Microeconomía II
Problem Set III

La fecha límite para entregar las respuestas es el miércoles 1 de Junio
La calificación de cada problema aparece al final del mismo

1. Consider a two-good, pure exchange economy under certainty. There are I identical consumers and the endowments of each of them are $\omega \in \mathbb{R}^2_{++}$. Consumers' preferences are strictly monotone but not necessarily convex. In addition, such preferences can be represented by a differentiable utility function. Argue that the symmetric allocation where each consumer gets her initial allocation is either a Walrasian equilibrium (for some price vector p^*) or, if not, then for I large enough it is not a Pareto optimum. [2.5pts]

2. Consider a three-period economy, $t = 0, 1, 2$, where at $t = 0$ the economy splits into two states, s_1 and s_2, one of which occurs at $t = 1$, and each of the two states at $t = 1$ splits again into two states (s_{11} and s_{12} for the two states originated from s_1, and s_{21} and s_{22} for the two states originated from s_1), one of which occurs at $t = 2$. There are L physical commodities and consumption can take place at the three dates. [Suggestion: Draw a tree describing how uncertainty evolves over time]
(a) Describe the Arrow-Debreu equilibrium problem for this economy.
(b) Describe the Radner equilibrium problem. Suppose that at $t = 0$ and $t = 1$ there are contingent markets for the delivery of one unit of the first physical good at the following date.
(c) Argue that the conclusion of the Theorem studied in class regarding the relation between Arrow-Debreu equilibrium and Radner equilibrium remains valid. [3pts]

3. Consider the asset trading fable presented in class. The only difference is that consumption is also possible at date $t = 0$. Suppose that there is a single physical good in each period and that the Bernoulli utility functions are state independent and additively separable across time, that is, for each consumer $i \in \mathcal{I}$ and each state $s \in \mathcal{S}$, we have $u_i(x_{i0}, x_{i1s}) = u_{i0}(x_{i0}) + u_{i1}(x_{i1s})$, where $x_{i0} \in \mathbb{R}_+$ denotes i's consumption at $t = 0$ and $x_{i1s} \in \mathbb{R}_+$ denotes i's consumption at $t = 1$ when the state is s. Write down the problem that solves a representative consumer in a Radner equilibrium and express the multipliers associated with this problem, μ_s, $s \in \mathcal{S}$, in terms of the marginal utilities of consumption. [2pts]

4. Consider a two-period economy under certainty with two consumers and a single physical good. The intertemporal utility function of each consumer $i = 1, 2$ is $u_i(x_{i0}, x_{i1}) = \ln(x_{i0}) + \ln(x_{i1})$, where x_{it} denotes agent i's consumption at date $t = 0, 1$. Consumers' intertemporal endowments are $\omega_1 = (10, 0)$ and $\omega_2 = (20, 5)$. The good is perfectly storable; what it is not consumed at $t = 0$ can be saved and consumed at $t = 1$.
(a) Suppose that the consumers cannot trade with each other. How much does each of

them consume in each date? What is the utility of each consumer?
(b) Suppose that there are “forward” and “spot” markets for the good. Let p_t be the price of the good in date $t = 0, 1$. What is the (Radner) equilibrium relative price p_1^*/p_0^*? [2.5pts]