Microeconomics II

CIDE, Spring 2011

List of Problems

1. There are three people, Amy (A), Bart (B) and Chris (C): A and B have hats. These three people are arranged in a room so that B can see everything that A does and C can see everything that B does, but the players can see nothing else. In particular, C cannot see what A does. First, A chooses either to put her hat on (abbreviated by H) or not (abbreviated by F). After observing A's move, B chooses between putting his hat on or not. If B puts his hat on, the game ends and everyone gets a payoff of 0. If B does not put his hat on, then C must guess whether A's hat is on her head by says "yes" (Y) or "no" (N). This ends the game. If C guesses correctly, then he gets a payoff of 1 and A gets -1. If he guesses incorrectly, then these payoffs are reversed. B's payoff is 0, regardless of what happens. Represent this game in extensive form and identify its formal elements.

2. There are two firms which initially earn zero profits. Firm 1 produces a gadget and is developing a new production process which is equally likely to have high (H) or low (L) costs. Firm 1 only can learn the costs when the process is already developed. Then, Firm 1 chooses whether to build a new plant or not. Firm 2 is not able to observe the costs of firm 1's new process but can observe whether firm 1 builds a new plant or not. Then, firm 2 must decide whether to enter the gadget market against firm 1 or not. Firm 2 earns \$2 million if it enters the gadget market and firm 1 has high costs but loses \$4 million if firm 1 has low costs. Firm 1 increases its profits by \$4 million if the production costs are lowered. Building the new plant adds \$2 million to firm 1's profits if the new process has low costs but subtracts \$4 million from firm 1's profits it the costs are high. In any event, firm 1's entry into the gadget market lowers firm 1's profits by \$6 million. Represent this game in extensive form and identify its formal elements.

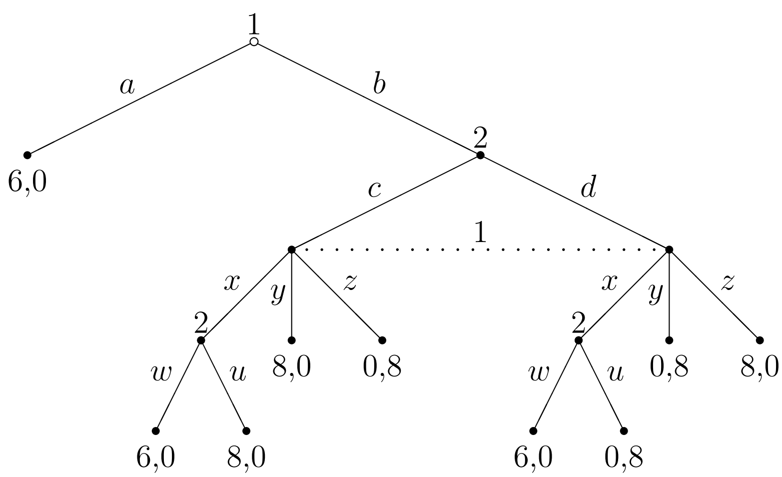
3. Two players in a card game begin by putting a dollar in the pot. Then, each player is handed a card, each player's card is equally likely to be high (H) or low (L), independent of the other player's card. Each player sees only her own card. Player 1 may see (S) or raise (R). If she sees, then the players compare their cards and the one with higher cards wins the pot; if the cards are the same, then each player takes back the dollar that she had put. If player 1 raises, then she has to add k dollars to the pot and player 2 may pass (P) or meet (M). If player 2 passes, then player 1 takes the money in the pot. If player 2 meets, then she adds k dollars to the pot and the players compare cards and the one with the higher cards wins the pot; if the cards are the same, then each players takes back the $1 + k$ dollars that she had put. Represent this game in extensive form and identify its formal elements.

4. There are two firms, denoted by $i = 1, 2$, which compete in a market by producing

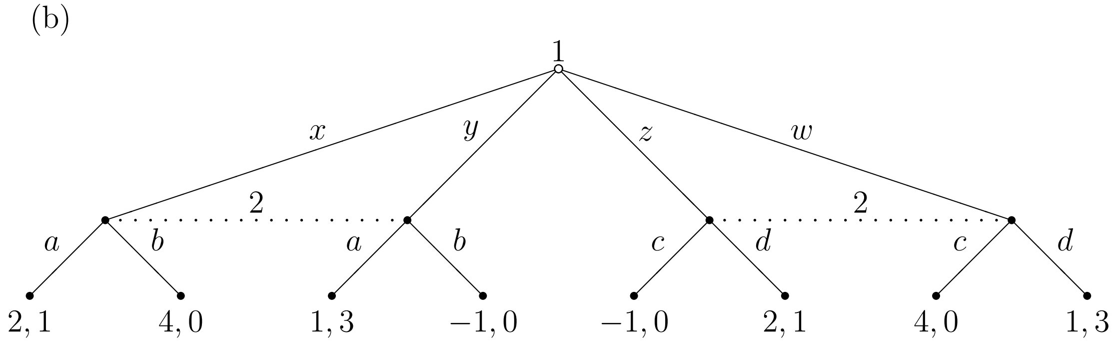
the same good. The firms simultaneously and independently choose quantities $q_i \geq 0$ to produce. The market price is given by $p = \max\{0, 2 - q_1 - q_2\}$. The cost of producing any quantity is zero for any firm. The payoff to any firm is simply its profit pq_i. Draw an extensive-form game describing this situation. Describe the strategic-form game of this game by expressing the strategy spaces and writing the payoffs as functions of the strategies.

5. Obtain the strategic-form games associated with the following extensive-form games.

(a)



(b)



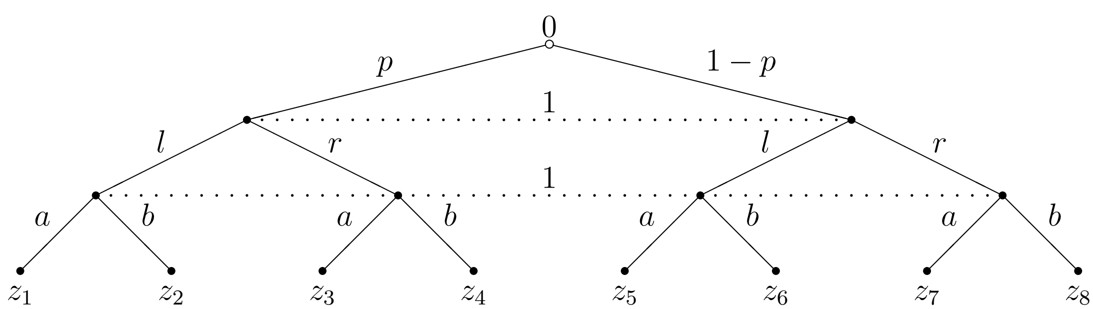
6. An incumbent faces the possibility of entry by a challenger. The challenger may stay out (O), prepare herself for combat and enter (R), or enter without making preparations (U). Preparation is costly but reduces de loss from a fight. The incumbent may either fight (F) or acquiesce to entry (A). A fight is less costly to the incumbent if the entrant is unprepared, but regardless of the entrant's preparation, the incumbent prefers to acquiesce

than to fight. The incumbent observes whether the challenger enters but not whether she is prepared. (a) Represent this situation using an extensive-form game and propose payoffs that capture the described preferences. (b) Obtain the strategic-form game associated with that extensive-form game.

7. Consider an extensive-form game with two players, 1 and 2, and perfect recall. Let $\Gamma = \langle \{1,2\}, \Delta(S_1) \times \Delta(S_2), (U_1, U_2) \rangle$ be the mixed extension of the strategic-form game associated with that extensive-form game. Let β_1 be a mixed strategy for player 1 and let s_2 be a pure strategy for player 2. Show that there exists a behavior strategy ρ_1 for player 1 such that

$$U_2(s_2, \rho_1) = U_2(s_2, \beta_1).$$

8. Consider the following (single-player) extensive form game with imperfect recall.



Show that there is a mixed strategy for player 1 which gives her a expected payoff

$$\frac{p}{3}[z_1 + 2z_4] + \frac{1-p}{3}[z_5 + 2z_8].$$

Show that there is no behavioral strategy for player 1 which gives her such a payoff.

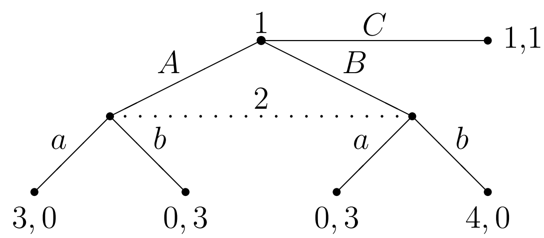
9. Evaluate the following payoffs for the strategic-form game

	L	C	R
U	$10,0$	$0,10$	$3,3$
M	$2,10$	$10,2$	$6,4$
D	$3,3$	$4,6$	$6,6$

(a) $U_1(\beta_1, C)$ for $\beta_1 = (2/3, 1/3, 0)$.
(b) $U_1(\beta_1, R)$ for $\beta_1 = (1/4, 1/2, 1/4)$.
(c) $U_2(\beta_1, R)$ for $\beta_1 = (1/3, 2/3, 0)$.
(d) $U_2(\beta_1, \beta_2)$ for $\beta_1 = (2/3, 1/3, 0)$ and $\beta_2 = (1/4, 1/4, 1/2)$.

10. There are two firms, denoted by $i = 1, 2$, which compete in a market by producing the same good. The firms simultaneously and independently choose quantities $q_i \geq 0$ to produce. The market price is given by $p = \max\{0, 100 - 2q_1 - 2q_2\}$. The cost of producing each unit of the good is 20 for any firm. The payoff to any firm is simply its profit $(p-20)q_i$. It is a ever a best response for firm 1 to choose $q_1 = 25$? Suppose that firm 1 believes that firm 2 is equally likely to select each of the quantities 6, 11, and 13. What is firm 1's best response?

11. In the following extensive-form game, is it rational for player 1 to select strategy C? Why?



12. Represent the rock-paper-scissors game in strategic-form and determine the following best-response sets
(a) $BR_1(\mu_2)$ for $\mu_2 = (1, 0, 0)$.
(b) $BR_1(\mu_2)$ for $\mu_2 = (1/2, 1/4, 1/4)$.
(c) $BR_1(\mu_2)$ for $\mu_2 = (1/3, 1/3, 1/3)$.

13. In the strategic-form game below, is player 1's strategy M dominated? If so, describe a strategy that dominates it. If not, describe a belief about player 2's behavior to which M is a best response.

	X	Y
K	$9, 2$	$1, 0$
L	$1, 0$	$6, 1$
M	$3, 2$	$4, 2$

14. Show that, in a finite two-player game, the set of (pure) strategies for a player that are not strictly dominated coincides with her set of (pure) strategies that are best responses against all her possible beliefs about the other player's behavior.

15. Find the set of rationalizable strategies for the following game.

	a	b	c	d
w	$5, 4$	$4, 4$	$4, 5$	$12, 2$
x	$3, 7$	$8, 7$	$5, 8$	$10, 6$
y	$2, 10$	$7, 6$	$4, 6$	$9, 5$
y	$4, 4$	$5, 9$	$4, 10$	$10, 9$

16. Consider a guessing game with ten players, $i = 1, \dots, 10$. Simultaneously and independently the players choose integers between 0 and 10. Then, player i's is given by $u_i = (\overline{s} - i - 1)s_i$, where s_i is the number chosen by player i and $\overline{s}$ is the average of the players' selections. What is the set of rationalizable strategies in this game?

17. There are two firms, denoted by $i = 1, 2$, which compete in a market by producing differentiated products. The firms simultaneously and independently choose prices $p_i \geq 0$ for their products. After the prices are set, consumers demand $\max\{0, 10 - p_1 + p_2\}$ units of the good produced by firm 1 and $\max\{0, 10 - p_2 + p_1\}$ units of the good produced by firm 2. Each firm must supply the number of units demanded and produces at zero cost.
(a) Compute firm 2's best-response correspondence (in terms of p_1).
(b) What is the set of rationalizable pure strategies in this game? (*Hint: Draw the graph of the best-response correspondences*).

18. Obtain the set of all Nash equilibria for the strategic-form game

	L	R
U	$5, 0$	$0, 4$
D	$1, 3$	$2, 0$

19. Consider the strategic-form game

	L	C	R
U	$3, 0$	$2, 2$	$1, 1$
M	$4, 4$	$0, 3$	$2, 2$
D	$1, 3$	$1, 0$	$0, 2$

Indicate which strategies survive iterative elimination of strictly dominated strategies. Compute all its Nash equilibria.

20. Find all Nash equilibria for the following strategic-form games

	L	C	R
U	$0, 4$	$5, 6$	$8, 7$
M	$2, 9$	$6, 5$	$5, 1$

	L	C	R
U	$0, 0$	$5, 4$	$4, 5$
M	$4, 5$	$0, 0$	$5, 4$
D	$5, 4$	$4, 5$	$0, 0$

21. Consider the strategic-form game

	L	R
U	$-5, -5$	$x - 10, 0$
D	$0, 15$	$10, 10$

where $x > 25$.
(a) Find the pure-strategy Nash equilibria of this game, if it has any.
(b) Compute the mixed-strategy Nash equilibria of the game.

22. Consider the following social problem. A pedestrian is hit by a car and requires immediate medical attention. There are n people around the accident. Simultaneously and independently each of the n bystanders decides whether or not to call 911. Each bystander obtains v units of utility if someone else makes the call. Those who call pay a personal cost of c, so that if a person calls for help, then she obtains a payoff $v-c$. If none of the n people calls for help, then each of them obtains a zero payoff. Assume $v > c > 0$.
(a) Find the symmetric Nash equilibria (those equilibria in which all players choose the same strategy, possibly mixed).
(b) Compute the probability that at least one person call 911 in equilibrium. Is this result intuitive?

23. Consider the perturbed version of the Battle of the Sexes game

	L	R
U	$3+\varepsilon t_1, 1$	$\varepsilon t_1, \varepsilon t_2$
D	$0, 0$	$1, 3+\varepsilon t_2$

where $\varepsilon \in (0,1)$, and t_1 and t_2 are realizations from independent random variables with a uniform distribution over the interval $[0,1]$. Each player i observes only t_i before choosing her action. Find three Bayes-Nash equilibria of this game.

24. Consider a Cournot duopoly with incomplete information. The demand of the good is given by $p = 1 - Q$, where Q is the total quantity produced in the market. Firm 1 selects a quantity q_1 and produces at zero cost. Firm 2's production cost is private information, selected by nature. With probability 1/2, firm 2 produces at zero cost. With probability 1/2, firm 2 produces with a marginal cost of 1/4. Let q_2^L and q_2^H be the quantities selected by the two types of firm 2. Find the Bayesian Nash equilibria of this market game in pure strategies.

25. Consider a game in which player 1 first selects between I and O. If player 1 chooses O, then the game ends with the payoff vector $(x, 1)$, where $x > 0$. If player 1 chooses I, then this selection is revealed to player 2 and then the players play simultaneously the subgame with matrix of payoffs

	A	B
A	$3, 1$	$0, 0$
B	$0, 0$	$1, 3$

(a) Find the Nash equilibria in pure strategies for this game.
(b) Calculate the Nash equilibria in mixed strategies for this game, and note how they

depend on x.

(b) Find the subgame perfect Nash equilibria in pure strategies for this game. Are there any Nash equilibria that are not subgame perfect?

(d) Calculate the subgame percect Nash equilibria in mixed strategies for this game.

26. Consider the following model (fable) of duopoly due to Stackelberg (1934). There are two firms which choose their quantities $q_1 \geq 0$ and $q_2 \geq 0$ of an homogeneous product. First, firm 1 selects q_1 and then firm 2, after observing the quantity chosen by firm 1, selects q_2. Both firms $i = 1, 2$ have identical production costs given by $C_i(q_i) = cq_i$, where $c > 0$. The market demand in given by $P(Q) = \max\{M - dQ, 0\}$, where $Q = q_1 + q_2$, and $M, d > 0$. Compute the subgame perfect Nash Equilibria in pure strategies for this game.

27. Consider the following model (that is, fable or fairy tale) of advertising. Two firms which offer the same product in a market compete in prices. The quantity demanded in the market is given by $Q = \max\{a - p, 0\}$, where $a \geq 0$ is the advertising level in the market and $p \geq 0$ is the price faced by the consumers. First, firm 1 selects an advertising level $a \geq 0$ and, after that, the firms simultaneously and independently select prices p_1 and p_2. The firm with the lowest price obtains all of the market demand at this price. If the firms charge the same price, then the market demand is split equally between them. The firms produce at zero cost and firm 1 must pay and advertising cost of $2a^3/81$. Find the subgame perfect Nash equilibrium in pure strategies for this game and explain why firm 1 advertises at that level that you obtain.

28. Consider a situation with a firm and a worker. The firm can be either of high quality (H), with probability $p \in (0, 1)$, or of low quality (L). The firm chooses either to offer a job to the worker (O) or not to offer the job (N). If no job is offered, then the game ends and both parties receive 0. If the firm offers a job, the worker either accepts (A) or rejects (R) the offer. The worker's job gives the firm a profit of 2. If the worker rejects an offer of employment, then the firm gets a payoff of -1. Rejecting an offer gives a payoff of 0 to the worker. Accepting a job offer yields the worker a payoff of 2 if the firm is of high quality and -1 if the firm is of low quality. The worker is uncertain about the quality of the firm.

(a) Is there a separating PBNE in this game? If so, specify the equilibrium and explain under what conditions it exists. If not, argue why.

(b) Is there a pooling PBNE in which both types of firms offer a job? If so, specify the equilibrium and explain under what conditions it exists. If not, argue why.

(c) Is there a pooling PBNE in which neither type of firm offers a job? If so, specify the equilibrium and explain under what conditions it exists. If not, argue why.

29. Consider the following bargaining game with incomplete information. Player 1 owns a television that she does not use and, therefore, its value for her is zero. Player 2 would like to have the television and its value for her is $v > 0$. Value v is privately known to player

2. Player 1 only knows that v is uniformly distributed between 0 and 1. These players engage in a two-period bargaining negotiation to establish whether player 1 will trade the television to player 2 for a price. The players discount the second period according to a discount factor $\delta \in (0,1)$. In both periods $t = 1,2$, player 1 proposes a price $p_t \in [0,1]$. In $t = 1$, after hearing player 1's proposal, player 2 either accepts (A) or rejects (R) the proposed price p_1. If player 2 accepts, then the television is traded at $t = 1$, player 1 gets p_1, and player 2 gets $v - p_1$. If player 2 rejects, then the play proceeds to period $t = 2$, at which player 2 either accepts or rejects price p_2. If player 2 accepts, then the television is traded at $t = 2$, player 1 gets δp_2, and player 2 gets $\delta(v - p_2)$. If no agreement is reached in the second period, then the television is not traded and each player receives a zero payoff. Calculate the PBNE in pure strategies for this game.

30. Consider a two-consumer, two-good, exchange economy under certainty and with no production. Suppose that utility functions are strictly increasing and prove that an allocation $\widehat{x}$ is Pareto optimal if and only if each $\widehat{x}_i$, $i = 1,2$, solves the problem

$$\begin{aligned} \max_{x_i \in X_i} \quad & u_i(x_i) \\ \text{s.t.:} \quad & u_j(x_j) \geq u_j(\widehat{x}_j) \\ & x_1 + x_2 = \omega_1 + \omega_2, \end{aligned}$$

for $j \neq i$.

31. Consider a two-consumer, one-good, exchange economy under uncertainty with two states of the world, s_1 and s_2. There is no production. Consumers' endowments are given by $\omega_1 = (18,4)$, $\omega_2 = (3,6)$, and their preferences are described by

$$U_1(x_{11}, x_{12}) = (x_{11}x_{12})^2,$$

for consumer 1, and by $\pi_{21} = 1/3$, $\pi_{22} = 2/3$, and

$$u_{2,s}(x_{2s}) = \ln(x_{2s}) \qquad \text{for each} \quad s \in \{s_1, s_2\},$$

for consumer 2.

(a) Characterize the set of Pareto optimal allocations as completely as possible.

(b) Compute the Arrow-Debreu equilibria of this economy.

32. Consider an Arrow-Debreu economy under uncertainty, with no production, and with one physical good. Consumers are risk averse and their preferences admit an expected utility representation. Suppose that the Bernoulli utility functions of each consumer for the good are identical across states and that subjective probabilities are the same across individuals. Consumers' endowments vary from state to state but aggregate endowment is constant across states. Set up the Arrow-Debreu trading problem and show that the allocation in which each consumer's consumption in each state is the average across states of her endowments is an equilibrium allocation.

33. Consider a two-good, two-consumer, economy under certainty, without production, and whose aggregate endowments are $(10, 20)$. Consumers' utility functions are

$$u_1(x_{11}, x_{12}) = x_{11}(x_{12})^2$$
$$u_2(x_{21}, x_{22}) = (x_{21})^2 x_{22}.$$

(a) A social planner wishes to allocate goods to maximize consumer 1's utility while holding consumer 2's utility at $u_2 = 8000/27$. Find the assignment of goods to consumers that solves the planner's problem and show that the solution is Pareto optimal.
(b) Suppose instead that the planner just divides the aggregate endowments so that $\omega_1 = (10, 0)$ and $\omega_2 = (0, 20)$ and then lets the consumers to trade through competitive markets. Find the Arrow-Debreu equilibrium of this economy.

34. A pure exchange economy under certainty and with no production has three consumers and three goods. Consumers' utility functions and initial endowments are

$$u_1(x_{11}, x_{12}, x_{13}) = \min\{x_{11}, x_{12}\}, \qquad \omega_1 = (1, 0, 0)$$
$$u_2(x_{21}, x_{22}, x_{23}) = \min\{x_{22}, x_{23}\}, \qquad \omega_2 = (0, 1, 0)$$
$$u_3(x_{31}, x_{32}, x_{33}) = \min\{x_{31}, x_{33}\}, \qquad \omega_3 = (0, 0, 1).$$

Find the Arrow-Debreu equilibrium of this economy.

35. There are 100 units of good 1 and 100 units of good 2. Consumers 1 and 2 are endowed each with 50 units of each good. Consumer 1 says "I love good 1, but can take or leave good 2." Consumer 2 says "I love good 2, but can take or leave good 1."
(a) Draw an Edgeworth box for these traders and sketch their preferences.
(b) Identify the set of Pareto optimal allocations of this economy.
(c) Find all Arrow-Debreu equilibria in this economy.

36. Consider a one-consumer economy with one consumption good. This consumer is endowed with none of the consumption good, y, and with 24 hours of time, h, so that $\omega = (24, 0)$. Her preferences are defined over $\mathbb{R}^2_+$ and represented by $u(h, y) = hy$. The production possibilities in this economy are described by

$$Y = \left\{(-h, y) \in \mathbb{R}^2 : 0 \le h \le b, \quad 0 \le y \le \sqrt{h}\right\},$$

where b is some large positive number. Let p_y and p_h be the prices of the consumption good and of leisure, respectively.
(a) Find relative prices p_y/p_h that clear the consumption and leisure markets simultaneously.
(b) Calculate the equilibrium consumption and production plans and sketch your results in $\mathbb{R}^2_+$. How many hours a day does the consumer work?

37. Consider a pure exchange economy with a single consumption good, two states, and two consumers. Expected utility functions are specified as

$$U_i(x_{i1}, x_{i2}) = \pi_{i1} u_i(x_{i1}) + \pi_{i2} u_i(x_{i2}), \qquad i = 1, 2,$$

where x_{is} is consumer i's consumption of the good in state s and π_{is} is the subjective probability of consumer i for state s. Assume that each function u_i is twice differentiable. The aggregate endowments of the two contingent commodities are $\tilde{\omega} = (\tilde{\omega}_1, \tilde{\omega}_2) \in \mathbb{R}^2_{++}$. Assume that each consumer gets half of the random variable $\tilde{\omega}$, that is $\omega_i = \frac{1}{2}\tilde{\omega}$.
(a) Suppose that u_1 is linear, u_2 is strictly concave, and both consumers have the same subjective probabilities ($\pi_{11} = \pi_{21}$). Show that, in an interior Arrow-Debreu equilibrium, consumer 2 insures completely, that is, $x^*_{21} = x^*_{22}$.
(b) Suppose that u_1 is linear, u_2 is strictly concave, and consumers' subjective probabilities are not the same (suppose in particular that $\pi_{11} > \pi_{21}$). Show that, in an interior Arrow-Debreu equilibrium, consumer 2 does not insure completely. In which state will consumer 2 consume a larger amount of the good?

38. Consider a sequential trade economy as the one presented in class. The only difference is that, for each state $s \in \mathcal{S}$, the contingent commodity pays 1 dollar (rather than one unit of the physical good 1) if and only if state s occurs. Write down the budget constraints corresponding to this fable and discuss which price normalizations are possible.

39. Formulate a fable similar to the sequential trade economy studied in class with the difference that consumption also takes place in period $t = 0$. Show that the result proved in class regarding the relation between Arrow-Debreu equilibrium and Radner equilibrium continues to hold.

40. Consider the asset trading fable presented in class. Assume that each return vector r_k is nonnegative and nonzero, that is, $r_k = (r_{k1}, \dots, r_{kS}) \in \mathbb{R}^S_+ \setminus \{\underline{0}\}$ for each $k \in \mathcal{K}$.
(a) Show that, for each vector $q^* = (q^*_1, \dots, q^*_K) \in \mathbb{R}^K$ of asset prices in a Radner equilibrium, we can find multipliers $\mu = (\mu_1, \dots, \mu_S) \in \mathbb{R}^S_+ \setminus \{\underline{0}\}$ such that $q_k = \sum_{s \in \mathcal{S}} \mu_s r_{ks}$ for each $k \in \mathcal{K}$.
(b) Suppose that there is a single physical good in each period. Express the multipliers μ_s in terms of the marginal utilities of consumption.

41. Consider a pure exchange economy, without production, and with two consumers who have asymmetric information. There are two equally likely states, $s \in \{0, 2\}$, and two physical goods. Uncertainty affects consumers' preferences and their endowments. Consumers' preferences are specified by the Bernoulli utility functions

$$u_{1,s}(x_{11s}, x_{12s}) = (2 + s)\ln(x_{11s}) + x_{12s},$$
$$u_{2,s}(x_{11s}, x_{12s}) = (4 - s)\ln(x_{21s}) + x_{22s}.$$

Consumers' endowments are $\omega_1 = (0, 0, a, b)$ and $\omega_2 = (6, 6 + \varepsilon, c, d)$, where a, b, c, and d are arbitrarily large real numbers. Take physical good 2 as the numeraire in each state and, therefore, fix its price equal to one. Denote the prices of the non-numeraire good in the two states as (p_1, p_2).

(a) Suppose that consumer 2 if fully informed of the state that occurs but consumer 1 is not informed (that is, she thinks that the two states are equally likely). Assuming that prices cannot transmit information to the consumers, determine the spot equilibrium prices $(\overline{p}_1(\varepsilon), \overline{p}_2(\varepsilon))$ in the two states.

(b) Again suppose that consumer 2 if fully informed of the state that occurs but consumer 1 is not informed. Assuming that prices can transmit information to the consumers, compute the rational expectations equilibrium prices $(p_1^*(\varepsilon), p_2^*(\varepsilon))$ in the two states when $\varepsilon \neq 0$.

(c) Show that if $\varepsilon = 0$, then there is no rational expectation equilibrium pair of prices.