the other agent either accepts or rejects it.

To be precise about the agents' interactions, we need to fix one of the agents in each given link $\{i, j\}$ as the reference agent for that link. The reference agent in a link is the agent who starts proposing a price in the first period, so that she has an ex-ante favorable bargaining position with respect to the other agent in the link. Let $M := \{m \in \{i, j\} : \{i, j\} \in g\} \subset N$ be the set of agents chosen as the reference agents for network g. If i is the reference agent in link $\{i, j\}$, then the terms of transaction between agents i and j are formally given by the ratio between the price of agent i's information over the sum of prices of both agents' pieces of information. By construction, this price, which we denote by q_{ij}, lies in the interval $[0, 1]$. Also, note that $q_{ji} = 1 - q_{ij}$. If the price offer is accepted, then the bargaining ends and the exchange takes place at the agreed price. If the price offer is rejected, then the play passes on to the next date, where the rejecting agent proposes in turn an agreement price. Bargaining continues in this way with no limits to the number of dates. Each agent is engaged from date $t = 1$ onwards simultaneously in this bilateral bargaining procedure with each of her neighbors. The bargaining procedures across different links are independent. Agents have perfect recall.

Given the bilateral bargaining procedures between linked agents in the network, each agent receives the revenue from exchanging her initial endowment of information with her neighbors. If i is the reference agent in the link $\{i, j\}$, then the payoff that accrues to her from the exchange of information with her neighbor j at price q_{ij} is the net revenue

$$r_{ij}(q_{ij}) = q_{ij} \cdot 1 - (1 - q_{ij}) \cdot 1 = 2q_{ij} - 1, \tag{1}$$

and the payoff to agent j from trading with i is given by

$$r_{ji}(q_{ij}) = (1 - q_{ij}) \cdot 1 - q_{ij} \cdot 1 = 1 - 2q_{ij}. \tag{2}$$

Clearly, $r_{ji}(q_{ij}) = -r_{ij}(q_{ij})$.

Agents are impatient and discount their future payoffs using a common discount factor, uniform across links, $\delta \in (0, 1)$.

We assume that each agent receives the payoffs due both to the use and exchange of information at the date at which the bargaining procedures with each of her neighbors ends. One way to interpret this assumption is by considering that each agent needs to aggregate all pieces of information gathered from her neighbors in order to be able to benefit from the use of information.

Assumption 1. For a given network g, each agent $i \in N$ can only benefit from the use and the exchange of information at the date in which the bargaining procedures with all of her neighbors finish.

The above assumption implies that an agent's optimal bargaining decisions depend on her relative position in the network. The agent cares about the date of agreement for each of her neighbors and, therefore, her bargaining decisions for two different neighbors must be correlated. Such a correlation depends on the number of her neighbors, on the number of neighbors that each of her neighbors has, and so on.

Finally, note that in order to specify completely this game of pairwise negotiations for a given network, one needs to label a reference agent for each link in the network. We denote by $\Gamma(g, M)$ the game of pairwise negotiations that we have described for a given network g when M is the set of reference agents.

We introduce now formally the elements needed to specify final payoffs and to define the equilibrium concept. Let A and R be two statements meaning, respectively, "Accept" and "Reject." Consider a network g and a given link $\{i, j\} \in g$, where agent i is taken as the reference agent in the bargaining procedure with agent j. A *strategy for agent i with respect to her neighbor j* is an infinite sequence with the form $s_{ij} = (q_{ij}^1, y^2, q_{ij}^3, y^4, \dots)$, where $q_{ij}^t \in [0, 1]$ and $y^t \in \{A, R\}$ for each $t \in T$. In this case, a strategy for agent j respect to agent i is an infinite sequence with the form $s_{ji} = (y^1, q_{ij}^2, y^3, q_{ij}^4, \dots)$. Let $s_i = (s_{ij})_{j \in N_i^g}$ be a *strategy* for agent i and let $s = (s_i)_{i \in N}$ be a *strategy profile*.[2]

Let $(s_{ij}, s_{ji})_\tau \in \mathbb{R}^2$ be the pair of coordinates in the τ-th position of the strategy pair (s_{ij}, s_{ji}). If $(s_{ij}, s_{ji})_\tau = (A, q_{ij}^\tau)$, then the price q_{ij}^τ is accepted by agent i at date τ. Analogously, if $(s_{ij}, s_{ji})_\tau = (q_{ij}^\tau, A)$, then the price q_{ij}^τ is accepted by agent j at date τ. Consider a strategy pair (s_{ij}, s_{ji}) and take a given date $t < \infty$. The acceptance date, starting from date t, in the bargaining procedure between agents i and j is given by

$$\tau_t^*(s_{ij}, s_{ji}) := \min_{\tau \geq t} \left\{ \tau \geq t \, : \, \text{either } (s_{ij}, s_{ji})_\tau = (A, q_{ij}^\tau) \text{ or } (s_{ij}, s_{ji})_\tau = (q_{ij}^\tau, A) \right\}.$$

Given Assumption 1, we are interested in the latest acceptance date for agent i across all her neighbors in the network. Starting from date t, this latest acceptance date is

$$\tau_t^*(i, s) := \max_{j \in N_i^g} \tau_t^*(s_{ij}, s_{ji}).$$

Note that the agreement dates $\tau_t^*(s_{ij}, s_{ji})$ specified above may not exist for each date t and each strategy profile s. This is the case when agents i and j do not reach an agreement starting from date t. If there is no agreement between agent i and one of her neighbors, the latest acceptance date $\tau_t^*(i, s)$ does not exist either. In this case, we write $\tau_t^*(i, s) = \infty$.

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[2] Notice that the specification of strategies also depends on the set M of agents who are chosen as the reference agents for the network.

Consider a given network g, and a given agent i who is taken as the reference agent in the bargaining procedure with each of her neighbors $k \in N_i^g$. For a strategy profile s, let $u_{i,t}^g(s)$ be the value at time t of the discounted aggregate payoff to agent i due to her bargaining with her neighbors in the network. We assume that $u_{i,t}^g(s) = 0$ if $\tau_t^*(i,s) = \infty$. The interpretation is that agent i receives a zero payoff at a given date if she does not reach an agreement with any of her neighbors from that date onwards. If, instead, $\tau_t^*(i,s)$ is a finite integer, then agent i exchanges her endowment of information with each of her neighbors, and obtains her payoffs both from the use and the exchange of information. Then, for a set M such that agent i is chosen as the reference agent in all links in which she is included, we have[3]

$$
u_{i,t}^g(s) = \begin{cases} 0 & \text{if } \tau_t^*(i,s) = \infty, \\ \delta^{\tau_t^*(i,s)-t} \left(v_i^g + \sum_{k \in N_i^g} r_{ik}(q_{ik}) \right) & \text{if } t \le \tau_t^*(i,s) < \infty. \end{cases} \tag{3}
$$

Note that, for a strategy profile s, the final payoff to agent i in the game $\Gamma(g, M)$ is given by $u_{i,1}^g(s)$. Then, let us simply write $u_i^g(s) = u_{i,1}^g(s)$ to ease notation.

Definition 1. Given a network g and a set of reference agents M for that network, a *subgame perfect Nash equilibrium (SPE)* of the game $\Gamma(g, M)$ is a strategy profile s^* such that for each agent $i \in N$ and each date $t \in T$, we have $u_{i,t}^g(s^*) \ge u_{i,t}^g(s_i, s_{-i}^*)$ for each s_i.

3. Main Results

The following proposition characterizes equilibrium prices and payoffs for the star network for two cases: (a) the peripheral agents are the reference agents, and (b) the central agent is the reference agent.

Proposition 1. *Consider the star network given by $g_S = \{\{1,2\}, \{1,3\}, \dots, \{1,n\}\}$. Suppose that either all the peripheral agents are chosen as the reference agents, i.e., $M = \{2,3,\dots,n\}$, or the central agent is chosen as the reference agent, i.e., $M = \{1\}$. Then, under Assumption 1, the game $\Gamma(g_S, M)$ has a unique SPE where the prices and the payoffs are given, respectively, by:*
(a) for $M = \{2,3,\dots,n\}$,

$$
q_{j1}^* = \frac{1 - v_j^{g_S}}{2} + \frac{V^{g_S}}{2(n-1+\delta)},
$$

$$
u_1^{g_S}(s^*) = \left[\frac{\delta}{n-1+\delta} \right] V^{g_S}, \quad \text{and} \quad u_j^{g_S}(s^*) = \left[\frac{1}{n-1+\delta} \right] V^{g_S} \quad \text{for } j \neq 1;
$$

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[3]This specification of payoffs is without loss of generality. Notice that, in many networks, not all agents can be chosen as the reference agent for all links in which they are included. If an agent i cannot be chosen as the reference agent for some neighbor j, then we simply need to change the price in the expression above from q_{ij} to q_{ji}.

(b) for $M = \{1\}$,

$$q_{j1}^{**} = \frac{1 - v_j^{gs}}{2} + \frac{\delta V^{gs}}{2[1 + (n-1)\delta]},$$

$$u_1^{gs}(s^{**}) = \left[\frac{1}{1 + (n-1)\delta}\right] V^{gs}, \quad and \quad u_j^{gs}(s^{**}) = \left[\frac{\delta}{1 + (n-1)\delta}\right] V^{gs} \quad for\ j \neq 1.$$

Note that, because $u_j^{gs}(s^*) = \delta u_1^{gs}(s^*)$ and $u_1^{gs}(s^{**}) = \delta u_j^{gs}(s^{**})$ for any peripheral agent j, agents who propose first take advantage in the bargaining processes regardless of their position in the network. In other words, the gain from proposing first overcomes any network effect.

Comparing the situations in which an agent proposes first and second, we can compute her payoff gain in the star. As in Rubinstein (1982), for the extreme case with only two agents the payoff gain is $(1 - \delta)V^{gs}/(1 + \delta)$. When more than two agents are connected in the star, the results in Proposition 1 imply that an agent obtains a payoff gain

$$\frac{(1 - \delta)(1 + \delta)V^{gs}}{(n - 1 + \delta)[1 + (n-1)\delta]}$$

for each link that she has. Therefore, this gain becomes very small in large populations. Suppose that we restrict agents in her order of proposal and, for the two possible cases of reference agents considered in Proposition 1, ask them about her preferred position in the star. If we impose that either the central agent or all the peripheral agents propose first in their negotiation processes, we have

$$u_1^{gs}(s^{**}) - u_j^{gs}(s^*) = \frac{(n - 2)(1 - \delta)}{(n - 1 + \delta)[1 + (n-1)\delta]} > 0,$$

so that an agent prefers to be a central agent. On the other hand, if we impose that either the central agent or all the peripheral agents propose second in their negotiation processes, we have

$$u_1^{gs}(s^*) - u_j^{gs}(s^{**}) = -\frac{\delta(n - 2)(1 - \delta)}{(n - 1 + \delta)[1 + (n-1)\delta]} < 0,$$

so that an agent prefers to be a peripheral agent. This is intuitive because there is a payoff gain for each link in which an agent moves from proposing second to proposing first.

Also notice that, at $\delta = 1$, i.e., all the agents are perfectly patient, $q_{j1}^* = q_{j1}^{**}$ for each $j \neq 1$, and in both cases, i.e., $M = \{2, 3, \ldots, n\}$ and $M = \{1\}$, all of them consume the same payoff equal to V^{gs}/n. When the agents are not at all impatient, they can wait forever for the bargaining procedures to end. This washes away both the advantages from being the reference agent(s) and from their relative positions in the network.

The next proposition characterizes the equilibrium when the agents are connected along a line in which they are ordered from left to right, and each agent is a reference agent with respect to her immediate successor in the line according to this order.[4]

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[4]Of course, the results in Proposition 2 continue to hold qualitatively if we reverse this order.

Proposition 2. *Consider the line network which is given by* $g_{\mathrm{L}} = \{\{1,2\}, \{2,3\}, \ldots, \{n-1, n\}\}$. *Suppose that* $M = \{1, 2, \ldots, n-1\}$. *Then, under Assumption 1, the game* $\Gamma(g_{\mathrm{L}}, M)$ *has a unique SPE such that each agent* $i = 1, \ldots, n-1$ *charges a price*

$$q^*_{i(i+1)} = \frac{1}{2} + \frac{\sum_{k=i+1}^{n} v_k^{g_{\mathrm{L}}} \sum_{j=0}^{i-1} \delta^j - \sum_{k=1}^{i} v_k^{g_{\mathrm{L}}} \sum_{j=i}^{n-1} \delta^j}{2 \sum_{j=0}^{n-1} \delta^j}$$

to each neighbor $i + 1$ *along the line. Moreover, in this SPE, the each agent* i's *payoff is given by*

$$u_i^{g_{\mathrm{L}}}(s^*) = \left[\frac{\delta^{i-1}}{\sum_{j=0}^{n-1} \delta^j} \right] V^{g_{\mathrm{L}}}.$$

Thus, the ex-ante relative bargaining power that any reference agent has over her immediate successor is transmitted along the line through the indirectly connected agents. This allows each agent to take advantage not only of the position of her immediate successor but also of the position of the indirectly connected agents who are located at her right-hand side. In other words, each agent extracts a surplus from the benefits that her neighbor obtains from her own neighbor, and so on.

Comparing the star with the line, we observe that, in the star network, when the number of agents tends to infinity, each agent's equilibrium payoffs vanish. However, in the line network, $\lim_{n \to \infty} u_i^{g_{\mathrm{L}}}(s^*) = \delta^{i-1}(1 - \delta)V^{g_{\mathrm{L}}}$, so that equilibrium payoffs remain positive for some agents, at least for those located on the left part of the line.

4. Concluding Comments

Equilibrium shares depend on the set of agents who are chosen as the reference agents. For the star network, agents prefer to be the central agent if they are restricted to propose first in each link, and they prefer to be the peripheral agents if they are restricted to propose second. However, they care more about the order in which they propose than about their position in the network. For the two possibilities of reference agents considered in Proposition 1, ex-ante bargaining power does not propagate through the star since no agent is a reference agent with respect to an agent who is a reference agent in another link. Otherwise, we should expect that the ex-ante bargaining power propagates through the network as in the line.

For the line network, one could propose a game in which some agents in the line are not chosen as reference agents in any of the links in which they are included. This amounts to give those agents less bargaining power ex-ante. It is straightforward to show that these agents are harmed more in the equilibrium than any agent in the game studied in Proposition 2. Instead, we have modeled a situation in which all agents, except those at the ends of the line, have ex-ante symmetric bargaining power as each of them is a reference agent in only one of the two links in which she is included.

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Although all agents have the same discount rate, their positions in the network, together with the assumption that they need to finish the procedures with all their neighbors in order to benefit from the use and trade of information, affects their relative bargaining power in the overall set of bilateral negotiations. As a result, the network structure influences their final payoffs.

Appendix

Proof of Proposition 1. Consider the star network $g_S = \{\{1, 2\}, \{1, 3\}, \ldots, \{1, n\}\}$ and fix a given link $\{1, j\} \in g_S$.

(a) Take the peripheral agent j as reference agent for the bargaining procedure with the central agent 1. Then, equation

$$\delta \left(v_1^{g_S} + r_{1j}(\tilde{q}_{j1}) + \sum_{k \neq 1, j} r_{1k}(q_{k1}^*) \right) = v_1^{g_S} + r_{1j}(q_{j1}^*) + \sum_{k \neq 1, j} r_{1k}(q_{k1}^*) \tag{4}$$

is the indifference condition for agent 1 between exchanging her endowment of information with agent j at price $\tilde{q}_{j1}$ in period $t = 2$ or at price q_{j1}^* in period $t = 1$. Consider the prices q_{k1}^*, for $k \neq 1, j$, as exogenously given for the moment. Also, equation

$$\delta \left(v_j^{g_S} + r_{j1}(q_{j1}^*) \right) = v_j^{g_S} + r_{j1}(\tilde{q}_{j1}) \tag{5}$$

is the indifference condition for agent j between trading her endowment of information with agent 1 at price q_{j1}^* in period $t = 2$ or at $\tilde{q}_{j1}$ price in period $t = 1$. By applying the expressions for agents' revenue in (1) and (2) to agents 1 and j, and by substituting price $\tilde{q}_{j1}$ from equation (5) into equation (4), we obtain

$$(1 + \delta)q_{j1}^* + \sum_{k \neq 1, j} q_{k1}^* = \frac{v_1^{g_S} - \delta v_j^{g_S} + n - 1 + \delta}{2}.$$

Now, consider simultaneously the bargaining procedures for all links $\{1, j\}$, $j \neq 1$. Then, we obtain $n - 1$ equations as the one above, one equation for each $j \neq 1$. This gives us a linear system whose solutions are the prices q_{j1}^*, $j \neq 1$. Using matrix notation, this system can be written as $A \cdot q^* = b$, where

$$A = \begin{pmatrix} (1 + \delta) & 1 & \ldots & 1 \\ 1 & (1 + \delta) & \ldots & 1 \\ \vdots & \vdots & \ddots & \vdots \\ 1 & 1 & \ldots & (1 + \delta) \end{pmatrix}, \quad q^* = \begin{pmatrix} q_{21}^* \\ q_{31}^* \\ \vdots \\ q_{n1}^* \end{pmatrix},$$

and

$$b = \frac{1}{2} \begin{pmatrix} v_1^{g_S} - \delta v_2^{g_S} + n - 1 + \delta \\ v_1^{g_S} - \delta v_3^{g_S} + n - 1 + \delta \\ \vdots \\ v_1^{g_S} - \delta v_n^{g_S} + n - 1 + \delta \end{pmatrix}.$$

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Each bilateral bargaining procedure within the star network corresponds to an infinite-horizon alternating-offers procedure between two agents. Then, the existence of a unique solution to the system $A \cdot q^* = b$ above implies Rubinstein's (1982) conditions for the existence of a SPE for the collection of all bilateral alternating offers procedures within the network. Application of Cramer's rule gives us

$$q_{j1}^* = \frac{1 - v_j^{g_S}}{2} + \frac{\sum_{k=1}^n v_k^{g_S}}{2(n - 1 + \delta)}.$$

By using the expression for payoffs in equation (3), we obtain that, in this SPE, the payoff to agent 1 is

$$u_1^{g_S}(s^*) = \left[\frac{\delta}{n - 1 + \delta} \right] \sum_{k=1}^n v_k^{g_S},$$

and the payoff to each peripheral agent $j \neq 1$ is

$$u_j^{g_S}(s^*) = \left[\frac{1}{n - 1 + \delta} \right] \sum_{k=1}^n v_k^{g_S}.$$

(b) Take the central agent 1 as reference agent for the bargaining procedure with the peripheral agent j. Then, equation

$$v_1^{g_S} + r_{1j}(\tilde{q}_{j1}) + \sum_{k \neq 1, j} r_{1k}(q_{k1}^{**}) = \delta \left(v_1^{g_S} + r_{1j}(q_{j1}^{**}) + \sum_{k \neq 1, j} r_{1k}(q_{k1}^{**}) \right) \tag{6}$$

is the indifference condition for agent 1 between exchanging her endowment of information with agent j at price $\tilde{q}_{j1}$ in period $t = 2$ or at price q_{j1}^{**} in period $t = 1$. Consider the prices q_{k1}^{**}, for $k \neq 1, j$, as exogenously given for the moment. Also, equation

$$v_j^{g_S} + r_{j1}(q_{j1}^{**}) = \delta \left(v_j^{g_S} + r_{j1}(\tilde{q}_{j1}) \right) \tag{7}$$

is the indifference condition for agent j between trading her endowment of information with agent 1 at price q_{j1}^{**} in period $t = 2$ or at $\tilde{q}_{j1}$ price in period $t = 1$. By applying the expressions for agents' revenue in (1) and (2) to agents 1 and j, and by substituting price $\tilde{q}_{j1}$ from equation (7) into equation (6), we obtain

$$(1 + \delta)q_{j1}^{**} + \delta \sum_{k \neq 1, j} q_{k1}^{**} = \frac{\delta v_1^{g_S} - v_j^{g_S} + 1 + (n - 1)\delta}{2}.$$

Now, consider simultaneously the bargaining procedures for all links $\{1, j\}$, $j \neq 1$. Then, we obtain $n - 1$ equations as the one above, one equation for each $j \neq 1$. This gives us a linear system whose solutions are the prices q_{j1}^{**}, $j \neq 1$. Using matrix notation, this system can be written as $A \cdot q^{**} = b$, where

$$A = \begin{pmatrix} (1 + \delta) & \delta & \dots & \delta \\ \delta & (1 + \delta) & \dots & \delta \\ \vdots & \vdots & \ddots & \vdots \\ \delta & \delta & \dots & (1 + \delta) \end{pmatrix}, \quad q^{**} = \begin{pmatrix} q_{21}^{**} \\ q_{31}^{**} \\ \vdots \\ q_{n1}^{**} \end{pmatrix},$$

and

$$b = \frac{1}{2} \begin{pmatrix} \delta v_1^{g_S} - v_2^{g_S} + 1 + (n-1)\delta \\ \delta v_1^{g_S} - v_3^{g_S} + 1 + (n-1)\delta \\ \vdots \\ \delta v_1^{g_S} - v_n^{g_S} + 1 + (n-1)\delta \end{pmatrix}.$$

Each bilateral bargaining procedure within the star network corresponds to an infinite-horizon alternating-offers procedure between two agents. Then, the existence of a unique solution to the system $A \cdot q^{**} = b$ above implies Rubinstein's (1982) conditions for the existence of a SPE for the collection of all bilateral alternating offers procedures within the network. Application of Cramer's rule gives us

$$q_{j1}^{**} = \frac{1 - v_j^{g_S}}{2} + \frac{\delta \sum_{k=1}^{n} v_k^{g_S}}{2[1 + (n-1)\delta]}.$$

By using the expression for payoffs in equation (3), we obtain that, in this SPE, the payoff to agent 1 is

$$u_1^{g_S}(s^{**}) = \left[\frac{1}{1 + (n-1)\delta} \right] \sum_{k=1}^{n} v_k^{g_S},$$

and the payoff to each peripheral agent $j \neq 1$ is

$$u_j^{g_S}(s^{**}) = \left[\frac{\delta}{1 + (n-1)\delta} \right] \sum_{k=1}^{n} v_k^{g_S}.$$

∎

Proof of Proposition 2. Consider the line network $g_L = \{\{1,2\}, \{2,3\}, \ldots, \{n-1,n\}\}$. Fix link $\{1,2\}$ and take agent 1 as reference agent in the bargaining procedure with agent 2. Then, equation

$$\delta \left(v_2^{g_L} + r_{21}(\tilde{q}_{12}) - r_{23}(q_{23}^*) \right) = v_2^{g_L} + r_{21}(q_{12}^*) - r_{23}(q_{23}^*) \tag{8}$$

specifies the indifference condition for agent 2 between exchanging her endowment of information with agent 1 at price $\tilde{q}_{12}$ in period $t = 2$ or at price q_{12}^* in period $t = 1$. Take price q_{23}^* as exogenously given for the moment. Analogously, equation

$$\delta \left(v_1^{g_L} + r_{12}(q_{12}^*) \right) = v_1^{g_L} + r_{12}(\tilde{q}_{12}) \tag{9}$$

gives us the indifference condition for agent 1 between exchanging her endowment of information with agent 1 at price q_{12}^* in period $t = 2$ or at price $\tilde{q}_{12}$ in period $t = 1$. By applying the expressions for agents' revenue in (1) and (2) to agents 1 and 2, and by substituting price $\tilde{q}_{12}$ from equation (9) into equation (8), we obtain

$$(1 + \delta)q_{12}^* - q_{23}^* = \frac{v_2^{g_L} - \delta v_1^{g_L} + \delta}{2}.$$

Now, fix a link $\{i, i+1\}$, connecting agents who are not at the ends of the line, i.e., $i = 2, \ldots, n-2$, and take agent i as the reference agent in the bargaining procedure with agent $i+1$. Suppose for the moment that the bargaining prices corresponding to any other link are exogenously given. By proceeding analogously as done above for link $\{1, 2\}$, we obtain

$$-\delta q^*_{(i-1)i} + (1+\delta)q^*_{i(i+1)} - q^*_{(i+1)(i+2)} = \frac{v^{g_L}_{i+1} - \delta v^{g_L}_i}{2}.$$

Finally, by doing the analogous computations for link $\{n-1, n\}$, we obtain

$$-\delta q^*_{(n-1)(n-1)} + (1+\delta)q^*_{(n-1)n} = \frac{v^{g_L}_n - \delta v^{g_L}_{n-1} + 1}{2}.$$

Now, consider simultaneously the bargaining procedures across all links $\{i, i+1\}$, $i = 1, \ldots, n-1$, in the network. Then, we obtain a linear system of $n-1$ equations with $n-1$ unknowns, $q^*_{i(i+1)}$, $i = 1, \ldots, n-1$. All prices are simultaneously obtained by solving this linear system. Using matrix notation, this system can be expressed as $A_{n-1} \cdot q^* = b$, where

$$A_{n-1} = \begin{pmatrix} (1+\delta) & -1 & 0 & \ldots & 0 & 0 \\ -\delta & (1+\delta) & -1 & \ldots & 0 & 0 \\ 0 & -\delta & (1+\delta) & \ldots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \ldots & (1+\delta) & -1 \\ 0 & 0 & 0 & \ldots & -\delta & (1+\delta) \end{pmatrix}, \quad q^* = \begin{pmatrix} q^*_{12} \\ q^*_{23} \\ q^*_{34} \\ \vdots \\ q^*_{(n-2)(n-1)} \\ q^*_{(n-1)n} \end{pmatrix},$$

and

$$b = \frac{1}{2} \begin{pmatrix} v^{g_L}_2 - \delta v^{g_L}_1 + \delta \\ v^{g_L}_3 - \delta v^{g_L}_2 \\ v^{g_L}_4 - \delta v^{g_L}_3 \\ \vdots \\ v^{g_L}_{n-1} - \delta v^{g_L}_{n-2} \\ v^{g_L}_n - \delta v^{g_L}_{n-1} + 1 \end{pmatrix}.$$

Each bilateral bargaining procedure within the line network corresponds to an infinite-horizon alternating-offers procedure between two agents. Then, the existence of a unique solution to the system $A_{n-1} \cdot q^* = b$ above implies Rubinstein's (1982) conditions for the existence of a PBE for the collection of all bilateral alternating offers procedures within the network.

Application of Cramer's rule gives us

$$q^*_{i(i+1)} = \frac{1}{2} + \frac{\sum_{k=i+1}^n v^{g_L}_k \sum_{j=0}^{i-1} \delta^j - \sum_{k=1}^i v^{g_L}_k \sum_{j=i}^{n-1} \delta^j}{2 \sum_{j=0}^{n-1} \delta^j}.$$

By using the expression for payoffs in equation (3), we obtain that, in this PBE, the payoff to each agent $i = 1, \ldots, n$ is given by

$$u_i^{g_{\mathrm{L}}}(s^*) = \left[\frac{\delta^{i-1}}{\sum_{j=0}^{n-1} \delta^j} \right] \sum_{k=1}^{n} v_k^{g_{\mathrm{L}}},$$

as stated. ∎

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