# REGRESSION MODELS FOR CATEGORICAL DEPENDENT VARIABLES USING STATA 

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## Preface

Our goal in writing this book was to make it routine to carry out the complex calculations necessary for the full interpretation of regression models for categorical outcomes. The interpretation of these models is made more complex because the models are nonlinear. Most software packages that estimate these models do not provide options that make it simple to compute the quantities that are useful for interpretation. In this book, we briefly describe the statistical issues involved in interpretation, and then we show how Stata can be used to make these computations. In reading this book, we strongly encourage you to be at your computer so that you can experiment with the commands as you read. To facilitate this, we include two appendices. Appendix A summarizes each of the commands that we have written for interpreting regression models. Appendix B provides information on the datasets that we use as examples.

Many of the commands that we discuss are not part of official Stata, but instead they are commands (in the form of ado-files) that we have written. To follow the examples in this book, you will have to install these commands. Details on how to do this are given in Chapter 2. While the book assumes that you are using Stata 7 or later, most commands will work in Stata 6, although some of the output will appear differently. Details on issues related to using Stata 6 are given at

```
www.indiana.edu/~ jsl650/spost.htm
```

The screen shots that we present are from Stata 7 for Windows. If you are using a different operating system, your screen might appear differently. See the StataCorp publication Getting Started with Stata for your operating system for further details. All of the examples, however, should work on all computing platforms that support Stata.

We use several conventions throughout the manuscript. Stata commands, variable names, filenames, and output are all presented in a typewriter-style font, e.g., logit lfp age wc hc k5. Italics are used to indicate that something should be substituted for the word in italics. For example, logit variablelist indicates that the command logit is to be followed by a specific list of variables. When output from Stata is shown, the command is preceded by a period (which is the Stata prompt). For example,

```
. logit lfp age wc hc k5, nolog
Logit estimates Number of obs = 753
    (output omitted)
```

If you want to reproduce the output, you do not type the period before the command. And, as just illustrated, when we have deleted part of the output we indicate this with (output omitted).

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Keystrokes are set in this font. For example, alt-f means that you are to hold down the alt key and press f. The headings for sections that discuss advanced topics are tagged with an *. These sections can be skipped without any loss of continuity with the rest of the book.

As we wrote this book and developed the accompanying software, many people provided their suggestions and commented on early drafts. In particular, we would like to thank Simon Cheng, Ruth Gassman, Claudia Geist, Lowell Hargens, and Patricia McManus. David Drukker at StataCorp provided valuable advice throughout the process. Lisa Gilmore and Christi Pechacek, both at StataCorp, typeset and proofread the book.

Finally, while we will do our best to provide technical support for the materials in this book, our time is limited. If you have a problem, please read the conclusion of Chapter 8 and check our web page before contacting us. Thanks.

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## Part I

## General Information

Our book is about using Stata for estimating and interpreting regression models with categorical outcomes. The book is divided into two parts. Part I contains general information that applies to all of the regression models that are considered in detail in Part II.

- Chapter 1 is a brief orienting discussion that also includes critical information on installing a collection of Stata commands that we have written to facilitate the interpretation of regression models. Without these commands, you will not be able to do many of the things we suggest in the later chapters.
- Chapter 2 includes both an introduction to Stata for those who have not used the program and more advanced suggestions for using Stata effectively for data analysis.
- Chapter 3 considers issues of estimation, testing, assessing fit, and interpretation that are common to all of the models considered in later chapters. We discuss both the statistical issues involved and the Stata commands that carry out these operations.

Chapters 4 through 7 of Part II are organized by the type of outcome being modeled. Chapter 8 deals primarily with complications on the right hand side of the model, such as including nominal variables and allowing interactions. The material in the book is supplemented on our web site at www.indiana.edu/~jsl650/spost.htm, which includes data files, examples, and a list of Frequently Asked Questions (FAQs). While the book assumes that you are running Stata 7, most of the information also applies to Stata 6; our web site includes special instructions for users of Stata 6.

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## 1 Introduction

### 1.1 What is this book about?

Our book shows you efficient and effective ways to use regression models for categorical and count outcomes. It is a book about data analysis and is not a formal treatment of statistical models. To be effective in analyzing data, you want to spend your time thinking about substantive issues, and not laboring to get your software to generate the results of interest. Accordingly, good data analysis requires good software and good technique.

While we believe that these points apply to all data analysis, they are particularly important for the regression models that we examine. The reason is that these models are nonlinear and consequently the simple interpretations that are possible in linear models are no longer appropriate. In nonlinear models, the effect of each variable on the outcome depends on the level of all variables in the model. As a consequence of this nonlinearity, which we discuss in more detail in Chapter 3, there is no single method of interpretation that can fully explain the relationship among the independent variables and the outcome. Rather, a series of post-estimation explorations are necessary to uncover the most important aspects of the relationship. In general, if you limit your interpretations to the standard output, that output constrains and can even distort the substantive understanding of your results.

In the linear regression model, most of the work of interpretation is complete once the estimates are obtained. You simply read off the coefficients, which can be interpreted as: for a unit increase in $x_{k}, y$ is expected to increase by $\beta_{k}$ units, holding all other variables constant. In nonlinear models, such as logit or negative binomial regression, a substantial amount of additional computation is necessary after the estimates are obtained. With few exceptions, the software that estimates regression models does not provide much help with these analyses. Consequently, the computations are tedious, time-consuming, and error-prone. All in all, it is not fun work. In this book, we show how post-estimation analysis can be accomplished easily using Stata and the set of new commands that we have written. These commands make sophisticated, post-estimation analysis routine and even enjoyable. With the tedium removed, the data analyst can focus on the substantive issues.

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### 1.2 Which models are considered?

Regression models analyze the relationship between an explanatory variable and an outcome variable while controlling for the effects of other variables. The linear regression model (LRM) is probably the most commonly used statistical method in the social sciences. As we have already mentioned, a key advantage of the LRM is the simple interpretation of results. Unfortunately, the application of this model is limited to cases in which the dependent variable is continuous. ${ }^{1}$ Using the LRM when it is not appropriate produces coefficients that are biased and inconsistent, and there is nothing advantageous about the simple interpretation of results that are incorrect.

Fortunately, a wide variety of appropriate models exists for categorical outcomes, and these models are the focus of our book. We cover cross-sectional models for four kinds of dependent variables. Binary outcomes (a.k.a, dichotomous or dummy variables) have two values, such as whether a citizen voted in the last election or not, whether a patient was cured after receiving some medical treatment or not, or whether a respondent attended college or not. Ordinal or ordered outcomes have more than two categories, and these categories are assumed to be ordered. For example, a survey might ask if you would be "very likely", "somewhat likely", or "not at all likely" to take a new subway to work, or if you agree with the President on "all issues", "most issues", "some issues", or "almost no issues". Nominal outcomes also have more than two categories but are not ordered. Examples include the mode of transportation a person takes to work (e.g., bus, car, train) or an individual's employment status (e.g., employed, unemployed, out of the labor force). Finally, count variables count the number of times something has happened, such as the number of articles written by a student upon receiving the Ph.D. or the number of patents a biotechnology company has obtained. The specific cross-sectional models that we consider, along with the corresponding Stata commands, are

Binary outcomes: binary logit (logit) and binary probit (probit).

Ordinal outcomes: ordered logit (ologit) and ordered probit (oprobit).

Nominal outcomes: multinomial logit (mlogit) and conditional logit (clogit).

Count outcomes: Poisson regression (poisson), negative binomial regression (nbreg), zeroinflated Poisson regression (zip), and zero-inflated negative binomial regression (zinb).

While this book covers models for a variety of different types of outcomes, they are all models for cross-sectional data. We do not consider models for survival or event history data,even though Stata has a powerful set of commands for dealing with these data (see the entry for st in the Reference Manual). Likewise, we do not consider any models for panel dataeven though Stata contains several commands for estimating these models (see the entry for xt in the Reference Manual).

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### 1.3 Who is this book for?

We expect that readers of this book will vary considerably in both their knowledge of statistics and their knowledge of Stata. With this in mind, we have tried to structure the book in a way that best accommodates the diversity of our audience. Minimally, however, we assume that readers have a solid familiarity with OLS regression for continuous dependent variables and that they are comfortable using the basic features of the operating system of their computer. While we have provided sufficient information about each model so that you can read each chapter without prior exposure to the models discussed, we strongly recommend that you do not use this book as your sole source of information on the models (Section 1.6 recommends additional readings). Our book will be most useful if you have already studied the models considered or are studying these models in conjunction with reading our book.

We assume that you have access to a computer that is running Stata 7 or later and that you have access to the Internet to download commands, datasets, and sample programs that we have written (see Section 1.5 for details on obtaining these). For information about obtaining Stata, see the StataCorp web site at www . stata. com. While most of the commands in later chapters also work in Stata 6, there are some differences. For details, check our web site at www.indiana.edu/~jsl650/spost.htm.

### 1.4 How is the book organized?

Chapters 2 and 3 introduce materials that are necessary for working with the models we present in the later chapters:

Chapter 2: Introduction to Stata reviews the basic features of Stata that are necessary to get new or inexperienced users up and running with the program. This introduction is by no means comprehensive, so we include information on how to get additional help. New users should work through the brief tutorial that we provide in Section 2.17. Those who are already skilled with Stata can skip this chapter, although even these readers might benefit from quickly reading it.

Chapter 3: Estimation, Testing, Fit, and Interpretation provides a review of using Stata for regression models. It includes details on how to estimate models, test hypotheses, compute measures of model fit, and interpret the results. We focus on those issues that apply to all of the models considered in Part II. We also provide detailed descriptions of the add-on commands that we have written to make these tasks easier. Even if you are an advanced user, we recommend that you look over this chapter before jumping ahead to the chapters on specific models.

Chapters 4 through 7 each cover models for a different type of outcome:
Chapter 4: Binary Outcomes begins with an overview of how the binary logit and probit models are derived and how they can be estimated. After the model has been estimated, we show

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how Stata can be used to test hypotheses, compute residuals and influence statistics, and calculate scalar measures of model fit. Then, we describe post-estimation commands that assist in interpretation using predicted probabilities, discrete and marginal change in the predicted probabilities, and, for the logit model, odds ratios. Because binary models provide a foundation on which some models for other kinds of outcomes are derived, and because Chapter 4 provides more detailed explanations of common tasks than later chapters do, we recommend reading this chapter even if you are mainly interested in another type of outcome.

Chapter 5: Ordinal Outcomes introduces the ordered logit and ordered probit models. We show how these models are estimated and how to test hypotheses about coefficients. We also consider two tests of the parallel regression assumption. In interpreting results, we discuss similar methods as in Chapter 4, as well as interpretation in terms of a latent dependent variable.

Chapter 6: Nominal Outcomes focuses on the multinomial logit model. We show how to test a variety of hypotheses that involve multiple coefficients and discuss two tests of the assumption of the independence of irrelevant alternatives. While the methods of interpretation are again similar to those presented in Chapter 4, interpretation is often complicated due to the large number of parameters in the model. To deal with this complexity, we present two graphical methods of representing results. We conclude the chapter by introducing the conditional logit model, which allows characteristics of both the alternatives and the individual to vary.

Chapter 7: Count Outcomes begins with the Poisson and negative binomial regression models, including a test to determine which model is appropriate for your data. We also show how to incorporate differences in exposure time into the estimation. Next we consider interpretation both in terms of changes in the predicted rate and changes in the predicted probability of observing a given count. The last half of the chapter considers estimation and interpretation of zero-inflated count models, which are designed to account for the large number of zero counts found in many count outcomes.

Chapter 8 returns to issues that affect all models:

Chapter 8: Additional Topics deals with several topics, but the primary concern is with complications among independent variables. We consider the use of ordinal and nominal independent variables, nonlinearities among the independent variables, and interactions. The proper interpretation of the effects of these types of variables requires special adjustments to the commands considered in earlier chapters. We then comment briefly on how to modify our commands to work with other estimation commands. Finally, we discuss several features in Stata that we think make data analysis easier and more enjoyable.

### 1.5 What software do you need?

To get the most out of this book, you should read it while at a computer where you can experiment with the commands as they are introduced. We assume that you are using Stata 7 or later. If you are running Stata 6, most of the commands work, but some things must be done differently and the

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output will look slightly different. For details, see www.indiana.edu/~jsl650/spost.htm. If you are using Stata 5 or earlier, the commands that we have written will not work.

Advice to New Stata Users If you have never used Stata, you might find the instructions in this section to be confusing. It might be easier if you only skim the material now and return to it after you have read the introductory sections of Chapter 2.

### 1.5.1 Updating Stata 7

Before working through our examples in later chapters, we strongly recommend that you make sure that you have the latest version of wstata. exe and the official Stata ado-files. You should do this even if you have just installed Stata, since the CD that you received might not have the latest changes to the program. If you are connected to the Internet and are in Stata, you can update Stata by selecting Official Updates from the Help menu. Stata responds with the following screen:


This screen tells you the current dates of your files. By clicking on http://www.stata.com, you can update your files to the latest versions. We suggest that you do this every few months. Or, if you encounter something that you think is a bug in Stata or in our commands, it is a good idea to update your copy of Stata and see if the problem is resolved.

### 1.5.2 Installing SPost

From our point of view, one of the best things about Stata is how easy it is to add your own commands. This means that if Stata does not have a command you need or some command does not

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work the way you like, you can program the command yourself and it will work as if it were part of official Stata. Indeed, we have created a suite of programs, referred to collectively as SPost (for Stata Post-estimation Commands), for the post-estimation interpretation of regression models. These commands must be installed before you can try the examples in later chapters.

What is an ado-file? Programs that add commands to Stata are contained in files that end in the extension .ado (hence the name, ado-files). For example, the file prvalue.ado is the program for the command prvalue. Hundreds of ado-files are included with the official Stata package, but experienced users can write their own ado-files to add new commands. However, for Stata to use a command implemented as an ado-file, the ado-file must be located in one of the directories where Stata looks for ado-files. If you type the command sysdir, Stata lists the directories that Stata searches for ado-files in the order that it searches them. However, if you follow our instructions below, you should not have to worry about managing these directories.

## Installing SPost using net search

Installation should be simple, although you must be connected to the Internet. In Stata 7 or later, type net search spost. The net search command accesses an on-line database that StataCorp uses to keep track of user-written additions to Stata. Typing net search spost brings up the names and descriptions of several packages (a package is a collection of related files) in the Results Window. One of these packages is labeled spostado from http://www.indiana.edu/~jsl650/stata. The label is in blue, which means that it is a link that you can click on. ${ }^{2}$ After you click on the link, a window opens in the Viewer (this is a new window that will appear on your screen) that provides information about our commands and another link saying "click here to install." If you click on this link, Stata attempts to install the package. After a delay during which files are downloaded, Stata responds with one of the following messages:
installation complete means that the package has been successfully installed and that you can now use the commands. Just above the "installation complete" message, Stata tells you the directory where the files were installed.
all files already exist and are up-to-date means that your system already has the latest version of the package. You do not need to do anything further.
the following files exist and are different indicates that your system already has files with the same names as those in the package being installed, and that these files differ from those in the package. The names of those files are listed and you are given several options. Assuming that the files listed are earlier versions of our programs, you should select the option "Force installation replacing already-installed files". This might sound ominous, but it is not.

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1.5 What software do you need?

Since the files on our web site are the latest versions, you want to replace your current files with these new files. After you accept this option, Stata updates your files to newer versions.
cannot write in directory directory-name means that you do not have write privileges to the directory where Stata wants to install the files. Usually, this occurs only when you are using Stata on a network. In this case, we recommend that you contact your network administrator and ask if our commands can be installed using the instructions given above. If you cannot wait for a network administrator to install the commands or to give you the needed write access, you can install the programs to any directory where you have write permission, including a zip disk or your directory on a network. For example, suppose you want to install SPost to your directory called d: \username (which can be any directory where you have write access). You should use the following commands:
. cd d:\username
d:\username
. mkdir ado
. sysdir set PERSONAL "d:\username\ado"
. net set ado PERSONAL
. net search spost
(contacting http://www.stata.com)
Then, follow the installation instructions that we provided earlier for installing SPost. If you get the error "could not create directory" after typing mkdir ado, then you probably do not have write privileges to the directory.

If you install ado-files to your own directory, each time you begin a new session you must tell Stata where these files are located. You do this by typing sysdir set PERSONAL directory, where directory is the location of the ado-files you have installed. For example,
. sysdir set PERSONAL d:\username

## Installing SPost using net install

Alternatively, you can install the commands entirely from the Command Window. (If you have already installed SPost, you do not need to read this section.) While you are on-line, enter
. net from http://www.indiana.edu/~jsl650/stata/
The available packages will be listed. To install spostado, type
. net install spostado
net get can be used to download supplementary files (e.g., datasets, sample do-files) from our web site. For example, to download the package spostst4, type
. net get spostst4
These files are placed in the current working directory (see Chapter 2 for a full discussion of the working directory).

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### 1.5.3 What if commands do not work?

This section assumes that you have installed SPost, but some of the commands do not work. Here are some things to consider:

1. If you get an error message unrecognized command, there are several possibilities.
(a) If the commands used to work, but do not work now, you might be working on a different computer (e.g., a different station in a computing lab). Since user-written ado-files work seamlessly in Stata, you might not realize that these programs need to be installed on each machine you use. Following the directions above, install SPost on each computer that you use.
(b) If you sent a do-file that contains SPost commands to another person and they cannot get the commands to work, let them know that they need to install SPost.
(c) If you get the error message unrecognized command: strangename after typing one of our commands, where strangename is not the name of the command that you typed, it means that Stata cannot find an ancillary ado-file that the command needs. We recommend that you install the SPost files again.
2. If you are getting an error message that you do not understand, click on the blue return code beneath the error message for more information about the error (this only works in Stata 7 or later).
3. You should make sure that Stata is properly installed and up-to-date. Typing verinst will verify that Stata has been properly installed. Typing update query will tell you if the version you are running is up-to-date and what you need to type to update it. If you are running Stata over a network, your network administrator may need to do this for you.
4. Often, what appears to be a problem with one of our commands is actually a mistake you have made (we know, because we make them too). For example, make sure that you are not using $=$ when you should be using $==$.
5. Since our commands work after you have estimated a model, make sure that there were no problems with the last model estimated. If Stata was not successful in estimating your model, then our commands will not have the information needed to operate properly.
6. Irregular value labels can cause Stata programs to fail. We recommend using labels that are less than 8 characters and contain no spaces or special characters other than _'s. If your variables (especially your dependent variable) do not meet this standard, try changing your value labels with the label command (details are given in Section 2.15).
7. Unusual values of the outcome categories can also cause problems. For ordinal or nominal outcomes, some of our commands require that all of the outcome values are integers between 0 and 99. For these type of outcomes, we recommend using consecutive integers starting with 1.

In addition to this list, we recommend that you check our Frequently Asked Questions (FAQ) page at www.indiana.edu/~jsl650/spost.htm. This page contains the latest information on problems that users have encountered.

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### 1.5.4 Uninstalling SPost

Stata keeps track of the packages that it has installed, which makes it easy for you to uninstall them in the future. If you want to uninstall our commands, simply type: ado uninstall spostado.

### 1.5.5 Additional files available on the web site

In addition to the SPost commands, we have provided other packages that you might find useful. For example, the package called spostst 4 contains the do-files and datasets needed to reproduce the examples from this book. The package spostrm4 contains the do-files and datasets to reproduce the results from Long (1997). To obtain these packages, type net search spost and follow the instructions you will be given. Important: if a package does not contain ado-files, Stata will download the files to the current working directory. Consequently, you need to change your working directory to wherever you want the files to go before you select "click here to get." More information about working directories and changing your working directory is provided in Section 2.5.

### 1.6 Where can I learn more about the models?

There are many valuable sources for learning more about the regression models that are covered in this book. Not surprisingly, we recommend

Long, J. Scott. 1997. Regression Models for Categorical and Limited Dependent Variables. Thousand Oaks, CA: Sage Publications.

This book provides further details on all of the models discussed in the current book. In addition, we recommend the following:

Cameron, A. C. and P. K. Trivedi. 1998. Regression Analysis of Count Data. Cambridge: Cambridge University Press. This is the definitive reference for count models.

Greene, W. C. 2000. Econometric Analysis. 4th ed. New York: Prentice Hall. While this book focuses on models for continuous outcomes, several later chapters deal with models for categorical outcomes.

Hosmer, D. W., Jr., and S. Lemeshow. 2000. Applied Logistic Regression. 2d ed. New York: John Wiley \& Sons. This book, written primarily for biostatisticians and medical researchers, provides a great deal of useful information on logit models for binary, ordinal, and nominal outcomes. In many cases the authors discuss how their recommendations can be executed using Stata.

Powers, D. A. and Y. Xie. 2000. Statistical Methods for Categorical Data Analysis. San Diego: Academic Press. This book considers all of the models discussed in our book, with the exception of count models, and also includes loglinear models and models for event history analysis.

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# REGRESSION MODELS FOR CATEGORICAL DEPENDENT VARIABLES USING STATA 

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## 2 Introduction to Stata

This book is about estimating and interpreting regression models using Stata, and to earn our pay we must get to these tasks quickly. With that in mind, this chapter is a relatively concise introduction to Stata 7 for those with little or no familiarity with the package. Experienced Stata users can skip this chapter, although a quick reading might be useful. We focus on teaching the reader what is necessary to work the examples later in the book and to develop good working techniques for using Stata for data analysis. By no means are the discussions exhaustive; in many cases, we show you either our favorite approach or the approach that we think is simplest. One of the great things about Stata is that there are usually several ways to accomplish the same thing. If you find a better way than we have shown you, use it!

You cannot learn how to use Stata simply by reading. Accordingly, we strongly encourage you to try the commands as we introduce them. We have also included a tutorial in Section 2.17 that covers many of the basics of using Stata. Indeed, you might want to try the tutorial first and then read our detailed discussions of the commands.

While people who are new to Stata should find this chapter sufficient for understanding the rest of the book, if you want further instruction, look at the resources listed in Section 2.3. We also assume that you know how to load Stata on the computer you are using and that you are familiar with your computer's operating system. By this, we mean that you should be comfortable copying and renaming files, working with subdirectories, closing and resizing windows, selecting options with menus and dialog boxes, and so on.
(Continued on next page)

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### 2.1 The Stata interface



Figure 2.1: Opening screen in Stata for Windows.

When you launch Stata, you will see a screen in which several smaller windows are located within the larger Stata window, as shown in Figure 2.1. This screen shot is for Windows using the default windowing preferences. If the defaults have been changed or you are running Stata under Unix or the MacOS, your screen will look slightly different. ${ }^{1}$ Figure 2.2 shows what Stata looks like after several commands have been entered and data have been loaded into memory. In both figures, four windows are shown. These are
(Continued on next page)

[^3]

Figure 2.2: Example of Stata windows after several commands have been entered and data have been loaded.

The Command Window is where you enter commands that are executed when you press Enter. As you type commands, you can edit them at any time before pressing Enter. Pressing PageUp brings the most recently used command into the Command Window; pressing PageUp again retrieves the command before that; and so on. Once a command has been retrieved to the Command Window, you can edit it and press Enter to run the modified command.

The Results Window contains output from the commands entered in the Command Window. The Results Window also echoes the command that generated the output, where the commands are preceded by a "." as shown in Figure 2.2. The scroll bar on the right lets you scroll back through output that is no longer on the screen. Only the most recent output is available this way; earlier lines are lost unless you have saved them to a log file (discussed below).

The Review Window lists the commands that have been entered from the Command Window. If you click on a command in this window, it is pasted into the Command Window where you can edit it before execution of the command. If you double-click on a command in the Review Window, it is pasted into the Command Window and immediately executed.

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The Variables Window lists the names of variables that are in memory, including both those loaded from disk files and those created with Stata commands. If you click on a name, it is pasted into the Command Window.

The Command and Results Windows illustrate the important point that Stata is primarily command based. This means that you tell Stata what to do by typing commands that consist of a single line of text followed by pressing Enter. ${ }^{2}$ This contrasts with programs where you primarily point-and-click options from menus and dialog boxes. To the uninitiated, this austere approach can make Stata seem less "slick" or "user friendly" than some of its competitors, but it affords many advantages for data analysis. While it can take longer to learn Stata, once you learn it, you should find it much faster to use. If you currently prefer using pull-down menus, stick with us and you will likely change your mind.

There are also many things that you can do in Stata by pointing and clicking. The most important of these are presented as icons on the toolbar at the top of the screen. While we on occasion mention the use of these icons, for the most part we stick with text commands. Indeed, even if you do click on an icon, Stata shows you how this could be done with a text command. For example, if you click on the browse button 国, Stata opens a spreadsheet for examining your data. Meanwhile, ". browse" is written to the Results Window. This means that instead of clicking the icon, you could have typed browse. Overall, not only is the range of things you can do with menus limited, but almost everything you can do with the mouse can also be done with commands, and often more efficiently. It is for this reason, and also because it makes things much easier to automate later, that we describe things mainly in terms of commands. Even so, readers are encouraged to explore the tasks available through menus and the toolbar and to use them when preferred.

## Changing the Scrollback Buffer Size

How far back you can scroll in the Results Window is controlled by the command

```
set scrollbufsize #
```

where $10,000 \leq \# \leq 500,000$. By default, the buffer size is 32,000 .

## Changing the Display of Variable Names in the Variable Window

The Variables Window displays both the names of variables in memory and their variable labels. By default, 32 columns are reserved for the name of the variable. The maximum number of characters to display for variable names is controlled by the command

```
set varlabelpos #
```

where $8 \leq \# \leq 32$. By default, the size is 32 . In Figure 2.2, none of the variable labels are shown since the 32 columns take up the entire width of the window. If you use short variable names, it is useful to set varlabelspos to a smaller number so that you can see the variable labels.

[^4]Tip: Changing Defaults We both prefer a larger scroll buffer and less space for variable names. We could enter the commands: set scrollbufsize 150000 and set varlabelpos 14 at the start of each Stata session, but it is easier to add the commands to profile.do, a file that is automatically run each time Stata begins. We show you how to do this in Chapter 8.

### 2.2 Abbreviations

Commands and variable names can often be abbreviated. For variable names, the rule is easy: any variable name can be abbreviated to the shortest string that uniquely identifies it. For example, if there are no other variables in memory that begin with a, then the variable age can be abbreviated as a or ag. If you have the variables income and income2 in your data, then neither of these variable names can be abbreviated.

There is no general rule for abbreviating commands, but, as one would expect, it is typically the most common and general commands whose names can be abbreviated. For example, four of the most often used commands are summarize, tabulate, generate, and regress, and these can be abbreviated as su, ta, g, and reg, respectively. From now on, when we introduce a Stata command that can be abbreviated, we underline the shortest abbreviation (e.g., generate). But, while very short abbreviations are easy to type, when you are getting started the short abbreviations can be confusing. Accordingly, when we use abbreviations, we stick with at least three-letter abbreviations.

### 2.3 How to get help

### 2.3.1 On-line help

If you find our description of a command incomplete or if we use a command that is not explained, you can use Stata's on-line help to get further information. The help, search, and net search commands, described below, can be typed in the Command Window with results displayed in the

Results Window. Or, you can open the Viewer by clicking on $<6$ Command where you can type commands such as help. The Viewer is particularly useful for reading help files that are long. Here is further information on commands for getting help:
help lists a shortened version of the documentation in the manual for any command. You can even type help help for help on using help. When using help for commands that can be abbreviated, you must use the full name of the command (e.g., help generate, not help gen). The output from help often makes reference to other commands, which are shown in blue. In Stata 7 or later, anything in the Results Window that is in blue type is a link that you can click on. In this case, clicking on a command name in blue type is the same as typing help for that command.

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search is handy when you do not know the specific name of the command that you need information about. search word [word...] searches Stata's on-line index and lists the entries that it finds. For example, search gen lists information on generate, but also many related commands. Or, if you want to run a truncated regression model but can not remember the name of the command, you could try search truncated to get information on a variety of possible commands. These commands are listed in blue, so you can click on the name and details appear in the Viewer. If you keep your version of Stata updated on the Internet (see Section 1.5 for details), search also provides current information from the Stata web site FAQ (i.e., Frequently Asked Questions) and articles in the Stata Journal (often abbreviated as SJ).
net search is a command that searches a database at www. stata.com for information about commands written by users (accordingly, you have to be on-line for this command to work). This is the command to use if you want information about something that is not part of official Stata. For example, when you installed the SPost commands, you used net search spost to find the links for installation. To get a further idea of how net search works, try net search truncated and compare the results to those from search truncated.

Tip: Help with error messages Error messages in Stata are terse and sometimes confusing. While the error message is printed in red, errors also have a return code (e.g., $r(199)$ ) listed in blue. Clicking on the return code provides a more detailed description of the error.

### 2.3.2 Manuals

The Stata manuals are extensive, and it is worth taking an hour to browse them to get an idea of the many features in Stata. In general, we find that learning how to read the manuals (and use the help system) is more efficient than asking someone else, and it allows you to save your questions for the really hard stuff. For those new to Stata, we recommend the Getting Started manual (which is specific to your platform) and the first part of the User's Guide. As you become more acquainted with Stata, the Reference Manual will become increasingly valuable for detailed information about commands, including a discussion of the statistical theory related to the commands and references for further reading.

### 2.3.3 Other resources

The User's Guide also discusses additional sources of information about Stata. Most importantly, the Stata web site (www.stata.com) contains many useful resources, including links to tutorials and an extensive FAQ section that discusses both introductory and advanced topics. You can also get information on the NetCourses offered by Stata, which are four- to seven-week courses offered over the Internet. Another excellent set of on-line resources is provided by UCLA's Academic Technology Services at www. ats.ucla.edu/stat/stata/.

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There is also a Statalist listserv that is independent of StataCorp, although many programmers/statisticians from StataCorp participate. This list is a wonderful resource for information on Stata and statistics. You can submit questions and will usually receive answers very quickly. Monitoring the listserv is also a quick way to pick up insights from Stata veterans. For details on joining the list, go to www. stata.com, follow the link to User Support, and click on the link to Statalist.

### 2.4 The working directory

The working directory is the default directory for any file operations such as using data, saving data, or logging output. If you type cd in the Command Window, Stata displays the name of the current working directory. To load a data file stored in the working directory, you just type use filename (e.g., use binlfp2). If a file is not in the working directory, you must specify the full path (e.g., use d: \spostdata $\backslash$ examples $\backslash$ binlfp2).

At the beginning of each Stata session, we like to change our working directory to the directory where we plan to work, since this is easier than repeatedly entering the path name for the directory. For example, typing cd d: \spostdata changes the working directory to $\mathrm{d}: \backslash$ spostdata. If the directory name includes spaces, you must put the path in quotation marks (e.g., cd "d: $\backslash$ my work $\$ ").

You can list the files in your working directory by typing dir or ls, which are two names for the same command. With this command you can use the $*$ wildcard. For example, dir $*$.dta lists all files with the extension .dta.

### 2.5 Stata file types

Stata uses and creates many types of files, which are distinguished by extensions at the end of the filename. The extensions used by Stata are

$$
\begin{array}{ll}
\text {.ado } & \text { Programs that add commands to Stata, such as the SPost commands. } \\
\text {.do } & \text { Batch files that execute a set of Stata commands. } \\
\text {.dta } & \text { Data files in Stata's format. } \\
\text {.gph } & \text { Graphs saved in Stata's proprietary format. } \\
\text {.hlp } & \begin{array}{l}
\text { The text displayed when you use the help command. For example, } \\
\text { fitstat.hlp has help for fitstat. }
\end{array} \\
& \begin{array}{l}
\text { Output saved as plain text by the log using command. }
\end{array} \\
\text {.log } & \text { Output saved in the SMCL format by the log using command. } \\
\text {.smcl } & \text { Graphs saved as Windows Metafiles. }
\end{array}
$$

The most important of these for a new user are the $. \operatorname{smcl}, . \log ,$. dta, and. do files, which we now discuss.

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### 2.6 Saving output to log files

Stata does not automatically save the output from your commands. To save your output to print or examine later, you must open a log file. Once a log file is opened, both the commands and the output they generate are saved. Since the commands are recorded, you can tell exactly how the results were obtained. The syntax for the log command is

```
log using filename [, append replace [text | smcl]]
```

By default, the log file is saved to your working directory. You can save it to a different directory by typing the full path (e.g., log using $d: \backslash$ project $\backslash m y l o g, ~ r e p l a c e) . ~$

## Options

replace indicates that you want to replace the log file if it already exists. For example, log using mylog creates the file mylog.smcl. If this file already exists, Stata generates an error message. So, you could use log using mylog, replace and the existing file would be overwritten by the new output.
append means that if the file exists, new output should be added to the end of the existing file.
smcl is the default option that requests that the log is written using the Stata Markup and Control Language (SMCL) with the file suffix .smcl. SMCL files contain special codes that add solid horizontal and vertical lines, bold and italic typefaces, and hyperlinks to the Result Window. The disadvantage of SMCL is that the special features can only be viewed within Stata. If you open a SMCL file in a text editor, your results will appear amidst a jumble of special codes.
text specifies that the log should be saved as plain text (ASCII), which is the preferred format for loading the log into a text editor for printing. Instead of adding the text option, such as log using mywork, text, you can specify plain text by including the.$l o g$ extension. For example, log using mywork.log.

Tip: Plain text logs by default We both prefer plain text for output rather than SMCL. Typing set logtype text at the beginning of a Stata session makes plain text the default for log files. In Chapter 8, we discuss using the profile.do file to have Stata run certain commands every time it launches. Both of us include set logtype text in our profile.do.

### 2.6.1 Closing a log file

To close a log file, type
. $\log$ close
Also, when you exit Stata, the log file closes automatically. Since you can only have one log file open at a time, any open log file must be closed before you can open a new one.

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### 2.6.2 Viewing a log file

Regardless of whether a $\log$ file is open or closed, a log file can be viewed by selecting File $\rightarrow$ Log $\rightarrow$ View from the menu, and the log file will be displayed in the Viewer. When in the Viewer, you can print the $\log$ by selecting File $\rightarrow$ Print Viewer... You can also view the log file by clicking on 88 , which opens the log in the Viewer. If the Viewer window "gets lost" behind other windows, you can click on to bring the Viewer to the front.

### 2.6.3 Converting from SMCL to plain text or PostScript

If you want to convert a log file in SMCL format to plain text, you can use the translate command. For example,

```
. translate mylog.smcl mylog.log, replace
(file mylog.log written in .log format)
```

tells Stata convert the SMCL file mylog.smcl to a plain text file called mylog.log. Or, you can convert a SMCL file to a PostScript file, which is useful if you are using $\mathrm{T}_{\mathrm{E}} \mathrm{X}$ or $\mathrm{ETE}_{\mathrm{E}} \mathrm{X}$ or if you want to convert your output into Adobe's Portable Document Format. For example,

```
. translate mylog.smcl mylog.ps, replace
```

(file mylog.ps written in .ps format)

Converting can also be done via the menus by selecting File $\rightarrow$ Log $\rightarrow$ Translate.

### 2.7 Using and saving datasets

### 2.7.1 Data in Stata format

Stata uses its own data format with the extension .dta. The use command loads such data into memory. Pretend we are working with the file nomocc2.dta in directory d: \spostdata. We can load the data by typing
. use d:\spostdata\nomocc2, clear
where the . dta extension is assumed by Stata. The clear option erases all data currently in memory and proceeds with loading the new data. Stata does not give an error if you include clear when there is no data in memory. If $\mathrm{d}: \backslash$ spostdata was our working directory, we could use the simpler command
. use nomocc2, clear
If you have changed the data by deleting cases, merging in another file, or creating new variables, you can save the file with the save command. For example,
. save d:\spostdata\nomocc3, replace
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where again we did not need to include the . dta extension. Also notice that we saved the file with a different name so that we can use the original data later. The replace option indicates that if the file nomocc3.dta already exists, Stata should overwrite it. If the file does not already exist, replace is ignored. If $\mathrm{d}: \backslash$ spostdata was our working directory, we could save the file with
. save nomocc3, replace
By default, save stores the data in a format that can only be read by Stata 7 or later. But, if you add the option old, the data is written so that it can be read with Stata 6. However, if your data contain variable names or value labels longer than 8 characters, features that only became available in Stata 7, Stata refuses to save the file with the old option.

Tip: compress before saving Before saving a file, run the compress command. compress checks each variable to determine if it can be saved in a more compact form. For instance, binary variables fit into the byte type, which takes up only one-fourth of the space of the float type. If you run compress, it might make your data file much more compact, and at worst it will do no harm.

### 2.7.2 Data in other formats

To load data from another statistical package, such as SAS or SPSS, you need to convert it into Stata's format. The easiest way to do this is with a conversion program such as Stat/Transfer (www.stattransfer.com) or DBMS/Copy (www.conceptual.com). We recommend obtaining one of these programs if you are using more than one statistical package or if you often share data with others who use different packages.

Alternatively, but less conveniently, most statistical packages allow you to save and load data in ASCII format. You can load ASCII data with the infile or infix commands and export it with the outfile command. The Reference Manual entry for infile contains an extensive discussion that is particularly helpful for reading in ASCII data, or, you can type help infile.

### 2.7.3 Entering data by hand

Data can also be entered by hand using a spreadsheet-style editor. While we do not recommend using the editor to change existing data (since it is too easy to make a mistake), we find that it is very useful for entering small datasets. To enter the editor, click on 費 or type edit on the command line. The Getting Started manual has a tutorial for the editor, but most people who have used a spreadsheet before will be immediately comfortable with the editor.

As you use the editor, every change that you make to the data is reported in the Results Window and is captured by the $\log$ file if it is open. For example, if you change age for the fifth observation to 32 , Stata reports replace age $=32$ in 5 . This tells you that instead of using the editor, you could

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have changed the data with a replace command. When you close the editor, you are asked if you really want to keep the changes or revert to the unaltered data.

### 2.8 Size limitations on datasets*

If you get the error message $r$ (900) : no room to add more observations when trying to load a dataset or the message r (901) : no room to add more variables when trying to add a new variable, you may need to allocate more memory. Typing memory shows how much memory Stata has allocated and how much it is using. You can increase the amount of memory by typing set memory \#k (for KB) or \#m (for MB). For example, set memory 32000k or set memory 32 m sets the memory to $32 \mathrm{MB} .{ }^{3}$ Note that if you have variables in memory, you must type clear before you can set the memory.

If you get the error $r$ (1000) : system limit exceeded--see manual when you try to load a dataset or add a variable, your dataset might have too many variables or the width of the dataset might be too large. Stata is limited to a maximum of 2047 variables, and the dataset can be no more than 8192 units wide (a binary variable has width 1, a double precision variable width 8 , and a string variable as much as width 80). File transfer programs such as Stat/Transfer and DBMS/Copy can drop specified variables and optimize variable storage. You can use these programs to create multiple datasets that each only contain the variables necessary for specific analyses.

## 2.9 do-files

You can execute commands in Stata by typing one command at a time into the Command Window and pressing Enter, as we have been doing. This interactive mode is useful when you are learning Stata, exploring your data, or experimenting with alternative specifications of your regression model. Alternatively, you can create a text file that contains a series of commands and then tell Stata to execute all of the commands in that file, one after the other. These files, which are known as do-files because they use the extension. do, have the same function as "syntax files" in SPSS or "batch files" in other statistics packages. For more serious or complex work, we always use do-files since they make it easier to redo the analysis with small modifications later and because they provide an exact record of what has been done.

To get an idea of how do-files work, consider the file example. do saved in the working directory:

```
log using example, replace text
use binlfp2, clear
tabulate hc wc, row nolabel
log close
```

[^5]To execute a do-file, you execute the command

## do dofilename

from the Command Window. For example, do example tells Stata to run each of the commands in example.do. (Note: If the do-file is not in the working directory, you need to add the directory, such as do d:\spostdata\example.) Executing example.do begins by opening the log example.log, then loads binlfp2.dta, and finally constructs a table with hc and wc. Here is what the output looks like:


### 2.9.1 Adding comments

Stata treats lines that begin with an asterisk $*$ or are located between a pair of $/ *$ and $* /$ as comments that are simply echoed to the output. For example, the following do-file executes the same commands as the one above, but includes comments:

```
/*
==> short simple do-file
==> for didactic purposes
*/
log using example, replace /* this comment is ignored */
* next we load the data
use binlfp2, clear
* tabulate husband's and wife's education
tabulate hc wc, row nolabel
* close up
log close
* make sure there is a cr at the end!
```

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If you look at the do-files on our web site that reproduce the examples in this book, you will see that we use many comments. They are extremely helpful if others will be using your do-files or log files or if there is a chance that you will use them again at a later time.

### 2.9.2 Long lines

Sometimes you need to execute a command that is longer than the text that can fit onto a screen. If you are entering the command interactively, the Command Window simply pushes the left part of the command off the screen as space is needed. Before entering a long command line in a do-file, however, you can use \#delimit ; to tell Stata to interpret ";" as the end of a command. After the long command is entered, you can enter \#delimit cr to return to using the carriage return as the end-of-line delimiter. For example,

```
#delimit ;
recode income91 1=500 2=1500 3=3500 4=4500 5=5500 6=6500 7=7500 8=9000
9=11250 10=13750 11=16250 12=18750 13=21250 14=23750 15=27500 16=32500
17=37500 18=45000 19=55000 20=67500 21=75000 *=. ;
#delimit cr
```

Tip: Long lines Instead of the \#delimit command, we could have used /**/ to comment out the carriage returns before the end of the command. Since Stata ignores everything between $/ *$ */, it ignores the carriage returns. For example,

```
recode income91 1=500 2=1500 3=3500 4=4500 5=5500 6=6500 7=7500 8=9000 /*
*/ 9=11250 10=13750 11=16250 12=18750 13=21250 14=23750 15=27500 16=32500 /*
*/ 17=37500 18=45000 19=55000 20=67500 21=75000 *=.
```


### 2.9.3 Stopping a do-file while it is running

If you are running a command or a do-file that you want to stop before it completes execution, click
 or press Ctrl-Break.

### 2.9.4 Creating do-files

## Using Stata's do-file editor

do-files can be created with Stata's built-in do-file editor. To use the editor, enter the command doedit to create a file to be named later or doedit filename to create or edit a file named file-
name.do. Alternatively, you can click on The do-file editor is easy to use and works like most text editors (see Getting Started for further details). After you finish your do-file, select Tools $\rightarrow$ Do
to execute the file or click on 氩 $\downarrow$ 。
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## Using other editors to create do-files

Since do-files are plain text files, you can create do-files with any program that creates text files. Specialized text editors work much better than word processors such as WordPerfect or Microsoft Word. Among other things, with word processors it is easy to forget to save the file as plain text. Our own preference for creating do-files is TextPad (www.textpad.com), which runs in Windows. This program has many features that make it faster to create do-files. For example, you can create a "clip library" that contains frequently entered material and you can obtain a syntax file from our web site that provides color coding of reserved words for Stata. TextPad also allows you to have several different files open at once, which is often handy for complex data analyses.

If you use an editor other than Stata's built-in editor, you cannot run the do file by clicking on an icon or selecting from a menu. Instead, you must switch from your editor and then enter the command do filename.

Warning Stata executes commands when it encounters a carriage return (i.e., the Enter key). If you do not include a carriage return after the last line in a do-file, that last line will not be executed. TextPad has a feature to enter that final, pesky carriage return automatically. To set this option in TextPad 4, select the option "Automatically terminate the last line of the file" in the preferences for the editor.

### 2.9.5 A recommended structure for do-files

This is the basic structure that we recommend for do-files:

```
* including version number ensures compatibility with later Stata releases
version }
* if a log file is open, close it
capture log close
* don't pause when output scrolls off the page
set more off
* log results to file myfile.log
log using myfile, replace text
** myfile.do - written 19 jan 2001 to illustrate do-files
*
** your commands go here
*
* close the log file.
log close
```

While the comments (that you can remove) should explain most of the file, there are a few points that we need to explain.

- The version 7 command indicates that the program was written for use in Stata 7. This command tells any future version of Stata that you want the commands that follow to work just as they did when you ran them in Stata 7. This prevents the problem of old do-files not running correctly in newer releases of the program.

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- The command capture log close is very useful. Suppose you have a do-file that starts with log using mylog, replace. You run the file and it "crashes" before reaching log close, which means that the log file remains open. If you revise the do-file and run it again, an error is generated when it tries to open the $\log$ file, because the file is already open. The prefix capture tells Stata not to stop the do-file if the command that follows produces an error. Accordingly, capture log close closes the log file if it is open. If it is not open, the error generated by trying to close an already closed file is ignored.

Tip: The command cmdlog is very much like the log command, except that it creates a text file with extension .txt that saves all subsequent commands that are entered in the Command Window (it does not save commands that are executed within a do-file). This is handy because it allows you to use Stata interactively and then make a do-file based on what you have done. You simply load the cmdlog that you saved, rename it to newname.do, delete commands you no longer want, and execute the new do-file. Your interactive session is now documented as a do-file. The syntax for opening and closing cmdlog files is the same as the syntax for log (i.e., cmdlog using to open and cmdlog close to close), and you can have log and cmdlog files open simultaneously.

### 2.10 Using Stata for serious data analysis

Voltaire is said to have written Candide in three days. Creative work often rewards such inspired, seat-of-the-pants, get-the-details-later activity. Data management does not. Instead, effective data management rewards forethought, carefulness, double- and triple-checking of details, and meticulous, albeit tedious, documentation. Errors in data management are astonishingly (and painfully) easy to make. Moreover, tiny errors can have disastrous implications that can cost hours and even weeks of work. The extra time it takes to conduct data management carefully is rewarded many times over by the reduced risk of errors. Put another way, it helps prevent you from getting incorrect results that you do not know are incorrect. With this in mind, we begin with some broad, perhaps irritatingly practical, suggestions for doing data analysis efficiently and effectively.

1. Ensure replicability by using do-files and log files for everything. For data analysis to be credible, you must be able to reproduce entirely and exactly the trail from the original data to the tables in your paper. Thus, any permanent changes you make to the data should be made by running do-files rather than in the interactive mode. If you work interactively, be sure that the first thing you do is open a log or cmdlog file. Then when you are done, you can use these files to create a do-file to reproduce your interactive results.
2. Document your do-files. The reasoning that is obvious today can be baffling in six months. We use comments extensively in our do-files, which are invaluable for remembering what we did and why we did it.

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3. Keep a research log. For serious work, you should keep a diary that includes a description of every program you run, the research decisions that are being made (e.g., the reasons for recoding a variable in a particular way), and the files that are created. A good research log allows you to reproduce everything you have done starting only with the original data. We cannot overemphasize how helpful such notes are when you return to a project that was put on hold, when you are responding to reviewers, or when you moving on to the next stage of your research.
4. Develop a system for naming files. Usually it makes the most sense to have each do-file generate one log file with the same prefix (e.g., clean_data.do, clean_data.log). Names are easiest to organize when brief, but they should be long enough and logically related enough to make sense of the task the file does. ${ }^{4}$ Scott prefers to keep the names short and organized by major task (e.g., recode01.do, recode02.do), while Jeremy likes longer names (e.g., make_income_vars.do, make_educ_variables.do). Either is fine as long as it works for you.
5. Use new names for new variables and files. Never change a data set and save it with the original name. If you drop three variables from pcoms1.dta and create two new variables, call the new file pcoms2.dta. When you transform a variable, give it a new name rather than simply replacing or recoding the old variable. For example, if you have a variable workmom with a five-point attitude scale and you want to create a binary variable indicating positive and negative attitudes, create a new variable called workmom2.
6. Use labels and notes. When you create a new variable, give it a variable label. If it is a categorical variable, assign value labels. You can add a note about the new variable using the notes command (described below). When you create a new dataset, you can also use notes to document what it is.
7. Double-check every new variable. Cross-tabulating or graphing the old variable and the new variable are often effective for verifying new variables. As we describe below, using list with a subset of cases is similarly effective for checking transformations. At the very least, be sure to look carefully at the frequency distributions and summary statistics of variables in your analysis. You would not believe how many times puzzling regression results turn out to involve miscodings of variables that would have been immediately apparent by looking at the descriptive statistics.
8. Practice good archiving. If you want to retain hard copies of all your analyses, develop a system of binders for doing so rather than a set of intermingling piles on one's desk. Back up everything. Make off-site backups and/or keep any on-site backups in a fireproof box. Should cataclysm strike, you will have enough other things to worry about without also having lost months or years of work.

[^6]
### 2.11 The syntax of Stata commands

Think about the syntax of commands in everyday, spoken English. They usually begin with a verb telling the other person what they are supposed to do. Sometimes the verb is the entire command: "Help!" or "Stop!" Sometimes the verb needs to be followed by an object that indicates who or what the verb is to be performed on: "Help Dave!" or "Stop the car!" In some cases, the verb is followed by a qualifier that gives specific conditions under which the command should or should not be performed: "Give me a piece of pizza if it doesn't have mushrooms" or "Call me if you get home before nine". Verbs can also be followed by adverbs that specify that the action should be performed in some way that is different from how it might normally be, such as when a teacher commands her students to "Talk clearly" or "Walk single file".

Stata follows an analogous logic, albeit with some additional wrinkles that we introduce later. The basic syntax of a command has four parts:

1. Command: What action do you want performed?
2. Names of Variables, Files, or other Objects: On what things is the command to be performed?
3. Qualifier on Observations: On which observations should the command be performed?
4. Options: What special things should be done in executing the command?

All commands in Stata require the first of these parts, just as it is hard to issue spoken commands without a verb. Each of the other three parts can be required, optional, or not allowed, depending on the particular command and circumstances. Here is an example of a command that features all four parts and uses binlfp2.dta that we loaded earlier:
. tabulate hc wc if age>40, row

| $\begin{array}{r} \text { Husband } \\ \text { College: } \\ 1=\text { yes } 0=\text { no } \end{array}$ | Wife Col NoCol | $\begin{array}{r} \text { e: } \begin{array}{r} 1=y e s \\ 0=n o \\ \text { College } \end{array} \end{array}$ | Total |
| :---: | :---: | :---: | :---: |
| NoCol | 263 | 23 | 286 |
|  | 91.96 | 8.04 | 100.00 |
| College | 58 | 91 | 149 |
|  | 38.93 | 61.07 | 100.00 |
| Total | 321 | 114 | 435 |
|  | 73.79 | 26.21 | 100.00 |

tabulate is a command for making one- or two-way tables of frequencies. In this example, we want a two-way table of the frequencies of variables hc by wc. By putting hc first, we make this the row variable and wc the column variable. By specifying if age $>40$, we specify that the frequencies should only include observations for those older than 40. The option row indicates that row percentages should be printed as well as frequencies. These allow us to see that in $61 \%$ of the cases in which the husband had attended college the wife had also done so, while wives had attended college only in $8 \%$ of cases in which the husband had not. Notice the comma preceding row: whenever options are specified, they are at the end of the command with a single comma to indicate where the list of options begins. The precise ordering of multiple options after the comma is never important.
Next, we provide more information on each of the four components.
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### 2.11.1 Commands

Commands define the tasks that Stata is to perform. A great thing about Stata is that the set of commands is deliciously open-ended. It expands not just with new releases of Stata, but also when users add their own commands, such as our SPost commands. Each new command is stored in its own file, ending with the extension .ado. Whenever Stata encounters a command that is not in its built-in library, it searches various directories for the appropriate ado-file. The list of the directories it searches (and the order that it searches them) can be obtained by typing adopath.

### 2.11.2 Variable lists

Variable names are case-sensitive. For example, you could have three different variables named income, Income, and inCome. Of course, this is not a good idea since it leads to confusion. To keep life simple, we stick exclusively to lowercase names. Starting with Stata 7, Stata allows variable names up to 32 characters long, compared to the 8 character maximum imposed by earlier versions of Stata and many other statistics packages. In practice, we try not to give variables names longer than 8 characters, as this makes it easier to share data with people who use other packages. Additionally, we recommend using short names because longer variable names become unwieldy to type. (Although variable names can be abbreviated to whatever initial set of characters identifies the variable uniquely, we worry that too much reliance on this feature might cause one to make mistakes.)

If you do not list any variables, many commands assume that you want to perform the operation on every variable in the dataset. For example, the summarize command provides summary statistics on the listed variables:
. sum age inc k5

| Variable | Obs | Mean | Std. Dev. | Min | Max |
| ---: | ---: | ---: | ---: | ---: | ---: |
| age | 753 | 42.53785 | 8.072574 | 30 | 60 |
| inc | 753 | 20.12897 | 11.6348 | -.0290001 | 96 |
| k5 | 753 | .2377158 | .523959 | 0 | 3 |

Alternatively, we could get summary statistics on every variable in our dataset by just typing

| . sum |
| :--- |
| Variable |
| Obs |
| lfp |

You can also select all variables that begin or end with the same letter(s) using the wildcard operator *. For example,

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. sum k*

| Variable | Obs | Mean | Std. Dev. | Min | Max |
| ---: | ---: | ---: | ---: | ---: | ---: |
| k5 | 753 | .2377158 | .523959 | 0 | 3 |
| k618 | 753 | 1.353254 | 1.319874 | 0 | 8 |

### 2.11.3 if and in qualifiers

Stata has two qualifiers that restrict the sample that is analyzed: if and in. in performs operations on a range of consecutive observations. Typing sum in 20/100 gives summary statistics based only on the 20th through 100th observations. in restrictions are dependent on the current sort order of the data, meaning that if you resort your data, the 81 observations selected by the restriction sum in $20 / 100$ might be different. ${ }^{5}$

In practice, if conditions are used much more often than in conditions. if restricts the observations to those that fulfill a specified condition. For example, sum if age $<50$ provides summary statistics for only those observations where age is less than 50 . Here is a list of the elements that can be used to construct logical statements for selecting observations with if:

| Operator | Definition | Example |
| :--- | :--- | :--- |
| $==$ | equal to | if female $==1$ |
| $\sim=$ | not equal to | if femal ${ }^{\sim}=1$ |
| $>$ | greater than | if age $>20$ |
| $>=$ | greater than or equal to | if age $>=21$ |
| $<$ | less than | if age $<66$ |
| $<=$ | less than or equal to | if age $<=65$ |
| $\&$ | and | if age $==21 \&$ female $==1$ |
| $\mid$ | or | if age $==21 \mid$ educ $>16$ |

There are two important things to note about the if qualifier:

1. To specify a condition to test, you use a double equal sign (e.g., sum if female==1). When assigning a value to something, such as when creating a new variable, you use a single equal sign (e.g., gen newvar=1). Putting these examples together, results in gen newvar=1 if female==1.
2. Stata treats missing cases as positive infinity when evaluating if expressions. In other words, if you type sum ed if age $>50$, the summary statistics for ed are calculated on all observations where age is greater than 50 , including cases where the value of age is missing. You must be careful of this when using if with $>$ or $>=$ expressions. If you type sum ed if age ${ }^{\sim}=$., Stata gives summary statistics for cases where age is not missing (Note: . means missing). Entering sum ed if age $>50$ \& age ${ }^{\sim}=$. provides summary statistics for those cases where age is greater than 50 and is not missing.
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## Examples of if qualifier

If we wanted summary statistics on income for only those respondents who were between the ages of 25 and 65 , we would type
. sum income if age $>=25$ \& age<=65
If we wanted summary statistics on income for only female respondents who were between the ages of 25 and 65 , we would type
. sum income if age>=25 \& age<=65 \& female==1
If we wanted summary statistics on income for the remaining female respondents, that is, those who are younger than 25 or older than 65 , we would type
. sum income if (age<25 | age>65) \& age ${ }^{\sim}=$. \& female==1
Notice that we need to include \& age ${ }^{\sim}=$. since Stata treats missing codes as positive infinity. The condition (age $<25 \mid$ age $>65$ ) would otherwise include those cases for which age is missing.

### 2.11.4 Options

Options are set off from the rest of the command by a comma. Options can often be abbreviated, although whether and how they can be abbreviated varies across commands. In this book we rarely cover all of the available options available for any given command, but you can check the manual or use help for further options that might be useful for your analyses.

### 2.12 Managing data

### 2.12.1 Looking at your data

There are two easy ways to look at your data.
browse opens a spreadsheet in which you can scroll to look at the data, but you cannot change the data. You can look and change data with the edit command, but this is risky. We much prefer making changes to our data using do files, even when we are only changing the value of one variable for one observation. The browser is also available by clicking on $\%$, while the data editor is available by clicking on 罪.
list creates a list of values of specified variables and observations. if and in qualifiers can be used to look at just a portion of the data, which is sometimes useful for checking that transformations of variables are correct. For example, if you want to confirm that the variable lninc has been correctly constructed as the natural log of inc, typing list inc lninc in $1 / 20$ lets you see the values of inc and lninc for the first 20 observations.

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### 2.12.2 Getting information about variables

There are several methods for obtaining basic information about your variables. Here are five commands that we find useful. Which one you use depends in large part on the kind and level of detail you need.
describe provides information on the size of the dataset and the names, labels, and types of variables. For example,

| ```Contains data obs: vars: size:``` | $\begin{gathered} \text { from bin } \\ 753 \\ 8 \\ 13,554 \end{gathered}$ | $\begin{aligned} & \text { nlfp2.dta } \\ & \text { (99.8\% of } \end{aligned}$ | memory free) | Data from 1976 PSID-T Mroz 15 Jan 2001 15:23 (_dta has notes) |
| :---: | :---: | :---: | :---: | :---: |
| variable name | storage type | display <br> format | value <br> label | variable label |
| lfp | byte | $\% 9.0 \mathrm{~g}$ | lfplbl | Paid Labor Force: 1=yes 0=no |
| k5 | byte | $\% 9.0 \mathrm{~g}$ |  | \# kids < 6 |
| k618 | byte | $\% 9.0 \mathrm{~g}$ |  | \# kids 6-18 |
| age | byte | $\% 9.0 \mathrm{~g}$ |  | Wife's age in years |
| wc | byte | $\% 9.0 \mathrm{~g}$ | collbl | Wife College: 1=yes 0=no |
| hc | byte | $\% 9.0 \mathrm{~g}$ | collbl | Husband College: 1=yes 0=no |
| lwg | float | $\% 9.0 \mathrm{~g}$ |  | Log of wife's estimated wages |
| inc | float | \%9.0g |  | Family income excluding wife's |

Sorted by: lfp
summarize provides summary statistics. By default, summarize presents the number of nonmissing observations, the mean, the standard deviation, the minimum values, and the maximum. Adding the detail option includes additional information. For example,

```
. sum age, detail
```

| Wife's age in years $^{c}$ |  |  |  |  |
| ---: | ---: | ---: | :--- | ---: |
|  | Percentiles | Smallest |  |  |
| $1 \%$ | 30 | 30 |  | 753 |
| $5 \%$ | 30 | 30 | Obs | 753 |
| $10 \%$ | 32 | 30 | Sum of Wgt. |  |
| $25 \%$ | 36 | 30 | Mean | 42.53785 |
| $50 \%$ | 43 |  | Std. Dev. | 8.072574 |
| $75 \%$ | 49 | Largest | 60 |  |
| $90 \%$ | 54 | 60 | Variance | 65.16645 |
| $95 \%$ | 56 | 60 | Skewness | .150879 |
| $99 \%$ | 59 |  | Kurtosis | 1.981077 |

tabulate creates the frequency distribution for a variable. For example,
. tab hc

| Husband <br> College: <br> 1=yes 0=no | Freq. | Percent | Cum. |
| ---: | ---: | ---: | ---: |
| NoCol <br> College | 458 | 60.82 | 60.82 |
| Total | 295 | 39.18 | 100.00 |
| 753 | 100.00 |  |  |

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If you do not want the value labels included, type
. tab hc, nolabel

| Husband <br> College: <br> 1=yes 0=no | Freq. | Percent | Cum. |
| ---: | ---: | ---: | ---: |
| 0 | 458 | 60.82 | 60.82 |
| 1 | 295 | 39.18 | 100.00 |
| Total | 753 | 100.00 |  |

If you want a two-way table, type
. tab hc wc

| Husband <br> College: | Wife College:1=yes <br> 0=no <br> 1=yes 0=no |  | NoCol |
| ---: | ---: | ---: | ---: | College $\quad$ Total

By default, tabulate does not tell you the number of missing values for either variable. Specifying the missing option includes missing values. We recommend this option whenever generating a frequency distribution to check that some transformation was done correctly. The options row, col, and cell request row, column, and cell percentages along with the frequency counts. The option chi 2 reports the chi-square for a test that the rows and columns are independent.
tab1 presents univariate frequency distributions for each variable listed. For example,
. tab1 hc wc
-> tabulation of hc

| Husband <br> College: <br> 1=yes 0=no | Freq. | Percent | Cum. |
| ---: | ---: | ---: | ---: |
| NoCol <br> College | 458 | 60.82 | 60.82 |
| Total | 295 | 39.18 | 100.00 |
| 753 | 100.00 |  |  |

-> tabulation of wc

| Wife <br> College: <br> $1=y e s ~ 0=$ no | Freq. | Percent | Cum. |
| ---: | ---: | ---: | ---: |
| NoCol <br> College | 541 | 71.85 | 71.85 |
| Total | 212 | 28.15 | 100.00 |
| 753 | 100.00 |  |  |

dotplot generates a quick graphical summary of a variable, which is very useful for quickly checking your data. For example, the command dotplot age leads to the following graph:

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This graph will appear in a new window called the Graph Window. Details on saving, printing, and enhancing graphs are given in Section 2.16.
codebook summarizes a variable in a format designed for printing a codebook. For example, codebook age produces

```
. codebook age
```



### 2.12.3 Selecting observations

As previously mentioned, you can select cases using with the if and in options. For example, summarize age if $w c==1$ provides summary statistics on age for only those observations where wc equals 1 . In some cases it is simpler to remove the cases with either the drop or keep commands. drop removes observations from memory (not from the . dta file) based on an if and/or in specification. The syntax is
drop [in range] [if exp]

Only observations that do not meet those conditions are left in memory. For example, drop if $W C==1$ keeps only those cases where wc is not equal to 1 , including observations with missing values on wc.

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keep has the same syntax as drop and deletes all cases except those that meet the condition. For example, keep if $\mathrm{wc}==1$ only keeps those cases where wc is 1 ; all other observations, including those with missing values for wc, are dropped from memory. After selecting the observations that you want, you can save the remaining variables to a new dataset with the save command.

### 2.12.4 Selecting variables

You can also select which variables you want to keep. The syntax is
drop variable_list
keep variable_list
With drop, all variables are kept except those that are explicitly listed. With keep, only those variables that are explicitly listed are kept. After selecting the variables that you want, you can save the remaining variables to a new dataset with the save command.

### 2.13 Creating new variables

The variables that you analyze are often constructed differently than the variables in the original dataset. In this section we consider basic methods for creating new variables. Our examples always create a new variable from an old variable rather than transforming an existing variable. Even though it is possible to simply transform an existing variable, we find that this leads to mistakes.

### 2.13.1 generate command

generate creates new variables. For example, to create age 2 that is an exact copy of age, type

```
. generate age2 = age
```

. summarize age2 age

| Variable | Obs | Mean | Std. Dev. | Min | Max |
| ---: | ---: | ---: | ---: | ---: | ---: |
| age2 | 753 | 42.53785 | 8.072574 | 30 | 60 |
| age | 753 | 42.53785 | 8.072574 | 30 | 60 |

The results of summarize show that the two variables are identical. Note that we used a single equal sign since we are making a variable equal to some value.

Observations excluded by if or in qualifiers in the generate command are coded as missing. For example, to generate age3 that equals age for those over 40 but is otherwise missing, type


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Whenever generate (or gen as it can be abbreviated) produces missing values, it tells you how many cases are missing.
generate can also create variables that are mathematical functions of existing variables. For example, we can create agesq that is the square of age and lnage that is the natural log of age:
. gen agesq = age~2
. gen lnage $=\ln ($ age $)$
For a complete list of the mathematical functions available in Stata, enter help functions. For quick reference, here is list of particularly useful functions:

| Function | Definition | Example |
| :--- | :--- | :--- |
| + | addition | gen $\mathrm{y}=\mathrm{a}+\mathrm{b}$ |
| - | subtraction | gen $\mathrm{y}=\mathrm{a}-\mathrm{b}$ |
| $/$ | division | gen density $=$ pop/area |
| $*$ | multiplication | gen $\mathrm{y}=\mathrm{a} * \mathrm{~b}$ |
| - | take to a power | gen $\mathrm{y}=\mathrm{a}$-3 |
| $\ln$ | natural log | gen lnwage $=\ln ($ wage $)$ |
| $\exp$ | exponential | gen $\mathrm{y}=\exp (\mathrm{a})$ |
| sqrt | square root | gen agesqrt $=\operatorname{sqrt}($ age $)$ |

### 2.13.2 replace command

replace has the same syntax as generate, but is used to change values of a variable that already exists. For example, say we want to make a new variable age 4 that equals age if age is over 40, but equals 40 for all persons aged 40 and under. First we create age 4 equal to age. Then, we replace those values we want to change:

```
. gen age4 = age
. replace age4 = 40 if age<40
(298 real changes made)
. sum age4 age
\begin{tabular}{r|rrrrr} 
Variable & Obs & Mean & Std. Dev. & Min & Max \\
\hline age4 & 753 & 44.85126 & 5.593896 & 40 & 60 \\
age & 753 & 42.53785 & 8.072574 & 30 & 60
\end{tabular}
```

Note that replace reports how many values were changed. This is useful in verifying that the command did what you intended. Also, summarize confirms that the minimum value of age is 30 and that age 4 now has a minimum of 40 as intended.

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Warning Of course, we could have simply changed the original variable: replace age $=40$ if age $<40$. But, if we did this and saved the data, there would be no way to return to the original values for age if we later needed them.

### 2.13.3 recode command

The values of existing variables can also be changed using the recode command. With recode you specify a set of correspondences between old values and new ones. For example, you might want old values of 1 and 2 to correspond to new values of 1 , old values of 3 and 4 to correspond to new values of 2 , and so on. This is particularly useful for combining categories. To use this command, we recommend that you start by making a copy of an existing variable. Then, recode the copy. Here are some examples:

To change 1 to 2 and 3 to 4 , but leave all other values unchanged, type

```
. gen myvar1 = origvar
. recode myvar1 1=2 3=4
(23 changes made)
```

To change 2 to 1 and change all other values (including missing) to 0 , type

```
. gen myvar2 = origvar
. recode myvar2 2=1 *=0
(100 changes made)
```

where the asterisk indicates all values, including missing values, that have not been explicitly recoded.

To change 2 to 1 and change all other values except missing to 0 , type

```
. gen myvar3 = origvar
. recode myvar3 2=1 *=0 if myvar3~}=\mathrm{ .
(89 changes made)
```

To change values from 1 to 4 inclusive to 2 and keep other values unchanged, type

```
. gen myvar4 = origvar
. recode myvar4 1/4=2
(40 changes made)
```

To change values 1, 3, 4 and 5 to 7 and keep other values unchanged, type

```
. gen myvar5 = origvar
. recode myvar5 1 3 4 5=7
(55 changes made)
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```

To change all values from the minimum through 5 to the minimum, type

```
. gen myvar6 = origvar
. recode myvar6 min/5=min
(56 changes made)
To change missing values to 9 , type
```

```
. gen myvar7 = origvar
. recode myvar7 .=9
(11 changes made)
```

To change values of -999 to missing, type
. gen myvar8 = origvar
. recode myvar8 -999=. (56 changes made)

### 2.13.4 Common transformations for RHS variables

For the models we discuss in later chapters, you can use many of the tricks you learned for coding right-hand-side (i.e., independent) variables in the linear regression model. Here are some useful examples. Details on how to interpret such variables in regression models are given in Chapter 8.

## Breaking a categorical variable into a set of binary variables

To use a $j$-category nominal variable as an independent variable in a regression model, you need to create a set of $j-1$ binary variables, also known as dummy variables or indicator variables. To illustrate how to do this, we use educational attainment (degree), which is coded as: $0=$ no diploma, $1=$ high school diploma, $2=$ associate's degree, $3=$ bachelor's degree, and $4=$ postgraduate degree, with some missing data. We want to make four binary variables with the "no diploma" category serving as our reference category. We also want observations that have missing values for degree to have missing values in each of the dummy variables that we create. The simplest way to do this is to use the generate option with tabulate:

| . tab degree, gen(edlevel) |  |  |
| ---: | ---: | ---: | ---: |
| rs highest |  |  |
| degree |  |  |$\quad$ Freq. $\quad$ Percent $\quad$ Cum.

[^8] either electronically or in printed form, to others.

The generate (name) option creates a new binary variable for each category of the specified variable. In our example, degree has 5 categories, so five new variables are created. These variables all begin with edlevel, the root that we specified with the generate (edlevel) option. We can check the five new variables by typing sum edlevel*:
. sum edlevel*

| Variable | Obs | Mean | Std. Dev. | Min | Max |
| ---: | ---: | ---: | ---: | ---: | ---: |
| edlevel1 | 4584 | .1747382 | .3797845 | 0 | 1 |
| edlevel2 | 4584 | .5292321 | .4991992 | 0 | 1 |
| edlevel3 | 4584 | .059555 | .2366863 | 0 | 1 |
| edlevel4 | 4584 | .1636126 | .369964 | 0 | 1 |
| edlevel5 | 4584 | .0728621 | .2599384 | 0 | 1 |

By cross-tabulating the new edlevel1 by the original degree, we can see that edlevel1 equals 1 for individuals with no high school diploma and equals 0 for everyone else except the 14 observations with missing values for degree:

```
. tab degree edlevel1, missing
```

| rs highest | degree==lt high school |  |  |  |
| ---: | ---: | ---: | ---: | ---: |
| degree | 0 | 1 | $\cdot$ | Total |
| lt high school | 0 | 801 | 0 | 801 |
| high school | 2426 | 0 | 0 | 2426 |
| junior college | 273 | 0 | 0 | 273 |
| bachelor | 750 | 0 | 0 | 750 |
| graduate | 334 | 0 | 0 | 334 |
| $\cdot$ | 0 | 0 | 14 | 14 |
| Total | 3783 | 801 | 14 | 4598 |

One limitation of using the generate (name) option of tab is that it only works when there is a one-to-one correspondence between the original categories and the dummy variables that we wish to create. So, let's suppose that we want to combine high school graduates and those with associate's degrees when creating our new binary variables. Say also that we want to treat those without high school diplomas as the omitted category. The following is one way to create the three binary variables that we need:

```
. gen hsdeg = (degree==1 | degree==2) if degree }\mp@subsup{}{~}{~}=
(14 missing values generated)
. gen coldeg = (degree==3) if degree~ = .
(14 missing values generated)
. gen graddeg = (degree==4) if degree }~=\mathrm{ .
(14 missing values generated)
. tab degree coldeg, missing
\begin{tabular}{r|rrr|r} 
rs highest \\
degree
\end{tabular}\(\quad\)\begin{tabular}{r} 
coldeg \\
0
\end{tabular}\(\quad 1 \quad . \quad\) Total
```

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To understand how this works, you need to know that when Stata is presented with an expression (e.g., degree==3) where it expects a value, it evaluates the expression and assigns it a value of 1 if true and 0 if false. Consequently, gen coldeg $=$ (degree==3) creates the variable coldeg that equals 1 whenever degree equals 3 and 0 otherwise. By adding if degree ${ }^{\sim}=$. to the end of the command, we assign these values only to observations in which the value of degree is not missing. If an observation has a missing value for degree, these cases are given a missing value.

## More examples of creating binary variables

Binary variables are used so often in regression models that it is worth providing more examples of generating them. In the dataset that we use in Chapter 5, the independent variable for respondent's education (ed) is measured in years. We can create a dummy variable that equals 1 if the respondent has at least 12 years of education and 0 otherwise:
. gen ed12plus $=(e d>=12)$ if ed $^{\sim}=$.
Alternatively, we might want to create a set of variables that indicates whether an individual has less than 12 years of education, between 13 and 16 years of education, or 17 or more years of education. This is done as follows:

```
. gen edlt13 = (ed<=12) if ed~}=\mathrm{ .
```

. gen ed1316 = (ed>=13 \& ed<=16) if ed ${ }^{\sim}=$.
. gen ed17plus $=(e d>17)$ if ed ${ }^{\sim}=$

Tip: Naming dummy variables Whenever possible, we name dummy variables so that 1 corresponds to "yes" and 0 to "no". With this convention, a dummy variable called female is coded 1 for women (i.e., yes the person is female) and 0 for men. If the dummy variable was named sex, there would be no immediate way to know what 0 and 1 mean.

The recode command can also be used to create binary variables. The variable warm contains responses to the question of whether working women can be as good of mothers as women who do not work: $1=$ strongly disagree, $2=$ disagree, $3=$ agree, and $4=$ strongly agree. To create a dummy indicating agreement as opposed to disagreement, type

| . recode wrmagree 1=0 2=0 3=1 4=1 (2293 changes made) |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| tab wrmagree warm |  |  |  |  |  |
| wrmagree |  | $\begin{gathered} \text { s was } \\ \text { D } \end{gathered}$ | $\begin{gathered} \text { tions } \\ \text { A } \end{gathered}$ | SA | Total |
| 0 | 297 | 723 | 0 | 0 | 1020 |
| 1 | 0 | 0 | 856 | 417 | 1273 |
| Total | 297 | 723 | 856 | 417 | 2293 |

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## Nonlinear transformations

Nonlinear transformations of the independent variables are commonly used in regression models. For example, researchers often include both age and age ${ }^{2}$ as explanatory variables to allow the effect of a one-year increase in age to change as one gets older. We can create a squared term as
. gen agesq = age*age
Likewise, income is often logged so that the impact of each additional dollar decreases as income increases. The new variable can be created as

- gen $\operatorname{lnincome~}=\ln$ (income)

We can use the minimum and maximum values reported by summarize as a check on our transformations:
. sum age agesq income lnincome

| Variable | Obs | Mean | Std. Dev. | Min | Max |
| ---: | ---: | ---: | ---: | ---: | ---: |
| age | 4598 | 46.12375 | 17.33162 | 18 | 99 |
| agesq | 4598 | 2427.72 | 1798.477 | 324 | 9801 |
| income | 4103 | 34790.7 | 22387.45 | 1000 | 75000 |
| lnincome | 4103 | 10.16331 | .8852605 | 6.907755 | 11.22524 |

## Interaction terms

In regression models, you can include interactions by taking the product of two independent variables. For example, we might think that the effect of family income differs for men and women. If gender is measured as the dummy variable female, we can construct an interaction term as follows:

```
. gen feminc = female * income
(495 missing values generated)
```

Tip: The xi command can be used to automatically create interaction variables. While this is a very powerful command that can save time, it is also a command that can be confusing unless you use the command frequently. Accordingly, we do not recommend it. Constructing interactions with generate is a good way to make sure you understand what the interactions mean.

### 2.14 Labeling variables and values

Variable labels provide descriptive information about what a variable measures. For example, the variable agesq might be given the label "age squared", or warm could have the label "Mother has a warm relationship". Value labels provide descriptive information about the different values of

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a categorical variable. For example, value labels might indicate that the values 1-4 correspond to survey responses of strongly agree, agree, disagree, and strongly disagree. Adding labels to variables and values is not much fun, but in the long run it can save a great deal of time and prevent misunderstandings. Also, many of the commands in SPost produce output that is more easily understood if the dependent variable has value labels.

### 2.14.1 Variable labels

The label variable command attaches a label of up to 80 characters to a variable. For example,

| . label variable agesq "Age squared" |  |  |  |  |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| . describe agesq |  |  |  |  |  |  |
| storage <br> type |  |  |  | display <br> format | value <br> label | variable label |
| variable name |  |  |  |  |  |  |

If no label is specified, any existing variable label is removed. For example,

| label variable agesq |
| :--- |
| . describe agesq |
| variable namestorage <br> typedisplay <br> format$\quad$value <br> label |
| agesq |

Tip: Use short labels While variable labels of up to 80 characters are allowed, output often does not show all 80 characters. We find it works best to keep variable labels short, with the most important information in the front of the label. That way, if the label is truncated, you will see the critical information.

### 2.14.2 Value labels

Beginners often find value labels in Stata confusing. The key thing to keep in mind is that Stata splits the process of labeling values into two steps: creating labels and then attaching the labels to variables.

Step 1 defines a set of labels without reference to a variable. Here are some examples of value labels:

- label define yesno 1 yes 0 no
. label define posneg4 1 veryN 2 negative 3 positive 4 veryP

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- label define agree4 1 StrongA 2 Agree 3 Disagree 4 StrongD
. label define agree5 1 StrongA 2 Agree 3 Neutral 4 Disagree 5 StrongD
Notice several things. First, each set of labels is given a unique name (e.g., yesno, agree4). Second, individual labels are associated with a specific value. Third, none of our labels have spaces in them (e.g., we use StrongA not Strong A). While you can have spaces if you place the label within quotes, some commands crash when they encounter blanks in value labels. So, it is easier not to do it. We have also found that the characters ., :, and \{ in value labels can cause similar problems. Fourth, our labels are 8 letters or shorter in length. Since some programs have trouble with value labels longer than 8 letters, we recommend keeping value labels short.

Step 2 assigns the value labels to variables. Let's say that variables female, black, and anykids all imply yes/no categories with 1 as yes and 0 as no. To assign labels to the values, we would use the following commands:

- label values female yesno
- label values black yesno
- label values anykids yesno
. describe female black anykids

| variable name | storage <br> type | display <br> format | value <br> label | variable label |
| :--- | :--- | :--- | :--- | :--- |
| female | byte | $\% 9.0 \mathrm{~g}$ | yesno | Female |
| black | byte | $\% 9.0 \mathrm{~g}$ | yesno | Black |
| anykids | byte | $\% 9.0 \mathrm{~g}$ | yesno | R have any children? |

The output for describe shows which value labels were assigned to which variables. The new value labels are reflected in the output from tabulate:
. tab anykids

| R have any <br> children? | Freq. | Percent | Cum. |
| ---: | ---: | ---: | ---: |
| no | 1267 | 27.64 | 27.64 |
| yes | 3317 | 72.36 | 100.00 |
| Total | 4584 | 100.00 |  |

For the degree variable that we looked at earlier, we assign labels with
. label define degree 0 "no_hs" 1 "hs" 2 "jun_col" 3 "bachelor" 4 "graduate"
. label values degree degree
. tab degree
rs highest
degree

no_hs
hs
jun_col

Notice that we used _s (underscores) instead of spaces.
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### 2.14.3 notes command

The notes command allows you to add notes to the dataset as a whole or to specific variables. Since the notes are saved in the dataset, the information is always available when you use the data. In the following example, we add one note describing the dataset and two that describe the income variable:
. notes: General Social Survey extract for Stata book
. notes income: self-reported family income, measured in dollars
. notes income: refusals coded as missing
We can review the notes by typing notes:

```
. notes
_dta:
    1. General Social Survey extract for Stata book
income:
    1. self-reported family income, measured in dollars
    2. refusals coded as missing
```

If we save the dataset after adding notes, the notes become a permanent part of the dataset.

### 2.15 Global and local macros

While macros are most often used when writing ado-files, they are also very useful in do-files. Later in the book, and especially in Chapter 8, we use macros extensively. Accordingly, we discuss them briefly here. Readers with less familiarity with Stata might want to skip this section for now and read it later when macros are used in our examples.

In Stata, you can assign values or strings to macros. Whenever Stata encounters the macro name, it automatically substitutes the contents of the macro. For example, pretend that you want to generate a series of two-by-two tables where you want cell percentages requiring the cell option, missing values requiring the missing option, values printed instead of value labels requiring the nolabel option, and the chi-squared test statistic requiring the chi2 option. Even if you use the shortest abbreviations, this would require typing ", ce m nol ch" at the end of each tab command. Alternatively, you could use the following command to define a global macro called myopt:

```
. global myopt = ", cell miss nolabel chi2"
```

Then, whenever you type \$myopt (the \$ tells Stata that myopt is a global macro), Stata substitutes , cell miss nolabel chi2. If you type
. tab lfp wc \$myopt
Stata interprets this as if you had typed
. tab lfp wc , cell miss nolabel chi2
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Global macros are "global" because once defined, you can use them for the rest of your Stata session. In do- (or ado-) files, you can also define "local" macros, which work only within the do-file. That is, as soon as the do-file is done running, the macro disappears. We prefer using local macros whenever possible because you do not have to worry about conflicts with other programs or do-files that try to use the same macro name for a different purpose. Local macros are defined using the local command, and they are referenced by placing the name of the local macro in single quotes such as 'myopt'. Notice that the two single quote marks use different symbols. If the operations we just performed were in a do-file, we could have produced the same output with the following lines:

```
. local opt = ", cell miss nolabel chi2"
. tab lfp wc `opt`
    (output omitted)
```

You can also define macros to equal the result of computations. After entering global four $=2+2$, the value 4 will be substituted for $\$$ four. In addition, Stata contains many macro functions in which items retrieved from memory are assigned to macros. For example, to display the variable label that you have assigned to the variable wc, you can type

```
. global wclabel : variable label wc
. display "$wclabel"
Wife College: 1=yes 0=no
```

We have only scratched the surface of the potential of macros. Macros are immensely flexible and are indispensable for a variety of advanced tasks that Stata can perform. Perhaps most importantly, macros are essential for doing any meaningful Stata programming. If you look at the ado-files for the commands we have written for this book, you will see many instances of macros, and even of macros within macros. For users interested in advanced applications, the macro entry in the Programming Manual should be read closely.

### 2.16 Graphics

Stata comes with an entire manual dedicated to graphics and is capable of making many more kinds of graphs than those used in this book. In this section, we provide only a brief introduction to graphics in Stata, focusing on the type of graph that we use most often in later chapters. Namely, we consider a plot of one or more outcomes against an independent variable using the command graph. In its simplest form, graph produces a simple scatterplot, but with options it becomes a line graph. While we do not consider other graph commands (e.g., dotplot), many of the options we present here can be used with other commands. For additional information on graphs, type help graph or see the Graphics Manual. ${ }^{6}$

Graphs that you create in Stata are drawn in their own window that should appear on top of the four windows we discussed above. If the Graph Window is hidden, you can bring it to the front by clicking on 畋. You can make the Graph Window larger or smaller by clicking and dragging the borders.

[^9]
### 2.16.1 The graph command

The type of graph that we use most often shows how the predicted probability of observing a given outcome changes as a continuous variable changes over a specified range. For example, in Chapter 4 we show you how to compute the predicted probability of a woman being in the labor force according to the number of children she has and the family income. In later chapters we show you how to compute these predictions, but for now you can simply load them with the command use lfpgraph2, clear. The variable income is family income measured in thousands of dollars, while the next three variables show the predicted probabilities of working for a woman who has no children under six (kid0p1), one child under six (kid1p1), or two children under six (kid2p1). Since there are only eleven values, we can easily list them:


We see that as annual income increases the predicted probability of being in the labor force decreases. Also, by looking across any row, we see that for a given level of income the probability of being in the labor force decreases as the number of young children increases. We want to display these patterns graphically.
graph can be used to draw a scatterplot in which the values of one or more $y$-variable are plotted against values of an $x$-variable. In our case, income is the $x$-variable and the predicted probabilities kid0p1, kid1p1, and kid2p1 are the $y$-variables. Thus, for each value of $x$ we have three values of $y$. In making scatterplots with graph, the $y$-variables are listed first and the $x$-variable is listed last. If we type,
. graph kidOp1 kid1p1 kid2p1 income
we obtain the following graph:
(Continued on next page)

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Our simple scatterplot shows the pattern of decreasing probabilities as income or number of children increases. Even so, there is much we can do to make this a more effective graph. These additions are made using some of the many options that are available for graph (see the Graphics Manual for others). The syntax for the options is

```
graph [varlist] ,[\underline{symbol(s...s) connect(c...c) xlabel(#,..,#) ylabel(#,..,#)}
    xtick(#,\ldots,#) ytick(#,\ldots,#) t1title(text) t2title(text) b1title(text)
    b2title(text) l1title(text) \underline{12title(text) r1title(text) r2title(text)}
    saving(filename [, replace])]
```

Each of these options is now considered.

Choosing symbols In our graph, the probabilities for women with no, one, and two young children are differentiated by using different symbols. The symbols in our first graph are the defaults chosen by Stata, but we can specify the symbols we want with the symbol ( $s \ldots s$ ) option. The codes for specifying symbols are

| 0 | Large circle | o | Small circle | i | Invisible |
| :--- | :--- | :---: | :--- | :---: | :--- |
| T | Large triangle | p | Small plus sign | . | Dot |
| S | Large square | d | Small diamond |  |  |

Since we are graphing three $y$-variables, we need to specify 3 symbols. If we use s (OTp), kid0p1 is plotted with a large circle, kid0p2 with a large triangle, and kid0p3 with a small plus sign.

Connecting points with lines connect (c...c) allows us to connect the points for each $y$-variable in the graph. For each $y$-variable, you specify how the points are to be connected. Connecting type option 1 (which is a lowercase L, not the number 1) connects points with straight lines; type $s$

[^10]connects the points using a smooth curve (which works well for the graphs we use); and . specifies that you do not want to connect points. For example, c(.ls) would not connect the first set of points (.), would connect the second set with straight lines (l), and would connect the third with a smoothed curve ( s ). We almost always use the $s$ option in our graphs; c (sss) tells Stata to connect all three sets of points with smoothed lines.

Choosing line styles We can also use line styles to distinguish the $y$-variables. For example, one variable could be plotted using a solid line, another using a dashed line, and the third using a line with alternative long and short dashes. This is done (beginning with Stata 7) using the same c() option that we used to specify how to connect the points. For each line, we indicate in brackets the patterns of line style we want after the letter that indicates how we wish to connect the points (e.g., $c(1[-])$ ). The following codes are used:

$$
\begin{array}{ll}
-\quad \text { A long dash (underscore) } & \text { A short dash (almost a dot) } \\
-\quad \text { A medium dash } & \# \text { A space }(\text { e.g., }-\#)
\end{array}
$$

If nothing is specified, a solid line is drawn. In our example, $c(11[-] 1[-]$.$) would tell Stata to$ use a solid line to connect the points for kid0p1, a line with medium dashes to connect the points for kid0p2, and a line with alternating dashes and dots to connect the points for kid0p3.

Labeling the values on the $\boldsymbol{x}$ - and $\boldsymbol{y}$-axes By default, graph labels only the minimum and maximum values on each axis. Usually, you will want more axis values to be labeled. xlabel (\#,... \#) and ylabel $(\#, \ldots, \#)$ specify which values to display along each axis, where the values can exceed the minimum and maximum if desired. If we use ylabel $(0, .2, .4, .6, .8,1)$, the $y$-axis is scaled from 0 to 1 with labels at increments of .2 . We can even abbreviate this to ylabel ( 0.2 to 1 ). xlabel $(10,30,50,70,90)$ specifies values on the $x$-axis ranging from 10 to 90 by increments of 20.

Tick marks Tick marks are placed wherever values are labeled. If you want additional tick marks, use xtick(\#,.,$\#)$ or $\operatorname{ytick}(\#, \ldots, \#)$. Typing xtick $(20,40,60,80)$ adds tick marks between the values that we just labeled using xlabel.

Adding titles graph allows as many as two titles on each of the four sides of the graph using the options t1title(), t2title(), l1title(), l2title(), b1title(), b2title(), r1title(), r2title(). Note that $t$ stands for top, $b$ for bottom, 1 for left, and $r$ for right. The easiest way to see what these commands do is to create a plot using these different options,

```
. graph kid0p1 kid1p1 kid2p1 income, l1title("l1title") l2title("l2title") /*
> */ t1title("t1title") t2title("t2title") b1title("b1title") /*
> */ b2title("b2title") r1title("r1title") r2title("r2title")
```

which yields
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There are several points to note:

1. b 1 titl e is printed in a larger font and is meant to be a title for the graph as a whole.
2. If you do not specify b2title(), Stata uses the variable label from the $x$-variable.
3. With a single $y$-variable, the variable label for $y$ is printed as l2title. With more than one $y$-variable, a legend is printed at the top that uses the variable labels for up to four $y$-variables. Variable names are used if no labels have been assigned. If you specify either t1title() or t2title(), the legend for the $y$-variables is not printed.
4. To add a single label to the $y$-axis, use l2title. For example, l2title("Probability") adds Probability to the left side of the graph.
5. Often the default placement of titles on the left sides of graphs is too far away. The gap() option allows you to specify this distance. We find that gap (4) often works well.

Controlling the size of labels This is not an option to graph, but is a separate command. The size of the labels in the graph is controlled by the command

```
set textsize #
```

where \# indicates how much smaller or larger you want the labels. For example, set textsize 125 makes the letters 25 percent larger. We find that graphs that look fine on the screen need to have larger letters when printed, so we routinely set the textsize to 125 .

Saving graphs Specifying saving(filename, replace) saves the graph in Stata's proprietary format (indicated by the suffix .gph) to the working directory. Including replace tells Stata to overwrite a file with that name if it exists. Saving graphs in .gph format is necessary when you want to combine graphs for display, as discussed below.

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Tip: Exporting graphs to other programs If you are using Windows and want to export graphs to another program, such as a word processor, we find that it works best to save them as a Windows Metafile, which has the . wmf extension. This can be done using the translate command. If the graph is currently in the graph window, you can save it in .wmf format with the command: translate @Graph filename.wmf (note the capital "G"). If the file is already saved in .gph format, you can translate it to .wmf format with the command: translate filename.gph filename.wmf. The replace option can be used with translate to automatically overwrite a graph of the same name, which is useful in do-files.

## Putting everything together

We can now put together several of the options discussed above and make a new graph by using the following command:

```
. set textsize 125
# #delimit ;
delimiter now ;
. graph kid0p1 kid1p1 kid2p1 income, s(OTp) c(lll) xlabel(10 30 to 90)
> xtick (20 40 60 80) ylabel (0 .2 to 1) l2title("Probability") gap(3)
> saving(mygraph, replace);
. #delimit cr
```

We have used the \#delimit command from Section 2.9.2 since the graph command was very long. The graph command we have just entered produces the following:


This graph is much more effective in illustrating that the probability of a woman being in the labor force declines as family income increases, and that the differences in predicted probabilities between women with no young children and those with one or two young children are greatest at the lowest levels of income.

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### 2.16.2 Printing graphs

When a graph is in the Graph Window, you may print it by selecting File $\rightarrow$ Print Graph from the menus or by clicking on . You can also print a graph in the Graph Window with the command print @Graph. You can print a saved graph with the command print filename.gph.

### 2.16.3 Combining graphs

Multiple graphs that have been saved in .gph format can be combined into a single graph. This is useful, for example, when you want to place two graphs side-by-side or one-above-the-other. In our example above, we created a graph that we saved as mygraph.gph. Now, we create a second graph that illustrates the use of line styles, and then use the graph using command to display the two graphs side by side:

```
. #delimit ;
delimiter now ;
. graph kid0p1 kid1p1 kid2p1 income, s(...) c(ll[-]l[-..]) xlabel(10 30 to 90)
> xtick (20 40 60 80) ylabel (0 .2 to 1) l2title("Probability") gap(3)
> saving(mygraph2, replace);
(note: file mygraph2.gph not found)
. #delimit cr
delimiter now cr
. graph using mygraph mygraph2, saving(mygraphsbs, replace)
```

This leads to


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Combining graphs to print one-over-the-other is a bit more difficult. The trick is to understand that when multiple graphs are combined, Stata divides the Graphics Window into a square array. When you combine from 2 to 4 graphs, Stata arranges them into a $2 \times 2$ array; if you combine 5 to 9 graphs, Stata arranges them into a $3 \times 3$ array; and so on. The graphs are arranged in the order in which the filenames are listed in the graph using command, with any leftover space left blank. When we displayed two graphs side-by-side, these were placed in the upper-left and the upper-right corners of the Graph Window, respectively. The bottom half of the window is empty since we did not specify a third or fourth graph to be combined.

To display two graphs one-over-the-other, we want these graphs to be placed in the upper-left and lower-left corners of the Graph Window, which means that we want to place an empty graph in the upper-right corner. The command graph using, saving (null, replace) creates an empty graph called null.gph, which we include as the second of three graphs we wish to combine:

- graph using, saving(null, replace)
. graph using mygraph null mygraph2, saving(mygraphvert, replace)
. translate mygraphvert.gph 02graphvert.wmf, replace
(file d:\spostdata\02graphvert.wmf written in Windows Metafile format)
This leads to


As we described earlier, translate can be used to save the graph as a Windows MetaFile that can be imported to a word processor or other program. More details on combining graphs can be found in the Stata Graphics Manual.

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Warning Do not use the saving, replace option to save a combined graph with the same name as one of the graphs that you are combining. Usually Stata catches this error, but sometimes it does not and the original graph is lost.

### 2.17 A brief tutorial

This tutorial uses the science2.dta data that is available from the book's web site. You can use your own dataset as you work through this tutorial, but you will need to change some of the commands to correspond to the variables in your data. In addition to our tutorial, StataCorp has useful tutorials that are described in the User's Guide and can be accessed by typing tutorial within Stata.

Open a $\log$ The first step is to open a $\log$ file for recording your results. Remember that all commands are case sensitive. The commands are listed with a period in front, but you do not type the period:

```
. capture log close
. log using tutorial, text
    log: d:\spostdata\tutorial.smcl
    log type: text
```

opened on: 9 Feb 2001, 21:18:15

Load the data We assume that science2.dta is in your working directory. clear tells Stata to "clear out" any existing data from memory before loading the new dataset:

```
. use science2, clear
(Note that some of the variables have been artificially constructed.)
```

The message after loading the data reflects that this dataset was created for teaching. While most of the variables contain real information, some variables have been artificially constructed.

Examine the dataset describe gives information about the dataset.


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| variable name | storage type | display <br> format | value <br> label | variable label |
| :---: | :---: | :---: | :---: | :---: |
| id | float | \%9.0g |  | ID Number. |
| cit1 | int | \%9.0g |  | Citations: PhD yr -1 to 1. |
| cit3 | int | \%9.0g |  | Citations: PhD yr 1 to 3. |
| cit6 | int | \%9.0g |  | Citations: PhD yr 4 to 6. |
| cit9 | int | \% 9.0g |  | Citations: PhD yr 7 to 9. |
| enrol | byte | \%9.0g |  | Years from BA to PhD. |
| fel | float | \%9.0g |  | Fellow or PhD prestige. |
| felclass | byte | \%9.0g | prstlb | * Fellow or PhD prestige class. |
| fellow | byte | $\% 9.0 \mathrm{~g}$ | fellbl | Postdoctoral fellow: 1=y,0=n. |
| female | byte | \%9.0g | femlbl | Female: 1=female, $0=$ male. |
| job | float | \%9.0g |  | Prestige of 1st univ job. |
| jobclass | byte | \%9.0g | prstlb | * Prestige class of 1st job. |
| mcit3 | int | \%9.0g |  | Mentor's 3 yr citation. |
| mcitt | int | \%9.0g |  | Mentor's total citations. |
| mmale | byte | \%9.0g | malelb | Mentor male: 1=male, $0=$ female . |
| mnas | byte | $\% 9.0 \mathrm{~g}$ | naslb | Mentor NAS: $1=y e s, 0=n o$. |
| mpub3 | byte | \%9.0g |  | Mentor's 3 year publications. |
| nopub1 | byte | \%9.0g | nopublb | $1=$ No pubs PhD yr -1 to 1. |
| nopub3 | byte | \%9.0g | nopublb | $1=$ No pubs PhD yr 1 to 3. |
| nopub6 | byte | \%9.0g | nopublb | $1=$ No pubs PhD yr 4 to 6. |
| nopub9 | byte | \%9.0g | nopublb | $1=$ No pubs PhD yr 7 to 9. |
| phd | float | \%9.0g |  | Prestige of Ph.D. department. |
| phdclass | byte | \%9.0g | prstlb | * Prestige class of Ph.D. dept. |
| pub1 | byte | \%9.0g |  | Publications: PhD yr -1 to 1. |
| pub3 | byte | \%9.0g |  | Publications: PhD yr 1 to 3. |
| pub6 | byte | \%9.0g |  | Publications: PhD yr 4 to 6. |
| pub9 | byte | \%9.0g |  | Publications: PhD yr 7 to 9. |
| work | byte | \%9.0g | worklbl | Type of first job. |
| workadmn | byte | \%9.0g | wadmnlb | Admin: 1=yes; 0=no. |
| worktch | byte | \%9.0g | wtchlb | * Teaching: 1=yes; $0=$ no. |
| workuniv | byte | \%9.0g | wunivlb | * Univ Work: 1=yes; 0=no. |
| wt | byte | \%9.0g |  |  |
| faculty | byte | \%9.0g | faclbl | 1=Faculty in University |
| jobrank | byte | \%9.0g | joblbl | Rankings of University Job. |
| totpub | byte | \%9.0g |  | Total Pubs in 9 Yrs post-Ph.D. <br> * indicated variables have notes |

Sorted by:

Examine individual variables A series of commands gives us information about individual variables. You can use whichever command you prefer, or all of them.
. sum work

| Variable | Obs | Mean | Std. Dev. | Min | Max |
| ---: | ---: | ---: | :---: | ---: | ---: |
| work | 302 | 2.062914 | 1.37829 | 1 | 5 |


| . tab work, missing |  |  |  |
| :--- | ---: | ---: | ---: |
| Type of <br> first job. | Freq. | Percent | Cum. |
| FacUniv | 160 | 52.98 | 52.98 |
| ResUniv | 53 | 17.55 | 70.53 |

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Graphing variables Graphs are also useful for examining data. The command
. dotplot work
creates the following graph:


Saving graphs To save the graph we have just made as a Windows Metafile, type

```
. translate @Graph myname.wmf, replace
(file d:\spostdata\myname.wmf written in Windows Metafile format)
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```

Adding comments To add comments to your output, which allows you to document your command files, type $*$ at the beginning of each comment line. The comments are listed in the log file:

```
. * saved graph as work.wmf
```

Creating a dummy variable Now, let's make a dummy variable with faculty in universities coded 1 and all others coded 0 . The command gen isfac $=$ (work==1) if work $\sim$. generates isfac as a dummy variable where isfac equals 1 if work is 1 , else 0 . The statement if work $\sim=$ makes sure that missing values are kept as missing in the new variable.
. gen isfac $=\left(\right.$ work==1) if work $^{\sim}=$.
( 6 missing values generated)
Six missing values were generated since work contained six missing observations.

Checking transformations One way to check a transformations is with a table. In general, it is best to look at the missing values, which requires the missing option:
. tab isfac work, missing

| isfac | Type of first job. |  |  |  |  |  |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
|  | FacUniv | ResUniv | ColTch | IndRes | Admin | Total |
| 0 | 0 | 53 | 26 | 36 | 27 | 142 |
| 1 | 160 | 0 | 0 | 0 | 0 | 160 |
| $\cdot$ | 0 | 0 | 0 | 0 | 0 | 6 |
| Total | 160 | 53 | 26 | 36 | 27 | 308 |


|  | Type of <br> first job. <br> isfac |  |
| ---: | ---: | ---: |
| 0 | 0 | Total |
| 1 | 0 | 142 |
| $\cdot$ | 6 | 160 |
| Total | 6 | 6 |

You can also graph the two variables. Since many cases have the same values of both variables, we need to add some noise to each observation. jitter (2) adds noise to the graph, where the larger the number, the more noise. The range of this number is from 0 to 30 :
. graph work isfac, jitter(2)
This creates the following graph:

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Labeling variables and values For many of the regression commands, value labels for the dependent variable are essential. We start by creating a variable label, then create faclbl to store the value labels, and finally assign the value labels to the variable isfac:
. label variable isfac "1=Faculty in University"
. label define isfac 0 "NotFac" 1 "Faculty"

- label values isfac isfac

Then, we can get labeled output:

| $\begin{array}{r} 1=\text { Faculty } \\ \text { in } \\ \text { University } \end{array}$ | Freq. | Percent | Cum. |
| :---: | :---: | :---: | :---: |
| NotFac | 142 | 47.02 | 47.02 |
| Faculty | 160 | 52.98 | 100.00 |
| Total | 302 | 100.00 |  |

Creating an ordinal variable The prestige of graduate programs is often referred to using the categories of adequate, good, strong, and distinguished. Here we create such an ordinal variable from the continuous variable for the prestige of the first job. missing tells Stata to show cases with missing values.

| . tab job, missing |  |  |  |
| :--- | ---: | ---: | ---: |
| Prestige of <br> 1st univ <br> job. | Freq. | Percent | Cum. |
| 1.01 | 1 | 0.32 | 0.32 |
| 1.2 | 1 | 0.32 | 0.65 |

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| 1.22 | 1 | 0.32 | 0.97 |
| ---: | ---: | ---: | ---: |
| 1.32 | 1 | 0.32 | 1.30 |
| 1.37 | 1 | 0.32 | 1.62 |
| (output omitted) |  |  |  |
| 3.97 | 6 | 1.95 | 48.38 |
| 4.18 | 2 | 0.65 | 49.03 |
| 4.42 | 1 | 0.32 | 49.35 |
| 4.5 | 6 | 1.95 | 51.30 |
| 4.69 | 5 | 1.62 | 52.92 |
| $\cdot$ | 145 | 47.08 | 100.00 |
| Total | 308 | 100.00 |  |
|  |  |  |  |

The recode command makes it easy to group the categories from job. Of course, we then label the variable:

```
. gen jobprst = job
(145 missing values generated)
. recode jobprst .=. 1/1.99=1 2/2.99=2 3/3.99=3 4/5=4
(162 changes made)
. label variable jobprst "Rankings of University Job"
. label define prstlbl 1 "Adeq" 2 "Good" 3 "Strong" 4 "Dist"
. label values jobprst prstlbl
```

Here is the new variable (note that we use the missing option so that missing values are included in the tabulation):

| . tab jobprst, missing |  |  |  |
| ---: | ---: | ---: | ---: |
| Rankings of <br> University <br> Job. | Freq. | Percent | Cum. |
| Adeq | 31 | 10.06 | 10.06 |
| Good | 47 | 15.26 | 25.32 |
| Strong | 71 | 23.05 | 48.38 |
| Dist | 14 | 4.55 | 52.92 |
| $\cdot$ | 145 | 47.08 | 100.00 |
| Total | 308 | 100.00 |  |

Combining variables Now we create a new variable by summing existing variables. If we add pub3, pub6, and pub9, we can obtain the scientist's total number of publications over the nine years following receipt of the Ph.D.
. gen pubsum = pub3+pub6+pub9
. label variable pubsum "Total Pubs in 9 Yrs post-Ph.D."
. sum pub3 pub6 pub9 pubsum
Variable
Vabs
pub3
pub6

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A scatterplot matrix graph can be used to simultaneously plot all pairs of variables. This is done with the command
. graph pubsum pub3 pub6 pub9, matrix half
which leads to


Saving the new data After you make changes to your dataset, it is a good idea to save the data with a new filename:
. save sciwork, replace
file sciwork.dta saved

Close the $\log$ file Last, we need to close the $\log$ file so that we can refer to it in the future.

```
. \(\log\) close
    log: d:\spostdata\tutorial.smcl
    log type: smcl
closed on: 17 Jan 2001, 9:55:13
```


## A batch version

If you have read Section 2.9, you know that a better idea is to create a batch (do) file, perhaps called tutorial.do: ${ }^{7}$

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```
version 7
capture log close
set more off
log using tutorial, replace
* load and describe the data
use science2, clear
describe
* check variable work
sum work
tab work, missing
codebook work
dotplot work
* saved graph as work.wmf
translate @Graph myname.wmf, replace
* dummy variable indicating faculty
gen isfac = (work==1) if work }=\mathrm{ =.
tab isfac work, missing
graph work isfac, jitter(2)
label variable isfac "1=Faculty in University"
label define isfac 0 "NotFac" 1 "Faculty"
label values isfac isfac
tab isfac
* clean and recode job variable
tab job, missing
gen jobprst=job
recode jobprst .=. 1/1.99=1 2/2.99=2 3/3.99=3 4/5=4
label variable jobprst "Rankings of University Job"
label define prstlbl 1 "Adeq" 2 "Good" 3 "Strong" 4 "Dist"
label values jobprst prstlbl
tab jobprst, missing
* create total publications variable
gen pubsum=pub3+pub6+pub9
label variable pubsum "Total Pubs in 9 Yrs post-Ph.D."
sum pub3 pub6 pub9 pubsum
graph pubsum pub3 pub6 pub9, matrix half
* save the new data
save sciwork, replace
* close the log
log close
```

Then type do tutorial in the Command Window or select File $\rightarrow$ Do... from the menu.

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# REGRESSION MODELS FOR CATEGORICAL DEPENDENT VARIABLES USING STATA 

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## 3 Estimation, Testing, Fit, and Interpretation

Our book deals with what we think are the most fundamental and useful cross-sectional regression models for categorical and count outcomes: binary logit and probit, ordinal logit and probit, multinomial and conditional logit, Poisson regression, negative binomial regression, and zero-inflated models for counts. ${ }^{1}$ While these models differ in many respects, they share common features:

1. Each model is estimated by maximum likelihood.
2. The estimates can be tested with Wald and LR tests.
3. Measures of fit can be computed.
4. Models can be interpreted by examining predicted values of the outcome.

As a consequence of these similarities, the same principles and commands can be applied to each model. Coefficients can be listed with listcoef. Wald and likelihood-ratio tests can be computed with test and lrtest. Measures of fit can be computed with fitstat, and our SPost suite of post-estimation commands for interpretation can be used to interpret the predictions.

Building on the overview that this chapter provides, later chapters focus on the application of these principles and commands to exploit the unique features of each model. Additionally, this chapter serves as a reference for the syntax and options for the SPost commands that we introduce here. Accordingly, we encourage you to read this chapter before proceeding to the chapter that covers the models of greatest interest to you.

### 3.1 Estimation

Each of the models that we consider is estimated by maximum likelihood (ML). ${ }^{2}$ ML estimates are the values of the parameters that have the greatest likelihood (i.e., the maximum likelihood)

[^12]of generating the observed sample of data if the assumptions of the model are true. To obtain the maximum likelihood estimates, a likelihood function calculates how likely it is that we would observe the data we actually observed if a given set of parameter estimates were the true parameters. For example, in linear regression with a single independent variable, we need to estimate both the slope $\beta$ and the intercept $\alpha$ (for simplicity, we are ignoring the parameter $\sigma^{2}$ ). For any combination of possible values for $\alpha$ and $\beta$, the likelihood function tells us how likely it is that we would have observed the data that we did observe if these values were the true population parameters. If we imagine a surface in which the range of possible values of $\alpha$ comprises one axis and the range of $\beta$ comprises another axis, the resulting graph of the likelihood function would look like a hill, and the ML estimates would be the parameter values corresponding to the top of this hill. The variance of the estimates corresponds roughly to how quickly the slope is changing near the top of the hill.

For all but the simplest models, the only way to find the maximum of the likelihood function is by numerical methods. Numerical methods are the mathematical equivalent of how you would find the top of a hill if you were blindfolded and only knew the slope of the hill at the spot where you are standing and how the slope at that spot is changing (which you could figure out by poking your foot in each direction). The search begins with start values corresponding to your location as you start your climb. From the start position, the slope of the likelihood function and the rate of change in the slope determine the next guess for the parameters. The process continues to iterate until the maximum of the likelihood function is found, called convergence, and the resulting estimates are reported. Advances in numerical methods and computing hardware have made estimation by numerical methods routine.

### 3.1.1 Stata's output for ML estimation

The process of iteration is reflected in the initial lines of Stata's output. Consider the first lines of the output from the logit model of labor force participation that we use as an example in Chapter 4:

```
. logit lfp k5 k618 age wc hc lwg inc
Iteration 0: log likelihood = -514.8732
Iteration 1: log likelihood = -454.32339
Iteration 2: log likelihood = -452.64187
Iteration 3: log likelihood = -452.63296
Iteration 4: log likelihood = -452.63296
Logit estimates
Number of obs = 753
    (output omitted)
```

The output begins with the iteration log, where the first line reports the value of the log likelihood at the start values, reported as iteration 0 . While earlier we talked about maximizing the likelihood equation, in practice, programs maximize the $\log$ of the likelihood, which simplifies the computations and yields the same ultimate result. For the probability models considered in this book, the $\log$ likelihood is always negative, because the likelihood itself is always between 0 and 1 . In our example, the log likelihood at the start values is -514.8732 . The next four lines in this example show the progress in maximizing the log likelihood, converging to the value of -452.63296 . The rest of the output is discussed later in this section.

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### 3.1.2 ML and sample size

Under the usual assumptions (see Cramer 1986 or Eliason 1993 for specific details), the ML estimator is consistent, efficient, and asymptotically normal. These properties hold as the sample size approaches infinity. While ML estimators are not necessarily bad estimators in small samples, the small sample behavior of ML estimators for the models we consider is largely unknown. With the exception of the logit and Poisson regression, which can be estimated using exact permutation methods with LogXact (Cytel Corporation 2000), alternative estimators with known small sample properties are generally not available. With this in mind, Long $(1997,54)$ proposed the following guidelines for the use of ML in small samples:

It is risky to use ML with samples smaller than 100 , while samples over 500 seem adequate. These values should be raised depending on characteristics of the model and the data. First, if there are many parameters, more observations are needed.... A rule of at least 10 observations per parameter seems reasonable.... This does not imply that a minimum of 100 is not needed if you have only two parameters. Second, if the data are ill-conditioned (e.g., independent variables are highly collinear) or if there is little variation in the dependent variable (e.g., nearly all of the outcomes are 1), a larger sample is required. Third, some models seem to require more observations [such as the ordinal regression model or the zero-inflated count models].

### 3.1.3 Problems in obtaining ML estimates

While the numerical methods used by Stata to compute ML estimates are highly refined and generally work extremely well, you can encounter problems. If your sample size is adequate, but you cannot get a solution or appear to get the wrong solution (i.e., the estimates do not make substantive sense), our experience suggests that the most common cause is that the data have not been properly "cleaned". In addition to mistakes in constructing variables and selecting observations, the scaling of variables can cause problems. The larger the ratio between the largest standard deviation among variables in the model and the smallest standard deviation, the more problems you are likely to encounter with numerical methods due to rounding. For example, if income is measured in units of $\$ 1$, income is likely to have a very large standard deviation relative to other variables. Recoding income to units of \$1,000 can solve the problem.

Overall, however, numerical methods for ML estimation work well when your model is appropriate for your data. Still, Cramer's $(1986,10)$ advice about the need for care in estimation should be taken very seriously:

Check the data, check their transfer into the computer, check the actual computations (preferably by repeating at least a sample by a rival program), and always remain suspicious of the results, regardless of the appeal.

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### 3.1.4 The syntax of estimation commands

All single-equation estimation commands have the same syntax: ${ }^{3}$

$$
\text { command depvar }[\text { indepvars }][\text { weight }][\text { if exp }][\text { in range }][\text {, option }(s)]
$$

Elements in [ ]'s are optional. Here are a few examples for a logit model with lfp as the dependent variable:

```
. logit lfp k5 k618 age wc lwg
    (output omitted)
. logit lfp k5 k618 age wc lwg if hc == 1
    (output omitted)
. logit lfp k5 k618 age wc lwg [pweight=wgtvar]
    (output omitted)
    logit lfp k5 k618 age wc lwg if hc == 1, level(90)
    (output omitted)
```

You can review the output from the last estimation by simply typing the command name again. For example, if the most recent model that you estimated was a logit model, you could have Stata replay the results by simply typing logit.

## Variable lists

depvar is the dependent variable. indepvars is a list of the independent variables. If no independent variables are given, a model with only the intercept(s) is estimated. Stata automatically corrects some mistakes in specifying independent variables. For example, if you include wc as an independent variable when the sample is restricted based on wc (e.g., logit lfp wc k5 k618 age hc if $\mathrm{wc}==1$ ), Stata drops wc from the list of variables. Or, suppose you recode a $k$-category variable into a set of $k$ dummy variables. Recall that one of the dummy variables must be excluded to avoid perfect collinearity. If you included all $k$ dummy variables in indepvars, Stata automatically excludes one of them.

## Specifying the estimation sample

if and in restrictions can be used to define the estimation sample (i.e., the sample used to estimate the model), where the syntax for if and in conditions follows the guidelines in Chapter 2. For example, if you want to estimate a logit model for only women who went to college, you could specify logit lfp k5 k618 age hc lwg if wc==1.

[^13]Missing data Estimation commands use listwise deletion to exclude cases in which there are missing values for any of the variables in the model. Accordingly, if two models are estimated using the same dataset but have different sets of independent variables, it is possible to have different samples. The easiest way to understand this is with a simple example. ${ }^{4}$ Suppose that among the 753 cases in the sample, 23 have missing data for at least one variable. If we estimated a model using all variables, we would get
. logit lfp k5 k618 age wc hc lwg inc, nolog

| Logit estimates |
| :--- |
| $\quad$ (output omitted) |$\quad$ Number of obs $=\quad 730$

Suppose that 7 of the missing cases were missing only for k 618 and that we estimate a second model that excludes this variable:
. logit lfp k5 age wc hc lwg inc, nolog

| Logit estimates |
| :--- |
| (output omitted) |$\quad$ Number of obs $=\quad 737$

The estimation sample for the second model has increased by 7 cases. Importantly, this means that you cannot compute a likelihood-ratio test comparing the two models (see Section 3.3) and that any changes in the estimates could be due either to changes in the model specification or to the use of different samples to estimate the models. When you compare coefficients across models, you want the samples to be exactly the same. If they are not, you cannot compute likelihood-ratio tests, and any interpretations of why the coefficients have changed must take into account differences between the samples.

While Stata uses listwise deletion when estimating models, this does not mean that this is the only or the best way to handle missing data. While the complex issues related to missing data are beyond the scope of our discussion (see Little and Rubin 1987; Schafer 1997; Allison forthcoming), we recommend that you make explicit decisions about which cases to include in your analyses, rather than let cases be dropped implicitly. Personally, we wish that Stata would issue an error rather than automatically dropping cases.
mark and markout commands make it simple to explicitly exclude missing data. mark markvar generates a new variable markvar that equals 1 for all cases. markout markvar varlist changes the values of markvar from 1 to 0 for any cases in which values of any of the variables in varlist are missing. The following example illustrates how this works (missing data were artificially created):
. mark nomiss
. markout nomiss lfp k5 k618 age wc hc lwg inc
. tab nomiss

| nomiss | Freq. | Percent | Cum. |
| ---: | ---: | ---: | ---: |
| 0 | 23 | 3.05 | 3.05 |
| 1 | 730 | 96.95 | 100.00 |
| Total | 753 | 100.00 |  |

[^14]

Since the if condition excludes the same cases from both equations, the sample size is the same for both models. Alternatively, after using mark and markout, we could have used drop if nomiss==0 to delete observations with missing values.

Post-estimation commands and the estimation sample Excepting predict, the post-estimation commands for testing, assessing fit, and making predictions that are discussed below use only observations from the estimation sample, unless you specify otherwise. Accordingly, you do not need to worry about if and in conditions or cases deleted due to missing data when you use these commands. Further details are given below.

Weights Weights indicate that some observations should be given more weight than others when computing estimates. The syntax for specifying weights is [type=varname], where the [ ]'s are part of the command, type is the abbreviation for the type of weight to be used, and varname is the weighting variable. Stata recognizes four types of weights:

1. fweights or frequency weights indicate that an observation represents multiple observations with identical values. For example, if an observation has an fweight of 5, this is equivalent to having 5 identical, duplicate observations. In very large datasets, fweights can substantially reduce the size of the data file. If you do not include a weight option in your estimation command, this is equivalent to specifying $f$ weight $=1$.
2. pweights or sampling weights denote the inverse of the probability that the observation is included due to the sampling design. For example, if a case has a pweight of 1200 , that case represents 1200 observations in the population.
3. aweights or analytic weights are inversely proportional to the variance of an observation. The variance of the $j$ th observation is assumed to be $\sigma^{2} / w_{j}$, where $w_{j}$ is the analytic weight. Analytic weights are used most often when observations are averages and the weights are the number of elements that gave rise to the average. For example, if each observation is the cell mean from a larger, unobserved dataset, the data are heteroskedastic. For some estimation problems, analytic weights can be used to transform the data to reinstate the homoskedasticity assumption.
4. iweights or importance weights have no formal statistical definition. They are used by programmers to facilitate certain types of computations under specific conditions.

The use of weights is a complex topic, and it is easy to apply weights incorrectly. If you need to use weights, we encourage you to read the detailed discussion in the Stata User's Guide

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([U] 14.1.6 weight; [U] 23.13 Weighted estimation). Winship and Radbill (1994) also provide a useful introduction to weights in the linear regression model.
svy Commands For more complex sampling designs that include sampling weights, strata, and PSU identifier variables, Stata provides a set of svy commands. For example, svylogit estimates a binary logit model with corrections for a complex sampling design. While the interpretation of the estimated coefficients is the same for these commands as their non-svy counterparts, we do not consider these commands further here. For further details, type help svy, see Hosmer and Lemeshow (2000, Chapter 6), or [R] svy estimators; [U] 30 Overview of survey estimators.

## Options

The following options apply to most regression models. Unique options for specific models are considered in later chapters.
noconstant constrains the intercept(s) to equal 0 . For example, in a linear regression the command regress yx1 x2, noconstant would estimate the model $y=\beta_{1} x_{1}+\beta_{2} x_{2}+\varepsilon$.
nolog suppresses the iteration history. While this option shortens the output, the iteration history might contain information that indicates problems with your model. If you use this option and you have problems in obtaining estimates, it is a good idea to re-estimate the model without this option and with the trace option.
trace lets you see the values of the parameters for each step of the iteration. This can be useful for determining which variables may be causing a problem if your model has difficulty converging.
level (\#) specifies the level of the confidence interval. By default, Stata provides $95 \%$ confidence intervals for estimated coefficients, meaning that the interval around the estimated $\widehat{\beta}$ would capture the true value of $\beta 95$ percent of the time if repeated samples were drawn. level allows you to specify other intervals. For example, level (90) specifies a $90 \%$ interval. You can also change the default level with the command set level 90 (for $90 \%$ confidence intervals).
cluster (varname) specifies that the observations are independent across the clusters that are defined by unique values of varname, but are not necessarily independent within clusters. Specifying this options lead to robust standard errors, as discussed below, with an additional correction for the effects of clustered data. See Hosmer and Lemeshow (2000, Section 8.3) for a detailed discussion of logit models with clustered data.
In some cases, observations share similarities that violate the assumption of independent observations. For example, the same person might provide information at more than one point in time. Or, several members of the same family might be in the sample, again violating independence. In these examples, it is reasonable to assume that the observations within the groups, which are known as clusters, are not independent. With clustering, the usual standard errors will be incorrect.

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robust replaces the traditional standard errors with robust standard errors, which are also known as Huber, White, or sandwich standard errors. These estimates are considered robust in the sense that they provide correct standard errors in the presence of violations of the assumptions of the model. For example, if the correct model is a binary logit model and a binary probit model is used, the model has been misspecified. The estimates obtained by fitting a logit model cannot be maximum likelihood estimates since an incorrect likelihood function is being used (i.e., a logistic probability density is used instead of the correct normal density). In this situation, the estimator is referred to by White (1982) as a minimum ignorance estimator since the estimators provide the best possible approximation to the true probability density function. When a model is misspecified in this way, the usual standard errors are incorrect. Arminger (1995) makes a compelling case for why robust standard errors should be used. He writes (where we have ellipsed some technical details): "If one keeps in mind that most researchers misspecify the model..., it is obvious that their estimated parameters can usually be interpreted only as minimum ignorance estimators and that the standard errors and test statistics may be far away from the correct asymptotic values, depending on the discrepancy between the assumed density and the actual density that generated the data." However, we have not seen any information on the small sample properties of robust standard errors for nonlinear models (i.e., how well these standard errors work in finite samples). Long and Ervin (2000) consider this problem in the context of the linear regression model, where they found that two small sample versions of the robust standard error work quite well, while the asymptotic version often does worse than the usual standard errors. ${ }^{5}$
Robust estimators are automatically used with svy commands and with the cluster() option. See the User's Guide ([U] 23.11 Obtaining robust variance estimates) and Gould and Sribney (1999, 1.3.4 Robust variance estimator) for a detailed discussion of how robust standard errors are computed in Stata; see Arminger (1995, 111-113) for a more mathematical treatment.

### 3.1.5 Reading the output

We have already discussed the iteration log, so in the following example we suppress it with the nolog option. Here we consider other parts of the output from estimation commands. While the sample output is from logit, our discussion generally applies to other regression models.

## (Continued on next page)

[^15]

## Header

1. Log likelihood $=-452.63296$ corresponds to the value of the $\log$ likelihood at convergence.
2. Number of obs is the number of observations, excluding those with missing values and after any if or in conditions have been applied.
3. LR chi2(7) is the value of a likelihood-ratio chi-squared for the test of the null hypothesis that all of the coefficients associated with independent variables are simultaneously equal to zero. The $p$-value is indicated by Prob $>$ chi2, where the number in parentheses is the number of coefficients being tested.
4. Pseudo R2 is the measure of fit also known as McFadden's $R^{2}$. Details on how this measure is computed are given below, along with a discussion of alternative measures of fit.

## Estimates and standard errors

1. The left column lists the variables in the model, with the dependent variable listed at the top. The independent variables are always listed in the same order as entered on the command line. The constant, labeled _cons, is last.
2. Column Coef . contains the ML estimates.
3. Column Std. Err . is the standard error of the estimates.
4. The resulting $z$-test, equal to the estimate divided by its standard error, is labeled $z$ with the two-tailed significance level listed as $P>|\mathrm{z}|$. A significance level listed as 0.000 means that $P<.0005$ (for example, .0006 is rounded to .001 , while .00049 is rounded to .000 ).
5. The start and end points for the confidence interval for each estimate are listed under [95\% Conf. Interval].

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### 3.1.6 Reformatting output with outreg

outreg is a user-written command (Gallup 2001). To install outreg, type net search outreg and follow the links. outreg can be used to reformat the output from an estimation command to look more like the tables that are seen in articles that use regression models. outreg also makes it easier to move estimation results into a word processor or spreadsheet to make a presentationquality table there. We strongly recommend using outreg or some other automated procedure rather than copying the numbers into a spreadsheet by hand. Not only is it much less tedious, but it also diminishes the possibility of errors. The syntax of outreg is

```
outreg [varlist] using filename [,options ]
```

where varlist contains the names of the independent variables in the order you want them presented in the table and where filename is the name of the file to contain the reformatted results. When you run outreg, the results are not posted in the Results Window, but are only written to this file.

After estimating the logit model that we presented above, we could run outreg as follows:

```
. logit lfp k5 k618 age wc hc lwg
    (output omitted)
. outreg using model1, replace
```

File model1. out is saved to the working directory. We needed to tinker with this file in a text editor to get the spacing just right, but our end result looks like this:

outreg is a very flexible command, with too many options to consider all of them here. Some of the most commonly used options are
replace specifies that filename. out should be overwritten if it already exists.
append indicates that the estimation output should be appended to that of an existing file. This allows you to construct tables with the results from several regressions, as we illustrate in Chapter 4.
se reports standard errors instead of $t / z$ statistics.
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### 3.1 Estimation

pvalue reports $p$-values instead of $t / z$ statistics.
title (text) is a title to be printed at the top of the output.
addnote (text) is a note to be printed at the bottom of the output.
nolabel specifies that variable names rather than variable labels should be printed in the output.
A full list of options and further information about using the command can be obtained by typing help outreg.

### 3.1.7 Alternative output with listcoef

The interpretation of regression models often involves transformations of the usually estimated parameters. For some official Stata estimation commands, there are options to list transformations of the parameters, such as the or option to list odds ratios in logit or the beta option to list standardized coefficients for regress. While Stata is commendably clear in explaining the meaning of the estimated parameters, in practice it is easy to be confused about proper interpretations. For example, the zip model (discussed in Chapter 8) simultaneously estimates a binary and count model, and it is easy to be confused regarding the direction of the effects.

For the estimation commands considered in this book (plus some not considered here), our command listcoef lists estimated coefficients in ways that facilitate interpretation. You can list coefficients selected by name or significance level, list transformations of the coefficients, and request help on proper interpretation. In fact, in many cases you won't even need the normal output from the estimation. You could suppress this output with the prefix quietly (e.g., quietly logit lfp k5 wc hc) and then use the listcoef command. The syntax is

where varlist indicates that coefficients for only these variables are to be listed. If no varlist is given, coefficients for all variables are listed.

## Options for types of coefficients

Depending on the model estimated and the specified options, listcoef computes standardized coefficients, factor changes in the odds or expected counts, or percent changes in the odds or expected counts. More information on these different types of coefficients is provided below, as well as in the chapters that deal with specific types of outcomes. The table below lists which options (details on these options are given below) are available for each estimation command. If an option is the default, it does not need to be specified.

|  | Option |  |  |
| :--- | :--- | :--- | :--- |
| Type 1: regress, probit, cloglog, oprobit, <br> tobit, cnreg, intreg | std | factor | percent |
| Type 2: logit, logistic ologit <br> Type 3: clogit, mlogit, poisson, nbreg, zip, zinb | No | No | No |

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factor requests factor change coefficients.
percent requests percent change coefficients instead of factor change coefficients.
std indicates that coefficients are to be standardized to a unit variance for the independent and/or dependent variables. For models with a latent dependent variable, the variance of the latent outcome is estimated.

## Other options

pvalue(\#) specifies that only coefficients significant at the \# significance level or smaller will be printed. For example, pvalue (.05) specifies that only coefficient significant at the .05 level of less should be listed. If pvalue is not given, all coefficients are listed.
constant includes the constant(s) in the output. By default, they are not listed.
help gives details for interpreting each coefficient.

## Standardized coefficients

std requests coefficients after some or all of the variables have been standardized to a unit variance. Standardized coefficients are computed as follows:
$x$-standardized coefficients The linear regression model can be expressed as

$$
\begin{equation*}
y=\beta_{0}+\beta_{1} x_{1}+\beta_{2} x_{2}+\varepsilon \tag{3.1}
\end{equation*}
$$

The independent variables can be standardized with simple algebra. Let $\sigma_{k}$ be the standard deviation of $x_{k}$. Then, dividing each $x_{k}$ by $\sigma_{k}$ and multiplying the corresponding $\beta_{k}$ by $\sigma_{k}$

$$
y=\beta_{0}+\left(\sigma_{1} \beta_{1}\right) \frac{x_{1}}{\sigma_{1}}+\left(\sigma_{2} \beta_{2}\right) \frac{x_{2}}{\sigma_{2}}+\varepsilon
$$

$\beta_{k}^{S_{x}}=\sigma_{k} \beta_{k}$ is an $x$-standardized coefficient. For a continuous variable, $\beta_{k}^{S_{x}}$ can be interpreted as
For a standard deviation increase in $x_{k}, y$ is expected to change by $\beta_{k}^{S_{x}}$ units, holding all other variables constant.

The same method of standardization can be used in all of the other models we consider in this book.
$y$ and $y^{*}$-standardized coefficients To standardize for the dependent variable, let $\sigma_{y}$ be the standard deviation of $y$. We can standardize $y$ by dividing Equation 3.1 by $\sigma_{y}$ :

$$
\frac{y}{\sigma_{y}}=\frac{\beta_{0}}{\sigma_{y}}+\frac{\beta_{1}}{\sigma_{y}} x_{1}+\frac{\beta_{2}}{\sigma_{y}} x_{2}+\frac{\varepsilon}{\sigma_{y}}
$$

Then $\beta_{k}^{S_{y}}=\beta_{k} / \sigma_{y}$ is a $y$-standardized coefficient that can be interpreted as
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For a unit increase in $x_{k}, y$ is expected to change by $\beta_{k}^{S_{y}}$ standard deviations, holding all other variables constant.

For a dummy variable,
Having characteristic $x_{k}$ (as opposed to not having the characteristic) results in an expected change in $y$ of $\beta_{k}^{S_{y}}$ standard deviations, holding all other variables constant.

In models with a latent dependent variable, the equation $y^{*}=\beta_{0}+\beta_{1} x_{1}+\beta_{2} x_{2}+\varepsilon$ can be divided by $\widehat{\sigma}_{y^{*}}$. To estimate the variance of the latent variable, the quadratic form is used:

$$
\widehat{\operatorname{Var}}\left(y^{*}\right)=\widehat{\beta}^{\prime} \widehat{\operatorname{Var}}(\mathbf{x}) \widehat{\beta}+\operatorname{Var}(\varepsilon)
$$

where $\widehat{\beta}$ is a vector of estimated coefficients and $\widehat{\operatorname{Var}}(\mathbf{x})$ is the covariance matrix for the $x$ 's computed from the observed data. By assumption, $\operatorname{Var}(\varepsilon)=1$ in probit models and $\operatorname{Var}(\varepsilon)=\pi^{2} / 3$ in logit models.

Fully standardized coefficients In the linear regression model it is possible to standardize both $y$ and the $x$ 's:

$$
\frac{y}{\sigma_{y}}=\frac{\beta_{0}}{\sigma_{y}}+\left(\frac{\sigma_{1} \beta_{1}}{\sigma_{y}}\right) \frac{x_{1}}{\sigma_{1}}+\left(\frac{\sigma_{2} \beta_{2}}{\sigma_{y}}\right) \frac{x_{2}}{\sigma_{2}}+\frac{\varepsilon}{\sigma_{y}}
$$

Then, $\beta_{k}^{S}=\left(\sigma_{k} \beta_{k}\right) / \sigma_{y}$ is a fully standardized coefficient that can be interpreted as follows:
For a standard deviation increase in $x_{k}, y$ is expected to change by $\beta_{k}^{S}$ standard deviations, holding all other variables constant.

The same approach can be used in models with a latent dependent variable $y^{*}$.

Example of listcoef for standardized coefficients Here we illustrate the computation of standardized coefficients for the regression model. Examples for other models are given in later chapters. The standard output from regress is
. use science2, clear
. regress job female phd mcit3 fellow pub1 cit1


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Now, we use listcoef:

```
. listcoef female cit1, help
regress (N=161): Unstandardized and Standardized Estimates
Observed SD: . }8826414
SD of Error: .78853764
```

| job | b | t | P>\|t| | bStdX | bStdY | bStdXY | SDofX |
| ---: | :---: | :---: | :---: | ---: | ---: | ---: | ---: |
| female | -0.12432 | -0.790 | 0.431 | -0.0534 | -0.1409 | -0.0605 | 0.4298 |
| cit1 | 0.00809 | 1.965 | 0.051 | 0.1719 | 0.0092 | 0.1947 | 21.2422 |

[^16]By default for regress, listcoef lists the standardized coefficients. Notice that we only requested information on two of the variables.

## Factor and percent change

In logit-based models and models for counts, coefficients can be expressed either as: (1) a factor or multiplicative change in the odds or the expected count (requested in listcoef by the factor option); or (2) the percent change in the odds or expected count (requested with the percent option). While these can be computed with options to some estimation commands, listcoef provides a single method to compute these. Details on these coefficients are given in later chapters for each specific model.

### 3.2 Post-estimation analysis

There are three types of post-estimation analysis that we consider in the remainder of this chapter. The first is statistical testing that goes beyond routine tests of a single coefficient. This is done with Stata's powerful test and lrtest commands. In later chapters, we present other tests of interest for a given model (e.g., tests of the parallel regression assumption for ordered regression models). The second post-estimation task is assessing the fit of a model using scalar measures computed by our command fitstat. Examining outliers and residuals for binary models is considered in Chapter 4. The third task, and the focus of much of this book, is interpreting the predictions from nonlinear models. We begin by discussing general issues that apply to all nonlinear models. We then discuss our SPost commands that implement these methods of interpretation.

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### 3.3 Testing

Coefficients estimated by ML can be tested with Wald tests using test and likelihood-ratio (LR) tests using lrtest. For both types of tests there is a null hypothesis $H_{0}$ that implies constraints on the model's parameters. For example, $H_{0}: \beta_{\mathrm{wc}}=\beta_{\mathrm{hc}}=0$ hypothesizes that two of the parameters are zero in the population.

The Wald test assesses $H_{0}$ by considering two pieces of information. First, all else being equal, the greater the distance between the estimated coefficients and the hypothesized values, the less support we have for $H_{0}$. Second, the greater the curvature of the log-likelihood function, the more certainty we have about our estimates. This means that smaller differences between the estimates and hypothesized values are required to reject $H_{0}$. The LR test assesses a hypothesis by comparing the log likelihood from the full model (i.e., the model that does not include the constraints implied by $H_{0}$ ) and a restricted model that imposes the constraints. If the constraints significantly reduce the $\log$ likelihood, then $H_{0}$ is rejected. Thus, the LR test requires estimation of two models. Even though the LR and Wald tests are asymptotically equivalent, in finite samples they give different answers, particularly for small samples. In general, it is unclear which test is to be preferred. Rothenberg (1984) and Greene (2000) suggest that neither test is uniformly superior, although many statisticians prefer the LR.

### 3.3.1 Wald tests

test computes Wald tests for linear hypotheses about parameters from the last model estimated. Here we consider the most useful features of this command for regression models. Information on features for multiple equation models, such as mlogit, zip, and zinb, are discussed in Chapters 6 and 7. Use help test for additional features and help testnl for testing nonlinear hypotheses.

The first syntax for test allows you to specify that one or more coefficients from the last estimation are simultaneously equal to 0

```
test varlist [, accumulate ]
```

where varlist contains names of one or more independent variable from the last estimation. The accumulate option will be discussed shortly.

Some examples should make this first syntax clear. With a single variable listed, k 5 in this case, we are testing $H_{0}$ : $\beta_{\mathrm{k} 5}=0$.

```
. logit lfp k5 k618 age wc hc lwg inc
(output omitted)
. test k5
( 1) k5 = 0.0
    chi2( 1) = 55.14
    Prob > chi2 = 0.0000
```

The resulting chi-squared test with 1 degree of freedom equals the square of the $z$-test in the output from the estimation command, and we can reject the null hypothesis.

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With two variables listed, we are testing $H_{0}: \beta_{\mathrm{k} 5}=\beta_{\mathrm{k} 618}=0$ :

```
. test k5 k618
(1) k5 = 0.0
( 2) k618 = 0.0
    chi2( 2) = 55.16
    Prob > chi2 = 0.0000
```

We can reject the hypothesis that the effects of young and older children are simultaneously zero.
In our last example, we include all of the independent variables.

```
. test k5 k618 age wc hc lwg inc
(1) k5 = 0.0
(2) }\textrm{k}618=0.
( 3) age = 0.0
(4) wc = 0.0
( 5) hc = 0.0
(6) }1\textrm{wg}=0.
(7) inc = 0.0
    chi2( 7) = 94.98
    Prob > chi2 = 0.0000
```

This is a test of the hypothesis that all of the coefficients except the intercept are simultaneously equal to zero. As noted above, a likelihood-ratio test of this same hypothesis is part of the standard output of estimation commands (e.g., LR chi2 $(7)=124.48$ from the earlier logit output).

The second syntax for test allows you to test hypotheses about linear combinations of coefficients:

```
test [exp=exp] [, accumulate ]
```

For example, to test that two coefficients are equal, for example $H_{0}: \beta_{\mathrm{k} 5}=\beta_{\mathrm{k} 618}$ :

```
. test k5=k618
( 1) k5 - k618 = 0.0
    chi2( 1) = 49.48
    Prob > chi2 = 0.0000
```

Because the test statistic is significant, we can reject the null hypothesis that the effect of young children on labor force participation is equal to the effect of having older children.

## The accumulate option

The accumulate option allows you to build more complex hypotheses based on the prior use of the test command. For example, you might begin with a test of $H_{0}: \beta_{\mathrm{k} 5}=\beta_{\mathrm{k} 618}$ :

```
. test k5=k618
    (1) k5 - k618 = 0.0
    chi2( 1) = 49.48
    Prob > chi2 = 0.0000
```

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Then, add the constraint that $\beta_{\mathrm{wc}}=\beta_{\mathrm{hc}}$ :
. test wc=hc, accumulate
(1) $\mathrm{k} 5-\mathrm{k} 618=0.0$
(2) $\mathrm{wc}-\mathrm{hc}=0.0$

$$
\begin{aligned}
\text { chi2 }(2) & =52.16 \\
\text { Prob }>\text { chi2 } & =0.0000
\end{aligned}
$$

This results in a test of $H_{0}: \beta_{\mathrm{k} 5}=\beta_{\mathrm{k} 618}, \beta_{\mathrm{wc}}=\beta_{\mathrm{hc}}$.

### 3.3.2 LR tests

lrtest compares nested models using an LR test. The syntax is
lrtest [, saving(name_recent) úsing(name_full) model(name_nested) ]

The first step is to save information from the last model that was estimated. For example,

```
. logit lfp k5 k618 age wc hc lwg inc, nolog
    (output omitted)
. lrtest, saving(0)
```

where we are using the name 0 to save information on the estimated model. While any name up to four characters (e.g., mod1) can be used, it is standard Stata practice to use 0 to name the first model. After you save the results, you estimate a model that is nested in the full model. A nested model is one that can be created by imposing constraints on the coefficients in the prior model. Most commonly, one excludes some of the variables that were included in the first model, which in effect constrains the coefficients of the these variables to be zero. For example, if we drop k5 and k618 from the last model

```
. logit lfp age wc hc lwg inc, nolog
    (output omitted)
. lrtest
Logit: likelihood-ratio test chi2(2) =
```

The result is an LR test of the hypothesis $H_{0}: \beta_{\mathrm{k} 5}=\beta_{\mathrm{k} 618}=0$. The significant chi-squared statistic means that we reject the null hypothesis that these two coefficients are simultaneously equal to zero.

There are two complications that are useful to know about. First, if you save the results of the full model with a name other than 0 , say lrtest, saving(mod1), you must indicate the name of the full model with using(). For example,

```
. logit lfp k5 k618 age wc hc lwg inc, nolog
    (output omitted)
. lrtest, saving(mod1)
. logit lfp age wc hc lwg inc, nolog
    (output omitted)
. lrtest, using(mod1)
Logit: likelihood-ratio test chi2(2) = 66.49
Prob > chi2 = 0.0000
```

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Second, if you estimate the nested model first, you need to save the results from both models and then use model () to specify the nested model. For example,

```
. * estimate nested model
. logit lfp age wc hc lwg inc, nolog
    (output omitted)
. lrtest, saving(0)
. * estimate full model
. logit lfp k5 k618 age wc hc lwg inc, nolog
    (output omitted)
. lrtest, saving(1)
. lrtest, using(1) model(0)
Logit: likelihood-ratio test chi2(2) = 66.49
```


## Avoiding invalid LR tests

lrtest does not prevent you from computing an invalid test. There are two things that you must check. First, the two models must be nested. Second, the two models must be estimated on exactly the same sample. If either of these conditions are violated, the results of lrtest are meaningless. For details on ensuring the same sample size, see our discussion of mark and markout in Section 3.1.4.

### 3.4 Measures of fit

Assessing fit involves both the analysis of the fit of individual observations and the evaluation of scalar measures of fit for the model as a whole. Regarding the former, Pregibon (1981) extended methods of residual and outlier analysis from the linear regression model to the case of binary logit and probit (see also Cook and Weisberg 1999, Part IV). These measures are considered in Chapter 4. Measures for many count models are also available (Cameron and Trivedi 1998) . Unfortunately, similar methods for ordinal and nominal outcomes are not available. Many scalar measures have been developed to summarize the overall goodness of fit for regression models of continuous, count, or categorical dependent variables. A scalar measure can be useful in comparing competing models and ultimately in selecting a final model. Within a substantive area, measures of fit can provide a rough index of whether a model is adequate. However, there is no convincing evidence that selecting a model that maximizes the value of a given measure results in a model that is optimal in any sense other than the model having a larger (or, in some instances, smaller) value of that measure. While measures of fit provide some information, it is only partial information that must be assessed within the context of the theory motivating the analysis, past research, and the estimated parameters of the model being considered.

[^18]
## Syntax of fitstat

Our command fitstat calculates a large number of fit statistics for the estimation commands we consider in this book. With its saving() and using () options, the command also allows the comparison of fit measures across two models. While fitstat duplicates some measures computed by other commands (e.g., the pseudo- $R^{2}$ in standard Stata output or lfit), fitstat adds many more measures and makes it convenient to compare measures across models. The syntax is

```
fitstat [, saving(name) using(name) save dif bic force]
```

While many of the calculated measures are based on values returned by the estimation command, for some measures it is necessary to compute additional statistics from the estimation sample. This is automatically done using the estimation sample from the last estimation command. fitstat can also be used when models are estimated with weighted data, with two limitations. First, some measures cannot be computed with some types of weights. Second, with pweights, we use the "pseudo-likelihoods" rather than the likelihood to compute our measures of fit. Given the heuristic nature of the various measures of fit, we see no reason why the resulting measures would be inappropriate.
fitstat terminates with an error if the last estimation command does not return a value for the $\log$ likelihood equation with only an intercept (i.e., if e(ll_0)=.). This occurs, for example, if the noconstant option is used to estimate a model.

## Options

saving (name) saves the computed measures in a matrix for subsequent comparisons. name must be four characters or shorter.
using (name) compares the fit measures for the current model with those of the model saved as name. name cannot be longer than four characters.
save and dif are equivalent to saving (0) and using (0).
bic presents only BIC and other information measures. When comparing two models, fitstat reports Raftery's (1996) guidelines for assessing the strength of one model over another.
force is required to compare two models when the number of observations or the estimation method varies between the two models.

## Models and measures

Details on the measures of fit are given below. Here we only summarize which measures are computed for which models. $\square$ indicates a measure is computed, and $\square$ indicates the measure is not computed.

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$\left.\begin{array}{llllllll}\hline \hline & & & & & & & \begin{array}{l}\text { gologit } \\ \text { nbreg }\end{array} \\ \text { poisson }\end{array}\right)$

1: For cloglog the log likelihood for the intercept-only model does not correspond to the first step in the iterations.
2: For zip and zinb, the log likelihood for the intercepts-only model is calculated by estimating zip | zinb depvar, $\inf$ (_cons).
3: The adjusted count $R^{2}$ is not defined for clogit.

## Example of fitstat

To compute fit statistics for a single model, we first estimate the model and then run fitstat:
. logit lfp k5 k618 age wc hc lwg inc, nolog
(output omitted)
. fitstat
Measures of Fit for logit of lfp
Log-Lik Intercept Only:
D(745):

McFadden's R2:
Maximum Likelihood R2:
McKelvey and Zavoina's R2:
Variance of y*:
Count R2:
AIC:
BIC:
fitstat is particularly useful for comparing two models. To do this, you begin by estimating a model and then save the results from fitstat. Here, we use quietly to suppress the output from fitstat since we list those results in the next step:

```
. logit lfp k5 k618 age wc hc lwg inc, nolog
    (output omitted)
. quietly fitstat, saving(mod1)
```

Next, we generate agesq which is the square of age. The new model adds agesq and drops k618, hc , and lwg. To compare the saved model to the current model, type

```
. generate agesq = age*age
. logit lfp k5 age agesq wc inc, nolog
    (output omitted)
```

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| Measures of Fit for logit of lfp |  |  |  |
| :---: | :---: | :---: | :---: |
|  | Current | Saved | Difference |
| Model: | logit | logit |  |
| N: | 753 | 753 | 0 |
| Log-Lik Intercept Only: | -514.873 | -514.873 | 0.000 |
| Log-Lik Full Model: | -461.653 | -452.633 | -9.020 |
| D: | 923.306(747) | 905.266(745) | 18.040 (2) |
| LR: | 106.441(5) | 124.480 (7) | 18.040 (2) |
| Prob > LR: | 0.000 | 0.000 | 0.000 |
| McFadden's R2: | 0.103 | 0.121 | -0.018 |
| McFadden's Adj R2: | 0.092 | 0.105 | -0.014 |
| Maximum Likelihood R2: | 0.132 | 0.152 | -0.021 |
| Cragg \& Uhler's R2: | 0.177 | 0.204 | -0.028 |
| McKelvey and Zavoina's R2: | 0.182 | 0.217 | -0.035 |
| Efron's R2: | 0.135 | 0.155 | -0.020 |
| Variance of y*: | 4.023 | 4.203 | -0.180 |
| Variance of error: | 3.290 | 3.290 | 0.000 |
| Count R2: | 0.677 | 0.693 | -0.016 |
| Adj Count R2: | 0.252 | 0.289 | -0.037 |
| AIC: | 1.242 | 1.223 | 0.019 |
| AIC*n : | 935.306 | 921.266 | 14.040 |
| BIC: | -4024.871 | -4029.663 | 4.791 |
| BIC ${ }^{-}$: | -73.321 | -78.112 | 4.791 |
| Difference of 4.791 in $\mathrm{BIC}^{-}$provides positive support for saved model. |  |  |  |
| Note: p-value for difference | ce in LR is only | alid if model | re nested. |

## Methods and formulas for fitstat

This section provides brief descriptions of each measure computed by fitstat. Full details along with citations to original sources are found in Long (1997). The measures are listed in the same order as the output above.

Log likelihood based measures Stata begins maximum likelihood iterations by computing the log likelihood of the model with all parameters but the intercept(s) constrained to zero, referred to as $L\left(M_{\text {Intercept }}\right)$. The log likelihood upon convergence, referred to as $M_{\text {Full }}$, is also listed. This information is usually presented as the first step of the iteration $\log$ and in the header for the estimation results. ${ }^{6}$

Chi-squared test of all coefficients An LR test of the hypothesis that all coefficients except the intercept(s) are zero can be computed by comparing the log likelihoods: $L R=2 \ln L\left(M_{\text {Full }}\right)-$ $2 \ln L\left(M_{\text {Intercept }}\right)$. This statistic is sometimes designated as $\mathrm{G}^{2} . L R$ is reported by Stata as LR $\operatorname{chi2}(7)=124.48$, where the degrees of freedom, (7), are the number of constrained parameters. fitstat reports this statistic as LR(7): 124.48. For zip and zinb, $L R$ tests that the coefficients in the count portion (not the binary portion) of the model are zero.

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Deviance The deviance compares a given model to a model that has one parameter for each observation so that the model reproduces perfectly the observed data. The deviance is defined as $D=-2 \ln L\left(M_{\text {Full }}\right)$, where the degrees of freedom equals $N$ minus the number of parameters. Note that $D$ does not have a chi-squared distribution.
$\mathbf{R}^{2}$ in the LRM For regress, fitstat reports the standard coefficient of determination which can be defined variously as

$$
\begin{equation*}
R^{2}=1-\frac{\sum_{i=1}^{N}\left(y_{i}-\widehat{y}_{i}\right)^{2}}{\sum_{i=1}^{N}\left(y_{i}-\bar{y}\right)^{2}}=\frac{\widehat{\operatorname{Var}}(\widehat{y})}{\widehat{\operatorname{Var}}(\widehat{y})+\widehat{\operatorname{Var}}(\widehat{\varepsilon})}=1-\left[\frac{L\left(M_{\text {Intercept }}\right)}{L\left(M_{\text {Full }}\right)}\right]^{2 / N} \tag{3.2}
\end{equation*}
$$

The adjusted $R^{2}$ is defined as

$$
\bar{R}^{2}=\left(R^{2}-\frac{K}{N-1}\right)\left(\frac{N-1}{N-K-1}\right)
$$

where $K$ is the number of independent variables.

Pseudo- $R^{2}$, $\mathbf{s}$ While each of the definitions of $R^{2}$ in equation 3.2 give the same numeric value in the LRM, they give different answers and thus provide different measures of fit when applied to the other models evaluated by fitstat.

McFadden's $R^{2} \quad$ McFadden's $R^{2}$, also known as the "likelihood-ratio index", compares a model with just the intercept to a model with all parameters. It is defined as

$$
R_{\mathrm{McF}}^{2}=1-\frac{\ln \widehat{L}\left(M_{\text {Full }}\right)}{\ln \widehat{L}\left(M_{\text {Intercept }}\right)}
$$

If model $M_{\text {Intercept }}=M_{\text {Full }}, R_{\mathrm{McF}}^{2}$ equals 0 , but $R_{\mathrm{McF}}^{2}$ can never exactly equal one. This measure, which is computed by Stata as Pseudo R2 $=0.1209$, is listed in fitstat as: McFadden's R2: 0.121 Since $R_{\text {McF }}^{2}$ always increases as new variables are added, an adjusted version is also available:

$$
\bar{R}_{\mathrm{McF}}^{2}=1-\frac{\ln \widehat{L}\left(M_{\text {Full }}\right)-K^{*}}{\ln \widehat{L}\left(M_{\text {Intercept }}\right)}
$$

where $K^{*}$ is the number of parameters (not independent variables).

Maximum likelihood $R^{2}$ Another analogy to $R^{2}$ in the LRM was suggested by Maddala:

$$
R_{\mathrm{ML}}^{2}=1-\left[\frac{L\left(M_{\text {Intercept }}\right)}{L\left(M_{\text {Full }}\right)}\right]^{2 / N}=1-\exp \left(-G^{2} / N\right)
$$

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Cragg \& Uhler's $R^{2} \quad$ Since $R_{\mathrm{ML}}^{2}$ only reaches a maximum of $1-L\left(M_{\text {Intercept }}\right)^{2 / N}$, Cragg and Uhler suggested a normed measure:

$$
R_{\mathrm{C} \& \mathrm{U}}^{2}=\frac{R_{\mathrm{ML}}^{2}}{\max R_{\mathrm{ML}}^{2}}=\frac{1-\left[L\left(M_{\text {Intercept }}\right) / L\left(M_{\mathrm{Full}}\right)\right]^{2 / N}}{1-L\left(M_{\mathrm{Intercept}}\right)^{2 / N}}
$$

Efron's $R^{2} \quad$ For binary outcomes, Efron's pseudo- $R^{2}$ defines $\widehat{y}=\widehat{\pi}=\widehat{\operatorname{Pr}}(y=1 \mid \mathbf{x})$ and equals

$$
R_{\text {Efron }}^{2}=1-\frac{\sum_{i=1}^{N}\left(y_{i}-\widehat{\pi}_{i}\right)^{2}}{\sum_{i=1}^{N}\left(y_{i}-\bar{y}\right)^{2}}
$$

$V\left(y^{*}\right), V(\varepsilon)$ and McKelvey and Zavoina's $R^{2} \quad$ Some models can be defined in terms of a latent variable $y^{*}$. This includes the models for binary or ordinal outcomes: logit, probit, ologit and oprobit, as well as some models with censoring: tobit, cnreg, and intreg. Each model is defined in terms of a regression on a latent variable $y^{*}$ :

$$
y^{*}=\mathbf{x} \beta+\varepsilon
$$

Using $\widehat{\operatorname{Var}}\left(\widehat{y}^{*}\right)=\widehat{\beta^{\prime}} \widehat{\operatorname{Var}}(\mathbf{x}) \widehat{\beta}$, McKelvey and Zavoina proposed

$$
R_{M \& Z}^{2}=\frac{\widehat{\operatorname{Var}}\left(\widehat{y}^{*}\right)}{\widehat{\operatorname{Var}}\left(y^{*}\right)}=\frac{\widehat{\operatorname{Var}}\left(\widehat{y}^{*}\right)}{\widehat{\operatorname{Var}}\left(\widehat{y}^{*}\right)+\operatorname{Var}(\varepsilon)}
$$

In models for categorical outcomes, $\operatorname{Var}(\varepsilon)$ is assumed to identify the model.

Count and adjusted count $R^{2}$ Observed and predicted values can be used in models with categorical outcomes to compute what is known as the count $R^{2}$. Consider the binary case where the observed $y$ is 0 or 1 and $\pi_{i}=\widehat{\operatorname{Pr}}\left(y=1 \mid \mathbf{x}_{i}\right)$. Define the expected outcome as

$$
\widehat{y}_{i}= \begin{cases}0 & \text { if } \widehat{\pi}_{i} \leq 0.5 \\ 1 & \text { if } \widehat{\pi}_{i}>0.5\end{cases}
$$

This allows us to construct a table of observed and predicted values, such as that produced for the logit model by the Stata command lstat:

| lstat |  |  |  |
| :---: | :---: | :---: | :---: |
| Logistic mod | lfp |  |  |
| Classified | D | $\sim^{\text {D }}$ | Total |
| + | 342 | 145 | 487 |
| - | 86 | 180 | 266 |
| Total | 428 | 325 | 753 |
| Classified + if predicted $\operatorname{Pr}(\mathrm{D})$ >= . 5 |  |  |  |
| True D defined as lfp $\backslash^{\sim}=0$ |  |  |  |

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From this output, we can see that positive responses were predicted for 487 observations, of which 342 of these were correctly classified because the observed response was positive ( $y=1$ ), while the other 145 were incorrectly classified because the observed response was negative $(y=0)$. Likewise, of the 266 observations for which a negative response was predicted, 180 were correctly classified, and 86 were incorrectly classified.

A seemingly appealing measure is the proportion of correct predictions, referred to as the count $R^{2}$,

$$
R_{\text {Count }}^{2}=\frac{1}{N} \sum_{j} n_{j j}
$$

where the $n_{j j}$ 's are the number of correct predictions for outcome $j$. The count $R^{2}$ can give the faulty impression that the model is predicting very well. In a binary model without knowledge about the independent variables, it is possible to correctly predict at least 50 percent of the cases by choosing the outcome category with the largest percentage of observed cases. To adjust for the largest row marginal,

$$
R_{\mathrm{AdjCount}}^{2}=\frac{\sum_{j} n_{j j}-\max _{r}\left(n_{r+}\right)}{N-\max _{r}\left(n_{r+}\right)}
$$

where $n_{r+}$ is the marginal for row $r$. The adjusted count $R^{2}$ is the proportion of correct guesses beyond the number that would be correctly guessed by choosing the largest marginal.

Information measures This class of measures can be used to compare models across different samples or to compare non-nested models.

AIC Akaike's (1973) information criteria is defined as AIC $=\frac{-2 \ln \widehat{L}\left(M_{k}\right)+2 P}{N}$, where $\widehat{L}\left(M_{k}\right)$ is the likelihood of the model and $P$ is the number of parameters in the model (e.g., $K+1$ in the binary regression model where $K$ is the number of regressors). All else being equal, the model with the smaller AIC is considered the better fitting model. Some authors define AIC as being $N$ times the value we report (see, e.g., the mlfit add-on command by Tobias and Campbell). We report this quantity as AIC $*$ n.

BIC and BIC' The Bayesian information criterion has been proposed by Raftery (1996 and the literature cited therein) as a measure of overall fit and a means to compare nested and non-nested models. Consider the model $M_{k}$ with deviance $D\left(M_{k}\right)$. BIC is defined as

$$
\mathrm{BIC}_{k}=D\left(M_{k}\right)-d f_{k} \ln N
$$

where $d f_{k}$ is the degrees of freedom associated with the deviance. The more negative the $\mathrm{BIC}_{k}$, the better the fit. A second version of BIC is based on the LR chi-square with $d f_{k}^{\prime}$ equal to the number of regressors (not parameters) in the model. Then,

$$
\mathrm{BIC}_{k}^{\prime}=-G^{2}\left(M_{k}\right)+d f_{k}^{\prime} \ln N
$$

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The more negative the $\mathrm{BIC}_{k}^{\prime}$ the better the fit. The difference in the BICs from two models indicates which model is more likely to have generated the observed data. Since $\mathrm{BIC}_{1}-\mathrm{BIC}_{2}=\mathrm{BIC}_{1}^{\prime}-\mathrm{BIC}_{2}^{\prime}$, the choice of which BIC measure to use is a matter of convenience. If $\mathrm{BIC}_{1}-\mathrm{BIC}_{2}<0$, then the first model is preferred. If $\mathrm{BIC}_{1}-\mathrm{BIC}_{2}>0$, then the second model is preferred. Raftery (1996) suggested guidelines for the strength of evidence favoring $M_{2}$ against $M_{1}$ based on a difference in BIC or BIC':

| Absolute <br> Difference | Evidence |
| :---: | :---: |
| $0-2$ | Weak |
| $2-6$ | Positive |
| $6-10$ | Strong |
| $>10$ | Very Strong |

### 3.5 Interpretation

Models for categorical outcomes are nonlinear. Understanding the implications of nonlinearity is fundamental to the proper interpretation of these models. In this section we begin with a heuristic discussion of the idea of nonlinearity and the implications of nonlinearity for the proper interpretation of these models. We then introduce a set of commands that facilitate proper interpretation. Later chapters contain the details for specific models.


Figure 3.1: A simple linear model.

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Linear models Figure 3.1 shows a simple, linear regression model, where $y$ is the dependent variable, $x$ is a continuous independent variable, and $d$ is a binary independent variable. The model being estimated is

$$
y=\alpha+\beta x+\delta d
$$

where for simplicity we assume that there is no error term. The solid line plots $y$ as $x$ changes holding $d=0$; that is, $y=\alpha+\beta x$. The dashed line plots $y$ as $x$ changes when $d=1$, which has the effect of changing the intercept: $y=\alpha+\beta x+\delta 1=(\alpha+\delta)+\beta x$.

The effect of $x$ on $y$ can be computed as the partial derivative or slope of the line relating $x$ to $y$, often called the marginal effect or marginal change:

$$
\frac{\partial y}{\partial x}=\frac{\partial(\alpha+\beta x+\delta d)}{\partial x}=\beta
$$

This equation is the ratio of the change in $y$ to the change in $x$, when the change in $x$ is infinitely small, holding $d$ constant. In a linear model, the marginal is the same at all values of $x$ and $d$. Consequently, when $x$ increases by one unit, $y$ increases by $\beta$ units regardless of the current values for $x$ and $d$. This is shown by the four small triangles with bases of length one and heights of $\beta$.

The effect of $d$ cannot be computed with a partial derivative since $d$ is discrete. Instead, we measure the discrete change in $y$ as $d$ changes from 0 to 1 , holding $x$ constant:

$$
\frac{\Delta y}{\Delta d}=(\alpha+\beta x+\delta 1)-(\alpha+\beta x+\delta 0)=\delta
$$

When $d$ changes from 0 to $1, y$ changes by $\delta$ units regardless of the level of $x$. This is shown by the two arrows marking the distance between the solid and dashed lines. As a consequence of the linearity of the model, the discrete change equals the partial change in linear models.

The distinguishing feature of interpretation in the LRM is that the effect of a given change in an independent variable is the same regardless of the value of that variable at the start of its change and regardless of the level of the other variables in the model. That is, interpretation only needs to specify which variable is changing, by how much, and that all other variables are being held constant.

Given the simple structure of linear models, such as regress, most interpretations only require reporting the estimates. In some cases, it is useful to standardize the coefficients, which can be obtained with listcoef as discussed earlier.
(Graph on next page)

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Figure 3.2: A simple nonlinear model.

Nonlinear models Figure 3.2 plots a logit model where $y=1$ if the outcome event occurs, say, if a person is in the labor force, else $y=0$. The curves are from the logit equation: ${ }^{7}$

$$
\begin{equation*}
\operatorname{Pr}(y=1)=\frac{\exp (\alpha+\beta x+\delta d)}{1+\exp (\alpha+\beta x+\delta d)} \tag{3.3}
\end{equation*}
$$

Once again, $x$ is continuous and $d$ is binary.
The nonlinearity of the model makes it more difficult to interpret the effects of $x$ and $d$ on the probability of an event occurring. For example, neither the marginal nor the discrete change with respect to $x$ are constant:

$$
\begin{aligned}
& \frac{\partial \operatorname{Pr}(y=1)}{\partial x} \neq \beta \\
& \frac{\Delta \operatorname{Pr}(y=1)}{\Delta d} \neq \delta
\end{aligned}
$$

This is illustrated by the triangles. Since the solid curve for $d=0$ and the dashed curve for $d=1$ are not parallel, $\Delta_{1} \neq \Delta_{4}$. And, the effect of a unit change in $x$ differs according to the level of both $d$ and $x: \Delta_{2} \neq \Delta_{3} \neq \Delta_{5} \neq \Delta_{6}$. In nonlinear models the effect of a change in a variable depends on the values of all variables in the model and is no longer simply equal to one of the parameters of the model.

[^20]
### 3.5.1 Approaches to interpretation

There are several general approaches for interpreting nonlinear models.

1. Predictions can be computed for each observation in the sample using predict.
2. The marginal or discrete change in the outcome can be computed at a representative value of the independent variables using prchange.
3. Predicted values for substantively meaningful "profiles" of the independent variables can be compared using prvalue, prtab, or prgen.
4. The nonlinear model can be transformed to a model linear in some other outcome. As we discuss in Chapter 4, the logit model in Equation 3.3 can be written as

$$
\ln \left(\frac{\operatorname{Pr}(y=1)}{1-\operatorname{Pr}(y=1)}\right)=\alpha+\beta x+\delta d
$$

which can then be interpreted with methods for linear model or the exponential of the coefficients can be interpreted in terms of factor changes in the odds.

The first three of these methods are now considered. Details on using these approaches for specific models are given in later chapters.

### 3.5.2 Predictions using predict

predict can be used to compute predicted values for each observation in the current dataset. While predict is a powerful command with many options, we consider only the simplest form of the command that provides all of the details that we need. For additional options, you can enter help predict. The simplest syntax for predict is
predict newvarname
where newvarname is the name or names of the new variables that are being generated. The quantity computed for newvarname depends on the model that was estimated, and the number of new variables created depends on the model. The defaults are listed in the following table.

| Estimation | Quantity <br> Computed |
| :--- | :--- |
| regress | Predicted value $\widehat{y}=\mathbf{x} \widehat{\beta}$. |
| logit, logistic, probit, cloglog, <br> ologit, oprobit, clogit, mlogit | Predicted probabilities $\widehat{\operatorname{Pr}}(y=k)$. |
| poisson, nbreg, zip, zinb | Predicted count or rate. |

In the following example, we generate predicted probabilities for a logit model of women's labor force participation. The values of pr1 generated by predict are the probabilities of a woman being in the labor force for each of the observations in the dataset:

[^21]3.5 Interpretation

```
logit lfp k5 k618 age wc hc lwg inc
(output omitted)
. predict pr1
(option p assumed; Pr(lfp))
. sum pr1
\begin{tabular}{r|rrrrr} 
Variable & Obs & Mean & Std. Dev. & Min & Max \\
\hline pr1 & 753 & .5683931 & .1944213 & .0139875 & .9621198
\end{tabular}
```

The summary statistics show that the predicted probabilities in the sample range from .013 to .962 , with an average probability of .568 . Further discussion of predicted probabilities for the logit model is provided in Chapter 4.

For models for ordinal or nominal categories, predict computes the predicted probability of an observation falling into each of the outcome categories. So, instead of specifying one variable for predictions, you specify as many names as there are categories. For example, after estimating a model for a nominal dependent variable with four categories, you can type predict pr1 pr2 pr3 pr4. These new variables contain the predicted probabilities of being in the first, second, third, and fourth categories, as ordered from the lowest value of the dependent variable to the highest.

For count models, predict computes predicted counts for each observation. Our command prcounts computes the predicted probabilities of observing specific counts (e.g., $\operatorname{Pr}(y=3)$ ) and the cumulative probabilities (e.g., $\operatorname{Pr}(y \leq 3)$ ). Further details are given in Chapter 7.

### 3.5.3 Overview of prvalue, prchange, prtab, and prgen

We have written the post-estimation commands prvalue, prchange, prtab, and prgen to make it simple to compute specific predictions that are useful for the interpretation of models for categorical and count outcomes. Details on installing these programs are given in Chapter 1.
prvalue computes predicted values of the outcomes for specified values of the independent variables, and can compute differences in predictions for two sets of values. prvalue is the most basic command. Indeed, it can be used to compute all of the quantities except marginal change from the next three commands.
prtab creates a table of predicted outcomes for a cross-classification of up to four categorical independent variables, while other independent variables are held at specified values.
prchange computes discrete and marginal changes in the predicted outcomes.
prgen computes predicted outcomes as a single independent variable changes over a specified range, holding other variables constant. New variables containing these values are generated which can then be plotted. prgen is limited in that it cannot handle complex model specifications in which a change in the value of the key independent variable implies a change in another independent variable, such as in models that include terms for both age and age squared. For these models, we have created the more general, but more complex, command praccum, which we describe in Chapter 8.

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The most effective interpretation involves using all of these commands in order to discover the most convincing way to convey the predictions from the model. This section is intended to give you an overview of the syntax and options for these commands. Many of the specific details might only be clear after reading the more detailed discussions in later chapters.

## Specifying the levels of variables

Each command computes predicted values for the last regression model that was estimated. To compute predicted values, you must specify values for all of the independent variables in the regression. By default, all variables are set to their means in the estimation sample. ${ }^{8}$ Using the $x()$ and rest () options, variables can be assigned to specific values or to a sample statistic computed from the data in memory.
x (variablel=value1 [...]) assigns variablel to value1, variable 2 to value2, and so on. While equal signs are optional, they make the commands easier to read. You can assign values to as many or as few variables as you want. The assigned value is either a specific number (e.g., female=1) or a mnemonic specifying the descriptive statistic (e.g., phd=mean to set variable phd to the mean; pub3=max to assign pub3 to the maximum value). Details on the mnemonics that can be used are given below.
rest (stat) sets the values of all variables not specified in x() to the sample statistic indicated by stat. For example, rest (mean) sets all variables to their mean. If $x()$ is not specified, all variables are set to stat. The value of stat can be calculated for the whole sample or can be conditional based on the values specified by $x()$. For example, if $x(f e m a l e=1)$ is specified, rest (grmean) specifies that all other variables should equal their mean in the sample defined by $f e m a l e=1$. This is referred to as a group statistic (i.e., statistics that begin with $g r$ ). If you specify a group statistic for rest(), only numeric values can be used for x() . For example, x (female=mean) rest (grmean) is not allowed. If rest () is not specified, it is assumed to be rest (mean).

The statistics that can be used with x () and rest () are mean, median, min, and max specify the unconditional mean, median, minimum, and maximum. By default, the estimation sample is used to compute these statistics. If the option all is specified, all cases in memory are used for computing descriptive statistics, regardless of whether they were used in the estimation. if or in conditions can also be used. For example, adding if female==1 to any of these commands restricts the computations of descriptive statistics to only women, even if the estimation sample included men and women.
previous sets values to what they were the last time the command was called; this can only be used if the set of independent variables is the same in both cases. This can be useful if you want only to change the value of one variable from the last time the command was used.

[^22]upper and lower set values to those that yield the maximum or minimum predicted values, respectively. These options can only be used for binary models.
grmean, grmedian, grmin, and grmax computes statistics that are conditional on the group specified in $x()$. For example, $x(f e m a l e=0)$ rest (grmean) sets female to 0 and all other variables to the means of the subsample in which female is 0 (i.e., the means of these other variables for male respondents).

## Options controlling output

nobase suppresses printing of the base values of the independent variables.
brief provides only minimal output.

### 3.5.4 Syntax for prchange

prchange computes marginal and discrete change coefficients. The syntax is

$$
\text { prchange }[\text { varlist }][\text { if exp }][\text { in range }][, \mathrm{x}(\text { variable } 1=\text { valuel }[\ldots]) \text { rest (stat) }
$$

all help fromto outcome(\#) delta(\#) uncentered nobase nolabel brief ]
varlist specifies that changes are to be listed only for these variables. By default, changes are listed for all variables.

## Options

help provides information explaining the output.
fromto specifies that the starting and ending probabilities from which the discrete change is calculated for prchange should also be displayed.
outcome (\#) specifies that changes will be printed only for the outcome indicated. For example, if ologit was run with outcome categories 1,2 , and 3 , outcome (1) requests that only changes in the probability of outcome 1 should be listed. For ologit, oprobit and mlogit, the default is to provide results for all outcomes. For the count models, the default is to present results with respect to the predicted rate; specifying an outcome number will provide changes in the probability of that outcome.
delta(\#) specifies the amount of the discrete change in the independent variable. The default is a 1 unit change (i.e., delta (1)).
uncentered specifies that the uncentered discrete change rather than the centered discrete change is to be computed. By default, the change in an independent variable is centered around its value.
nolabel uses values rather than value labels in the output.
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### 3.5.5 Syntax for prgen

prgen computes a variable containing predicted values as one variable changes over a range of values, which is useful for constructing plots. The syntax is

```
prgen varname [if exp] [in range], generate(prefix) [年rom(#) to(#)
    ncases(#) x(variablel=valuel[...]) rest(stat) maxcnt(#) brrief all ]
```


## Options

varname is the name of the variable that changes while all other variables are held at specified values. generate (prefix) sets the prefix for the new variables created by prgen. Choosing a prefix that is different than the beginning letters of any of the variables in your dataset makes it easier to examine the results. For example, if you choose the prefix abcd then you can use the command sum abcd* to examine all newly created variables.
from(\#) and to(\#) are the start and end values for varname. The default is for varname to range from the observed minimum to the observed maximum of varname.
ncases (\#) specifies the number of predicted values prgen computes as varname varies from the start value to the end value. The default is 11 .
$\operatorname{maxcnt}(\#)$ is the maximum count value for which a predicted probability is computed for count models. The default is 9 .

## Variables generated

prgen constructs variables that can be graphed. The observations contain predicted values and/or probabilities for a range of values for the variable varname, holding the other variables at the specified values. $n$ observations are created, where $n$ is 11 by default or specified by ncases(). The new variables all start with the prefix specified by gen(). The variables created are

| For which models | Name | Content |
| :---: | :---: | :---: |
| All models | prefixx | The values of varname from from(\#) to to(\#). |
| logit, probit | prefixp0 | Predicted probability $\operatorname{Pr}(y=0)$. |
|  | prefixp1 | Predicted probability $\operatorname{Pr}(y=1)$. |
| ologit, oprobit | prefixpk | Predicted probability $\operatorname{Pr}(y=k)$ for all outcomes. |
|  | prefixsk | Cumulative probability $\operatorname{Pr}(y \leq k)$ for all outcomes. |
| mlogit | prefixpk | Predicted probability $\operatorname{Pr}(y=\bar{k})$ for all outcomes. |
| poisson, nbreg, zip, zinb | prefixmu | Predicted rate $\mu$. |
|  | prefixpk | Predicted probability $\operatorname{Pr}(y=k)$, for $0 \leq k \leq \operatorname{maxcnt}$ (). |
|  | prefixsk | Cumulative probability $\operatorname{Pr}(y \leq k)$, for $0 \leq k \leq \operatorname{maxcnt}$ (). |
| zip, zinb | prefixinf | Predicted probability $\operatorname{Pr}($ Always $0=1)=\operatorname{Pr}$ (inflate) . |
| regress, tobit, cnreg, intreg | prefixxb | Predicted value of $y$. |

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### 3.5.6 Syntax for prtab

prtab constructs a table of predicted values for all combinations of up to three variables. The syntax is
prtab rowvar [colvar [supercolvar]] [if exp] [in range] [, by (superrowvar)
$\mathrm{x}($ variablel $=$ valuel $[\ldots]$ ) rest (stat) outcome(\#) base novarlbl novallbl brief

## Options

rowvar, colvar, supercolvar, and superrowvar are independent variables from the previously estimated model. These define the table that is constructed.
outcome(\#) specifies that changes are printed only for the outcome indicated. The default for ologit, oprobit, and mlogit is to provide results for all outcomes. For the count models, the default is to present results with respect to the predicted rate; specifying an outcome number provides changes in the probability of that outcome.
by (superrowvar) specifies the categorical independent variable that is to be used to form the superrows of the table.
novarlbl uses a variable's name rather than the variable label in the output. Sometimes this is more readable.
novallbl uses a variable's numerical values rather than value labels in the output. Sometimes this is more readable.

### 3.5.7 Syntax for prvalue

prvalue computes the predicted outcome for a single set of values of the independent variables. The syntax is

```
prvalue [if exp] [in range] [, x(variablel=valuel [...]) rest(stat)
    level(#) maxcnt(#) save dif all nobase nolabel brief ]
```


## Options

level (\#) specifies the level or percent for confidence intervals. The default is level (95) or as set by the Stata command set level.
$\max \operatorname{cnt}(\#)$ is the maximum count value for which a predicted probability is computed for count models. The default is 9 .
save preserves predicted values computed by prvalue for subsequent comparison.
dif compares predicted values computed by prvalue to those previously preserved with the save option.

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### 3.5.8 Computing marginal effects using mfx compute

Stata 7 introduced the mfx command for calculating marginal effects. Recall from above that the marginal effect is the partial derivative of $y$ with respect to $x_{k}$. For the nonlinear models, the value of the marginal depends on the specific values of all of the independent variables. After estimating a model, mfx compute will compute the marginal effects for all of the independent variables, evaluated at values that are specified using the at () option. at () is similar to the x () and rest () syntax used in our commands. To compute the marginal effects while holding age at 40 and female at 0 , the command is $m f x$ compute, at (age=40 female=0). As with our commands for working with predicted values, unspecified independent variables are held at their mean by default.
$m f x$ has several features that make it worth exploring. For one, it works after many different estimation commands. For dummy independent variables, mfx computes the discrete change rather than the marginal effect. Of particular interest for economists, the command optionally computes elasticities instead of marginal effects. And, mfx also computes standard errors for the effects. The derivatives are calculated numerically, which means that the command can take a very long time to execute when there are many independent variables and observations, especially when used with mlogit. While we do not provide further discussion of mfx in this book, readers who are interested in learning more about this command are encouraged to examine its entry in the Reference Manual.

### 3.6 Next steps

This concludes our discussion of the basic commands and options that are used for the estimation, testing, assessing fit, and interpretation of regression models. In the next four chapters we illustrate how each of the commands can be applied for models relevant to one particular type of outcome. While Chapter 4 has somewhat more detail than later chapters, readers should be able to proceed from here to any of the chapters that follow.

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# REGRESSION MODELS FOR CATEGORICAL DEPENDENT VARIABLES USING STATA 

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## Part II

## Models for Specific Kinds of Outcomes

In Part II, we provide information on the models appropriate for different kinds of dependent outcomes.

- Chapter 4 considers binary outcomes. Models for binary outcomes are the most basic type that we consider and, to some extent, they provide a foundation for the models in later chapters. For this reason, Chapter 4 has more detailed explanations and we recommend that all readers review this chapter even if they are mainly interested other types of outcomes. We show how to estimate the binary regression model, how to test hypotheses, how to compute residuals and influence statistics, and how to calculate scalar measures of model fit. Following this, we focus on interpretation, describing how these models can be interpreted using predicted probabilities, discrete and marginal change in these probabilities, and odds ratios.

Chapters 5, 6, and 7 can be read in any combination or order, depending on the reader's interests. Each chapter provides information on estimating the relevant models, testing hypotheses about the coefficients, and interpretation in terms of predicted probabilities. In addition,

- Chapter 5 on ordered outcomes describes the parallel regression assumption that is made by the ordered logit and probit models and shows how this assumption can be tested. We also discuss interpretation in terms of the underlying latent variable and odds ratios.
- Chapter 6 on nominal outcomes introduces the multinomial logit model. We show how to test the assumption of the independence of irrelevant alternatives and present two graphical methods of interpretation. We conclude by introducing the conditional logit model.
- Chapter 7 on count outcomes presents the Poisson and negative binomial regression models. We show how to test the Poisson model's assumption of equidispersion and how to incorporate differences in exposure time into the models. We also describe versions of these models for data with a high frequency of zero counts.
- Chapter 8 covers additional topics that extend material presented earlier. We discuss the use and interpretation of categorical independent variables, interactions, and nonlinear terms. We also provide tips on how to use Stata more efficiently and effectively.

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## 4 Models for Binary Outcomes

Regression models for binary outcomes are the foundation from which more complex models for ordinal, nominal, and count models can be derived. Ordinal and nominal regression models are equivalent to the simultaneous estimation of a series of binary outcomes. While the link is less direct in count models, the Poisson distribution can be derived as the outcome of a large number of binary trials. More importantly for our purposes, the zero-inflated count models that we discuss in Chapter 7 merge a binary logit or probit with a standard Poisson or negative binomial model. Consequently, the principles of estimation, testing, and interpretation for binary models provide tools that can be readily adapted to models in later chapters. Thus, while each chapter is largely self-contained, this chapter provides somewhat more detailed explanations than later chapters. As a result, even if your interests are in models for ordinal, nominal, or count outcomes, we think that you will benefit from reading this chapter.

Binary dependent variables have two values, typically coded as 0 for a negative outcome (i.e., the event did not occur) and 1 as a positive outcome (i.e., the event did occur). Binary outcomes are ubiquitous and examples come easily to mind. Did a person vote? Is a manufacturing firm unionized? Does a person identify as a feminist or non-feminist? Did a start-up company go bankrupt? Five years after a person was diagnosed with cancer, is he or she still alive? Was a purchased item returned to the store or kept?

Regression models for binary outcomes allow a researcher to explore how each explanatory variable affects the probability of the event occurring. We focus on the two most frequently used models, the binary logit and binary probit models, referred to jointly as the binary regression model (BRM). Since the model is nonlinear, the magnitude of the change in the outcome probability that is associated with a given change in one of the independent variables depends on the levels of all of the independent variables. The challenge of interpretation is to find a summary of the way in which changes in the independent variables are associated with changes in the outcome that best reflect the key substantive processes without overwhelming yourself or your readers with distracting detail.

The chapter begins by reviewing the mathematical structure of binary models. We then examine statistical testing and fit, and finally, methods of interpretation. These discussions are intended as a review for those who are familiar with the models. For a complete discussion, see Long (1997). You can obtain sample do-files and data files that reproduce the examples in this chapter by downloading the spostst4 package (see Chapter 1 for details).

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### 4.1 The statistical model

There are three ways to derive the BRM, with each method leading to the same mathematical model. First, an unobserved or latent variable can be hypothesized along with a measurement model relating the latent variable to the observed, binary outcome. Second, the model can be constructed as a probability model. Third, the model can be generated as random utility or discrete choice model. This last approach is not considered in our review; see Long (1997, 155-156) for an introduction or Pudney (1989) for a detailed discussion.

### 4.1.1 A latent variable model

Assume a latent or unobserved variable $y^{*}$ ranging from $-\infty$ to $\infty$ that is related to the observed independent variables by the structural equation,

$$
y_{i}^{*}=\mathbf{x}_{i} \beta+\varepsilon_{i}
$$

where $i$ indicates the observation and $\varepsilon$ is a random error. For a single independent variable, we can simplify the notation to

$$
y_{i}^{*}=\alpha+\beta x_{i}+\varepsilon_{i}
$$

These equations are identical to those for the linear regression model with the important difference that the dependent variable is unobserved.

The link between the observed binary $y$ and the latent $y^{*}$ is made with a simple measurement equation:

$$
y_{i}= \begin{cases}1 & \text { if } y_{i}^{*}>0 \\ 0 & \text { if } y_{i}^{*} \leq 0\end{cases}
$$

Cases with positive values of $y^{*}$ are observed as $y=1$, while cases with negative or zero values of $y^{*}$ are observed as $y=0$.

Imagine a survey item that asks respondents if they agree or disagree with the proposition that "a working mother can establish just as warm and secure a relationship with her children as a mother who does not work". Obviously, respondents vary greatly in their opinions on this issue. Some people very adamantly agree with the proposition, some very adamantly disagree, and still others have only weak opinions one way or the other. We can imagine an underlying continuum of possible responses to this item, with every respondent having some value on this continuum (i.e., some value of $y^{*}$ ). Those respondents whose value of $y^{*}$ is positive answer "agree" to the survey question $(y=1)$, and those whose value of $y^{*}$ is 0 or negative answer "disagree" $(y=0)$. A shift in a respondent's opinion might move them from agreeing strongly with the position to agreeing weakly with the position, which would not change the response we observe. Or, the respondent might move from weakly agreeing to weakly disagreeing, in which case we would observe a change from $y=1$ to $y=0$.

Consider a second example, which we use throughout this chapter. Let $y=1$ if a woman is in the paid labor force and $y=0$ if she is not. The independent variables include variables such as number of children, education, and expected wages. Not all women in the labor force $(y=1)$ are there with the same certainty. One woman might be close to leaving the labor force, while another

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woman could be firm in her decision to work. In both cases, we observe $y=1$. The idea of a latent $y^{*}$ is that an underlying propensity to work generates the observed state. Again, while we cannot directly observe the propensity, at some point a change in $y^{*}$ results in a change in what we observe, namely, whether the woman is in the labor force.


Figure 4.1: Relationship between latent variable $y^{*}$ and $\operatorname{Pr}(y=1)$ for the BRM.

The latent variable model for binary outcomes is illustrated in Figure 4.1 for a single independent variable. For a given value of $x$, we see that

$$
\operatorname{Pr}(y=1 \mid x)=\operatorname{Pr}\left(y^{*}>0 \mid x\right)
$$

Substituting the structural model and rearranging terms,

$$
\begin{equation*}
\operatorname{Pr}(y=1 \mid x)=\operatorname{Pr}(\varepsilon>-[\alpha+\beta x] \mid x) \tag{4.1}
\end{equation*}
$$

This equation shows that the probability depends on the distribution of the error $\varepsilon$.
Two distributions of $\varepsilon$ are commonly assumed, both with an assumed mean of 0 . First, $\varepsilon$ is assumed to be distributed normally with $\operatorname{Var}(\varepsilon)=1$. This leads to the binary probit model, in which Equation 4.1 becomes

$$
\operatorname{Pr}(y=1 \mid x)=\int_{-\infty}^{\alpha+\beta x} \frac{1}{\sqrt{2 \pi}} \exp \left(-\frac{t^{2}}{2}\right) d t
$$

Alternatively, $\varepsilon$ is assumed to be distributed logistically with $\operatorname{Var}(\varepsilon)=\pi^{2} / 3$, leading to the binary logit model with the simpler equation

$$
\begin{equation*}
\operatorname{Pr}(y=1 \mid x)=\frac{\exp (\alpha+\beta x)}{1+\exp (\alpha+\beta x)} \tag{4.2}
\end{equation*}
$$

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The peculiar value assumed for $\operatorname{Var}(\varepsilon)$ in the logit model illustrates a basic point about the identification of models with latent outcomes. In the LRM, $\operatorname{Var}(\varepsilon)$ can be estimated since $y$ is observed. For the BRM , the value of $\operatorname{Var}(\varepsilon)$ must be assumed since the dependent variable is unobserved. The model is unidentified unless an assumption is made about the variance of the errors. For probit, we assume $\operatorname{Var}(\varepsilon)=1$ since this leads to a simple form of the model. If a different value was assumed, this would simply change the values of the structural coefficients in a uniform way. In the logit model, the variance is set to $\pi^{2} / 3$ since this leads to the very simple form in Equation 4.2. While the value assumed for $\operatorname{Var}(\varepsilon)$ is arbitrary, the value chosen does not affect the computed value of the probability (see Long 1997, 49-50 for a simple proof). In effect, changing the assumed variance affects the spread of the distribution, but not the proportion of the distribution above or below the threshold.


Figure 4.2: Relationship between the linear model $y^{*}=\alpha+\beta x+\varepsilon$ and the nonlinear probability model $\operatorname{Pr}(y=1 \mid x)=F(\alpha+\beta x)$.

For both models, the probability of the event occurring is the cumulative density function (cdf) of $\varepsilon$ evaluated at given values of the independent variables:

$$
\begin{equation*}
\operatorname{Pr}(y=1 \mid \mathbf{x})=F(\mathbf{x} \beta) \tag{4.3}
\end{equation*}
$$

where $F$ is the normal cdf $\Phi$ for the probit model and the logistic cdf $\Lambda$ for the logit model. The relationship between the linear latent variable model and the resulting nonlinear probability model is shown in Figure 4.2 for a model with a single independent variable. Panel A shows the error distribution for nine values of $x$, which we have labeled $1,2, \ldots, 9$. The area where $y^{*}>0$ corresponds to $\operatorname{Pr}(y=1 \mid x)$ and has been shaded. Panel B plots $\operatorname{Pr}(y=1 \mid x)$ corresponding to the shaded

[^23]regions in Panel A. As we move from 1 to 2, only a portion of the thin tail crosses the threshold in Panel A, resulting in a small change in $\operatorname{Pr}(y=1 \mid x)$ in Panel B. As we move from 2 to 3 to 4, thicker regions of the error distribution slide over the threshold and the increase in $\operatorname{Pr}(y=1 \mid x)$ becomes larger. The resulting curve is the well-known S-curve associated with the BRM.

### 4.1.2 A nonlinear probability model

Can all binary dependent variables be conceptualized as observed manifestations of some underlying latent propensity? While philosophically interesting, perhaps, the question is of little practical importance, as the BRM can also be derived without appealing to a latent variable. This is done by specifying a nonlinear model relating the $x$ 's to the probability of an event. Following Theil (1970), the logit model can be derived by constructing a model in which the predicted $\operatorname{Pr}(y=1 \mid \mathbf{x})$ is forced to be within the range 0 to 1 . For example, in the linear probability model,

$$
\operatorname{Pr}(y=1 \mid \mathbf{x})=\mathbf{x} \beta+\varepsilon
$$

the predicted probabilities can be greater than 1 and less than 0 . To constrain the predictions to the range 0 to 1 , first transform the probability into the odds,

$$
\Omega(\mathbf{x})=\frac{\operatorname{Pr}(y=1 \mid \mathbf{x})}{\operatorname{Pr}(y=0 \mid \mathbf{x})}=\frac{\operatorname{Pr}(y=1 \mid \mathbf{x})}{1-\operatorname{Pr}(y=1 \mid \mathbf{x})}
$$

which indicate how often something happens $(y=1)$ relative to how often it does not happen $(y=0)$, and range from 0 when $\operatorname{Pr}(y=1 \mid \mathbf{x})=0$ to $\infty$ when $\operatorname{Pr}(y=1 \mid \mathbf{x})=1$. The log of the odds, or logit, ranges from $-\infty$ to $\infty$. This suggests a model that is linear in the logit:

$$
\ln \Omega(\mathbf{x})=\mathbf{x} \beta
$$

This equation can be shown to be equivalent to the logit model from Equation 4.2. Interpretation of this form of the logit model often focuses on factor changes in the odds, which is discussed below.

Other binary regression models are created by choosing functions of $\mathbf{x} \beta$ that range from 0 to 1 . Cumulative distribution functions have this property and readily provide a number of examples. For example, the cdf for the standard normal distribution results in the probit model.

### 4.2 Estimation using logit and probit

Logit and probit can be estimated with the commands:

```
logit depvar [indepvars] [weight] [if exp] [in range] [, nolog or level(#)
    noconstant cluster(varname) robust ]
probit depvar [indepvars] [weight] [if exp] [in range] [, nolog or level(#)
    noconstant cluster(varname) robust ]
```

We have never had a problem with either of these models converging, even with small samples and data with wide variation in scaling.

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## Variable lists

depvar is the dependent variable. indepvars is a list of independent variables. If indepvars is not included, Stata estimates a model with only an intercept.

Warning For binary models, Stata defines observations in which depvar $=0$ as negative outcomes and observations in which depvar equals any other non-missing value (including negative values) as positive outcomes. To avoid possible confusion, we urge you to explicitly create a 0/1 variable for use as depvar.

## Specifying the estimation sample

if and in qualifiers can be used to restrict the estimation sample. For example, if you want to estimate a logit model for only women who went to college (as indicated by the variable wc), you could specify: logit lfp k5 k618 age hc lwg if wc==1.

Listwise deletion Stata excludes cases in which there are missing values for any of the variables in the model. Accordingly, if two models are estimated using the same dataset but have different sets of independent variables, it is possible to have different samples. We recommend that you use mark and markout (discussed in Chapter 3) to explicitly remove cases with missing data.

## Weights

Both logit and probit can be used with fweights, pweights, and iweights. In Chapter 3, we provide a brief discussion of the different types of weights and how weighting variables are specified.

## Options

nolog suppresses the iteration history.
or reports the "odds ratios" defined as $\exp (\widehat{\beta})$. Standard errors and confidence intervals are similarly transformed. Alternatively, our listcoef command can be used.
level (\#) specifies the level of the confidence interval. By default, Stata provides $95 \%$ confidence intervals for estimated coefficients. You can also change the default level, say to a $90 \%$ interval, with the command set level 90.
noconstant specifies that the model should not have a constant term. This would rarely be used for these models.
cluster (varname) specifies that the observations are independent across the groups specified by unique values of varname but not necessarily within the groups.

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robust indicates that robust variance estimates are to be used. When cluster () is specified, robust standard errors are automatically used. We provide a brief general discussion of these options in Chapter 3.

## Example

Our example is from Mroz's (1987) study of the labor force participation of women, using data from the 1976 Panel Study of Income Dynamics. ${ }^{1}$ The sample consists of 753 white, married women between the ages of 30 and 60 . The dependent variable lfp equals 1 if a woman is employed and else equals 0 . Since we have assigned variable labels, a complete description of the data can be obtained using describe and summarize:

```
. use binlfp2, clear
(Data from 1976 PSID-T Mroz)
. desc lfp k5 k618 age wc hc lwg inc
```

| variable name | storage type | display <br> format | value <br> label | variable label |
| :---: | :---: | :---: | :---: | :---: |
| lfp | byte | \%9.0g | lfplbl | $1=$ in paid labor force; 0 not |
| k5 | byte | $\% 9.0 \mathrm{~g}$ |  | \# kids < 6 |
| k618 | byte | $\% 9.0 \mathrm{~g}$ |  | \# kids 6-18 |
| age | byte | $\% 9.0 \mathrm{~g}$ |  | Wife's age in years |
| wc | byte | $\% 9.0 \mathrm{~g}$ | collbl | Wife College: 1=yes, $0=$ no |
| hc | byte | $\% 9.0 \mathrm{~g}$ | collbl | Husband College: 1=yes, $0=$ no |
| lwg | float | $\% 9.0 \mathrm{~g}$ |  | Log of wife's estimated wages |
| inc | float | \%9.0g |  | Family income excluding wife's |

. summarize lfp k5 k618 age wc hc lwg inc

| Variable | Obs | Mean | Std. Dev. | Min | Max |
| ---: | ---: | ---: | ---: | ---: | ---: |
| lfp | 753 | .5683931 | .4956295 | 0 | 1 |
| k5 | 753 | .2377158 | .523959 | 0 | 3 |
| k618 | 753 | 1.353254 | 1.319874 | 0 | 8 |
| age | 753 | 42.53785 | 8.072574 | 30 | 60 |
| wc | 753 | .2815405 | .4500494 | 0 | 1 |
| hc | 753 | .3917663 | .4884694 | 0 | 1 |
| lwg | 753 | 1.097115 | .5875564 | -2.054124 | 3.218876 |
| inc | 753 | 20.12897 | 11.6348 | -.0290001 | 96 |

Using these data, we estimated the model

$$
\begin{aligned}
\operatorname{Pr}(\operatorname{lfp}=1)= & F\left(\beta_{0}+\beta_{\mathrm{k} 5} \mathrm{k} 5+\beta_{\mathrm{k} 618} \mathrm{k} 618+\beta_{\text {age }}\right. \text { age } \\
& \left.+\beta_{\mathrm{wc}} \mathrm{Wc}+\beta_{\mathrm{hc}} \mathrm{hc}+\beta_{\text {lwg }} \text { lwg }+\beta_{\text {inc }} \text { inc }\right)
\end{aligned}
$$

with both the logit and probit commands, and then we created a table of results with outreg: ${ }^{2}$

[^24]| . logit lfp k5 k618 age wc hc lwg inc, nolog |  |  |  |
| :--- | :--- | :--- | :--- |
| Logit estimates | Number of obs | $=$ | 753 |
|  | LR chi2 (7) | $=$ | 124.48 |
| Log likelihood $=-452.63296$ |  | Prob > chi2 | $=$ |
|  | Pseudo R2 | $=$ | 0.0000 |
|  |  |  |  |


| lfp | Coef. | Std. Err. | z | $\mathrm{P}>\|\mathrm{z}\|$ | [95\% Conf. Interval] |  |
| ---: | ---: | ---: | :---: | :---: | ---: | ---: |
| k 5 | -1.462913 | .1970006 | -7.43 | 0.000 | -1.849027 | -1.076799 |
| k 618 | -.0645707 | .0680008 | -0.95 | 0.342 | -.1978499 | .0687085 |
| age | -.0628706 | .0127831 | -4.92 | 0.000 | -.0879249 | -.0378162 |
| wc | .8072738 | .2299799 | 3.51 | 0.000 | .3565215 | 1.258026 |
| hc | .1117336 | .2060397 | 0.54 | 0.588 | -.2920969 | .515564 |
| lwg | .6046931 | .1508176 | 4.01 | 0.000 | .3090961 | .9002901 |
| inc | -.0344464 | .0082084 | -4.20 | 0.000 | -.0505346 | -.0183583 |
| _cons | 3.18214 | .6443751 | 4.94 | 0.000 | 1.919188 | 4.445092 |

. outreg using 04lgtpbt, replace
. probit lfp k5 k618 age wc hc lwg inc, nolog

| Probit estimates | Number of obs | $=$ | 753 |
| :--- | :--- | :--- | :--- |
|  | LR chi2 $(7)$ | $=$ | 124.36 |
|  | Prob $>$ chi2 | $=$ | 0.0000 |
| Log likelihood $=-452.69496$ | Pseudo R2 | $=$ | 0.1208 |


| lfp | Coef. | Std. Err. | z | $\mathrm{P}>\|\mathrm{z}\|$ | [95\% Conf. Interval] |  |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| k 5 | -.8747112 | .1135583 | -7.70 | 0.000 | -1.097281 | -.6521411 |
| k 618 | -.0385945 | .0404893 | -0.95 | 0.340 | -.117952 | .0407631 |
| age | -.0378235 | .0076093 | -4.97 | 0.000 | -.0527375 | -.0229095 |
| wc | .4883144 | .1354873 | 3.60 | 0.000 | .2227642 | .7538645 |
| hc | .0571704 | .1240052 | 0.46 | 0.645 | -.1858754 | .3002161 |
| lwg | .3656287 | .0877792 | 4.17 | 0.000 | .1935847 | .5376727 |
| inc | -.020525 | .0047769 | -4.30 | 0.000 | -.0298875 | -.0111626 |
| _cons | 1.918422 | .3806536 | 5.04 | 0.000 | 1.172355 | 2.66449 |

. outreg using 04lgtpbt, append
While the iteration log was suppressed by the nolog option, the value of the log likelihood at convergence is listed as Log likelihood. The information in the header and table of coefficients is in the same form as discussed in Chapter 3.

By using the append option the second time we call outreg, the combined results for the logit and probit models are put in the file 04lgtpbt. out. After making a few edits to the file, we get
(Continued on next page)

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|  | Logit | Probit |
| :---: | :---: | :---: |
| \# kids <= 5. | -1.463 | -0.875 |
|  | (7.43) | (7.70) |
| \# kids 6-18. | -0.065 | -0.039 |
|  | (0.95) | (0.95) |
| Wife's age in years. | -0.063 | -0.038 |
|  | (4.92) | (4.97) |
| Wife College: 1=yes, $0=$ no. | 0.807 | 0.488 |
|  | (3.51) | (3.60) |
| Husband College: 1=yes, $0=$ no. | 0.112 | 0.057 |
|  | (0.54) | (0.46) |
| Log of wife's estimated wages. | 0.605 | 0.366 |
|  | (4.01) | (4.17) |
| Family income excluding wife's | -0.034 | -0.021 |
|  | (4.20) | (4.30) |
| Observations | 753 | 753 |
| Absolute value of $z$-statistics | n paren |  |

The estimated coefficients differ from logit to probit by a factor of about 1.7. For example, the ratio of the logit to probit coefficient for k 5 is 1.67 and for inc is 1.68 . This illustrates how the magnitudes of the coefficients are affected by the assumed $\operatorname{Var}(\varepsilon)$. The exception to the ratio of 1.7 is the coefficient for hc. This estimate has a great deal of sampling variability (i.e., a large standard error), and in such cases, the 1.7 rule often does not hold. Values of the $z$-tests are quite similar since they are not affected by the assumed $\operatorname{Var}(\varepsilon)$. The $z$-test statistics are not exactly the same because the two models assume different distributions of the errors.

### 4.2.1 Observations predicted perfectly

ML estimation is not possible when the dependent variable does not vary within one of the categories of an independent variable. For example, say that you are estimating a logit model predicting whether a person voted in the last election, vote, and that one of the independent variables is whether you are enrolled in college, college. If you had a small number of college students in your sample, it is possible that none of them voted in the last election. That is, vote==0 every time college==1. The model cannot be estimated since the coefficient for college is effectively negative infinity. Stata's solution is to drop the variable college along with all observations where college==1. For example,

```
. logit vote college phd, nolog
Note: college }~=0 predicts failure perfectly
    college dropped and 4 obs not used
Logit estimates Number of obs = 299
    (output omitted)
```


### 4.3 Hypothesis testing with test and Irtest

Hypothesis tests of regression coefficients can be conducted with the $z$-statistics in the estimation output, with test for Wald tests of simple and complex hypotheses, and with lrtest for the corresponding likelihood-ratio tests. We consider the use of each of these to test hypotheses involving

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only one coefficient, and then we show you how both test and lrtest can be used to test hypotheses involving multiple coefficients.

### 4.3.1 Testing individual coefficients

If the assumptions of the model hold, the ML estimators (e.g., the estimates produced by logit or probit) are distributed asymptotically normally:

$$
\widehat{\beta}_{k} \stackrel{a}{\sim} \mathcal{N}\left(\beta_{k}, \sigma_{\widehat{\beta}_{k}}^{2}\right)
$$

The hypothesis $H_{0}: \beta_{k}=\beta^{*}$ can be tested with the $z$-statistic:

$$
z=\frac{\widehat{\beta}_{k}-\beta^{*}}{\widehat{\sigma}_{\widehat{\beta}_{k}}}
$$

$z$ is included in the output from logit and probit. Under the assumptions justifying ML, if $H_{0}$ is true, then $z$ is distributed approximately normally with a mean of zero and a variance of one for large samples. This is shown in the following figure, where the shading shows the rejection region for a two-tailed test at the .05 level:


For example, consider the results for variable $k 5$ from the logit output generated in Section 4.2:

| lfp | Coef. | Std. Err. | z | P>\|z| | [95\% Conf. Interval] |  |
| ---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| k 5 | -1.462913 | .1970006 | -7.43 | 0.000 | -1.849027 | -1.076799 |

(output omitted)
We conclude that

Having young children has a significant effect on the probability of working ( $z=$ $-7.43, p<0.01$ for a two-tailed test).

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## One and two-tailed tests

The probability levels in the output for estimation commands are for two-tailed tests. That is, the result corresponds to the area of the curve that is either greater than $|z|$ or less than $-|z|$. When past research or theory suggests the sign of the coefficient, a one-tailed test can be used, and $H_{0}$ is only rejected when $z$ is in the expected tail. For example, assume that my theory proposes that having children can only have a negative effect on labor force participation. For $\mathrm{k} 618, z=-0.950$ and $\mathrm{P}>|\mathrm{z}|$ is .342 . This is the proportion of the sampling distribution for $z$ that is less than -0.950 or greater than 0.950 . Since we want a one-tailed test and the coefficient is in the expected direction, we only want the proportion of the distribution less than -0.950 , which is $.342 / 2=.171$. We conclude that

Having older children does not significantly affect a woman's probability of working ( $z=-0.95, p=.17$ for a one-tailed test).

You should only divide $P>|z|$ by 2 when the estimated coefficient is in the expected direction. For example, suppose I am testing a theory that having a husband who went to college has a negative effect on labor force participation, but the estimated coefficient is positive with $z=0.542$ and $P>|\mathrm{z}|$ is .588 . The one-tailed significance level would be the percent of the distribution less than .542 (not the percent less than -.542 ), which is equal to $1-(.588 / 2)=.706$, not $.588 / 2=.294$. We conclude that

Having a husband who attends college does not significantly affect a woman's probability of working ( $z=0.542, p=.71$ for a one-tailed test).

## Testing single coefficients using test

The $z$-test included in the output of estimation commands is a Wald test, which can also be computed using test. For example, to test $H_{0}: \beta_{\mathrm{k} 5}=0$,

```
. test k5
( 1) }\textrm{k}5=0.
    chi2( 1) = 55.14
    Prob > chi2 = 0.0000
```

We can conclude that
The effect of having young children on the probability of entering the labor force is significant at the . 01 level $\left(X^{2}=55.14, d f=1, p<.01\right)$

The value of a chi-squared test with 1 degree of freedom is identical to the square of the corresponding $z$-test. For example, using Stata's display as a calculator

```
. display sqrt(55.14)
7.4256313
```

This corresponds to -7.426 from the logit output. Some packages, such as SAS, present chisquared tests rather than the corresponding $z$-test.

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## Testing single coefficients using Irtest

An LR test is computed by comparing the log likelihood from a full model to that of a restricted model. To test a single coefficient, we begin by estimating the full model:

```
. logit lfp k5 k618 age wc hc lwg inc, nolog
```

| Logit estimates | Number of obs | $=$ | 753 |
| :--- | :--- | :--- | :--- |
|  | LR chi2 $(7)$ | $=$ | 124.48 |
| Log likelihood $=-452.63296$ | Prob $>$ chi2 | $=$ | 0.0000 |
| (output omitted) | Pseudo R2 | $=$ | 0.1209 |
| . lrtest, saving (0) |  |  |  |

Then we estimate the model without k5:
. logit lfp k618 age wc hc lwg inc, nolog
Logit estimates
Log likelihood $=-485.87503$
(output omitted)
. lrtest
Logit: likelihood-ratio test

| Number of obs | $=$ | 753 |
| :--- | :--- | ---: |
| LR chi2 (6) | $=$ | 58.00 |
| Prob > chi2 | $=$ | 0.0000 |
| Pseudo R2 | $=$ | 0.0563 |
|  |  |  |
| chi2(1) | $=$ | 66.48 |
| Prob > chi2 | $=$ | 0.0000 |

The resulting LR test can be interpreted as

The effect of having young children is significant at the .01 level $\left(L R X^{2}=66.48\right.$, $d f=1, p<.01$ ).

### 4.3.2 Testing multiple coefficients

In many situations, one wishes to test complex hypotheses that involve more than one coefficient. For example, we have two variables that reflect education in the family, hc and wc. The conclusion that education has (or does not have) a significant effect on labor force participation cannot be based on a pair of tests of single coefficients. But, a joint hypothesis can be tested using either test or lrtest.

## Testing multiple coefficients using test

To test that the effect of the wife attending college and of the husband attending college on labor force participation are both equal to $0, H_{0}: \beta_{\mathrm{wc}}=\beta_{\mathrm{hc}}=0$, we estimate the full model and then

```
. test hc wc
```

(1) $h c=0.0$
(2) wc $=0.0$

$$
\begin{array}{rlr}
\operatorname{chi2}\left(\begin{array}{rl}
2) & = \\
\hline & 17.66 \\
\text { Prob }>\text { chi2 } & = \\
\hline
\end{array}\right) 0.0001
\end{array}
$$

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### 4.3 Hypothesis testing with test and lrtest

We conclude that

The hypothesis that the effects of the husband's and the wife's education are simultaneously equal to zero can be rejected at the . 01 level ( $X^{2}=17.66, d f=2, p<.01$ ).

This form of the test command can be readily extended to hypotheses regarding more than two independent variables by listing more variables; for example, test wc hc k5.
test can also be used to test the equality of coefficients. For example, to test that the effect of the wife attending college on labor force participation is equal to the effect of the husband attending college, $H_{0}: \beta_{\mathrm{wc}}=\beta_{\mathrm{hc}}$ :
. test hc=wc
(1) $-\mathrm{wc}+\mathrm{hc}=0.0$

$$
\begin{array}{rll}
\operatorname{chi2}\left(\begin{array}{rl}
1) & =
\end{array}\right. & 3.54 \\
\text { Prob }>\operatorname{chi} 2 & = & 0.0600
\end{array}
$$

Note that test has translated $\beta_{\mathrm{wc}}=\beta_{\mathrm{hc}}$ into the equivalent expression $-\beta_{\mathrm{wc}}+\beta_{\mathrm{hc}}=0$. We conclude that

The null hypothesis that the effects of husband's and wife's education are equal is marginally significant at the .05 level $\left(X^{2}=3.54, d f=1, p=.06\right)$. This suggests that we have weak evidence that the effects are not equal.

## Testing multiple coefficients using Irtest

To compute an LR test of multiple coefficients, we first estimate the full model and then save the results using the command: lrtest, saving(0). Then, to test the hypothesis that the effect of the wife attending college and of the husband attending college on labor force participation are both equal to zero, $H_{0}: \beta_{\mathrm{wc}}=\beta_{\mathrm{hc}}=0$, we estimate the model that excludes these two variables and then run lrtest:
. logit lfp k5 k618 age wc hc lwg inc, nolog
Logit estimates
$\underset{\text { (output omitted) }}{\text { Log likelihood }=}-452.63296$
. lrtest, saving(0)
. logit lfp k5 k618 age lwg inc, nolog
$\quad$ (output omitted)
. lrtest
Logit: likelihood-ratio test

| Number of obs | $=$ | 753 |
| :--- | :--- | ---: |
| LR chi2 $(7)$ | $=$ | 124.48 |
| Prob > chi2 | $=$ | 0.0000 |
| Pseudo R2 | $=$ | 0.1209 |

(output omitted)

- lrtest, saving(0)
. logit lfp k5 k618 age lwg inc, nolog
(output omitted)


We conclude that

The hypothesis that the effects of the husband's and the wife's education are simultaneously equal to zero can be rejected at the .01 level $\left(L R X^{2}=18.50, d f=2, p<.01\right)$.

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This logic can be extended to exclude other variables. Say we wish to test the null hypothesis that all of the effects of the independent variables are simultaneously equal to zero. We do not need to estimate the full model again since the results are still saved from our use of lrtest, saving (0) above. We estimate the model with no independent variables and run lrtest:
Logit estimates
Log likelihood = -514.8732
(output omitted)
. lrtest
Logit: likelihood-ratio test

| Number of obs | $=$ | 753 |
| :--- | :--- | ---: |
| LR chi2 (0) | $=$ | 0.00 |
| Prob > chi2 | $=$ | . |
| Pseudo R2 | $=$ | 0.0000 |
|  |  |  |
| chi2(7) | $=$ | 124.48 |
| Prob > chi2 | $=$ | 0.0000 |

We can reject the hypothesis that all coefficients except the intercept are zero at the . 01 level $\left(L R X^{2}=124.48, d f=7, p<.01\right)$.

Note that this test is identical to the test in the header of the logit output: LR $\operatorname{chi2}(7)=124.48$.

### 4.3.3 Comparing LR and Wald tests

While the LR and Wald tests are asymptotically equivalent, their values differ in finite samples. For example,

|  |  | LR Test |  |  | Wald Test |  |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| Hypothesis | $d f$ | $G^{2}$ | $p$ |  | $W$ | $p$ |
| $\beta_{\mathrm{k} 5}=0$ | 1 | 66.48 | $<0.01$ |  | 55.14 | $<0.01$ |
| $\beta_{\mathrm{wc}}=\beta_{\mathrm{hc}}=0$ | 2 | 18.50 | $<0.01$ |  | 17.66 | $<0.01$ |
| All slopes $=0$ | 7 | 124.48 | $<0.01$ |  | 95.0 | $<0.01$ |

Statistical theory is unclear on whether the LR or Wald test is to be preferred in models for categorical outcomes, although many statisticians, ourselves included, prefer the LR test. The choice of which test to use is often determined by convenience, personal preference, and convention within an area of research.

### 4.4 Residuals and influence using predict

Examining residuals and outliers is an important way to assess the fit of a regression model. Residuals are the difference between a model's predicted and observed outcome for each observation in the sample. Cases that fit poorly (i.e., have large residuals) are known as outliers. When an observation has a large effect on the estimated parameters, it is said to be influential.

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Not all outliers are influential, as Figure 4.3 illustrates. In the top panel Figure 4.3, we show a scatterplot of some simulated data, and we have drawn the line that results from the linear regression of $y$ on $x$. The residual of any observation is its vertical distance from the regression line. The observation highlighted by the box has a very large residual and so is an outlier. Even so, it is not very influential on the slope of the regression line. In the bottom panel, the only observation whose value has changed is the highlighted one. Now, the magnitude of the residual for this observation is much smaller, but it is very influential; its presence is entirely responsible for the slope of the new regression line being positive instead of negative.



Figure 4.3: The distinction between an outlier and an influential observation.
Building on the analysis of residuals and influence in the linear regression model (see Fox 1991 and Weisberg 1980, Chapter 5 for details), Pregibon (1981) extended these ideas to the BRM.

### 4.4.1 Residuals

If we define the predicted probability for a given set of independent variables as

$$
\pi_{i}=\operatorname{Pr}\left(y_{i}=1 \mid \mathbf{x}_{i}\right)
$$

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then the deviations $y_{i}-\pi_{i}$ are heteroskedastistic, with

$$
\operatorname{Var}\left(y_{i}-\pi_{i} \mid \mathbf{x}_{i}\right)=\pi_{i}\left(1-\pi_{i}\right)
$$

This implies that the variance in a binary outcome is greatest when $\pi_{i}=.5$ and least as $\pi_{i}$ approaches 0 or 1 . For example, $.5(1-.5)=.25$ and $.01(1-.01)=.0099$. In other words, there is heteroskedasticity that depends on the probability of a positive outcome. This suggests the Pearson residual which divides the residual $y-\widehat{\pi}$ by its standard deviation:

$$
r_{i}=\frac{y_{i}-\widehat{\pi}_{i}}{\sqrt{\widehat{\pi}_{i}\left(1-\widehat{\pi}_{i}\right)}}
$$

Large values of $r$ suggest a failure of the model to fit a given observation. Pregibon (1981) showed that the variance of $r$ is not one, since $\operatorname{Var}\left(y_{i}-\widehat{\pi}_{i}\right) \neq \widehat{\pi}_{i}\left(1-\widehat{\pi}_{i}\right)$, and proposed the standardized Pearson residual

$$
r_{i}^{\mathrm{Std}}=\frac{r_{i}}{\sqrt{1-h_{i i}}}
$$

where

$$
\begin{equation*}
h_{i i}=\widehat{\pi}_{i}\left(1-\widehat{\pi}_{i}\right) \mathbf{x}_{i} \widehat{\operatorname{Var}}(\widehat{\beta}) \mathbf{x}_{i}^{\prime} \tag{4.4}
\end{equation*}
$$

While $r^{\text {Std }}$ is preferred over $r$ due to its constant variance, we find that the two residuals are often similar in practice. But, since $r^{\text {Std }}$ is simple to compute in Stata, we recommend that you use this measure.

## Example

An index plot is a useful way to examine residuals by simply plotting them against the observation number. The standardized residuals can be computed by specifying the rs option with predict. For example,

```
. logit lfp k5 k618 age wc hc lwg inc, nolog
    (output omitted)
- predict rstd, rs
. label var rstd "Standardized Residual"
. sort inc
. generate index = _n
. label var index "Observation Number"
```

In this example, we first estimate the logit model. Second, we use the rs option for predict to specify that we want standardized residuals, which are placed in a new variable that we have named rstd. Third, we sort the cases by income, so that observations are ordered from lowest to highest incomes. This results in a plot of residuals in which cases are ordered from low income to high income. The next line creates a new variable index whose value for each observation is that observation's number (i.e., row) in the dataset. Note that $\_$n on the right side of generate inserts the observation number. All that remains is to plot the residuals against the index using the commands ${ }^{3}$

[^25]```
. graph rstd index, s(o) xscale(0,800) yscale(-4,4) yline(0) gap(4) /*
> */ xlabel (0,200,400,600,800) ylabel (-4,-2,0,2,4) b2("Observation Number")
```

which produces the following index plot of standardized Pearson residuals:


There is no hard-and-fast rule for what counts as a "large" residual. Indeed, in their detailed discussion of residuals and outliers in the binary regression model, Hosmer and Lemeshow (2000, 176) sagely caution that it is impossible to provide any absolute standard: "in practice, an assessment of 'large' is, of necessity, a judgment call based on experience and the particular set of data being analyzed".

One way to search for problematic residuals is to sort the residuals by the value of a variable that you think may be a problem for the model. In our example, we sorted the data by income before plotting. If this variable was primarily responsible for the lack of fit of some observations, the plot would show a disproportionate number of cases with large residuals among either the low income or high income observations in our model. However, this does not appear to be the case for these data.

Still, in our plot, several residuals stand out as being large relative to the others. In such a cases, it is important to identify the specific observations with large residuals for further inspection. We can do this by instructing graph to use the observation number to label each point in our plot. Recall that we just created a new variable called index whose value is equal to the observation number for each observation. If we replace option $s(o)$, which plots points as small circles, with $s$ ([index]), then the values of index will be used as the symbol for each point in the plot. For example, the command ${ }^{4}$

```
. graph rstd index, s([index]) xscale(0,800) yscale(-4,4) yline(0) gap(4) /*
```

$>\quad * / \mathrm{xlabel}(0,200,400,600,800)$ ylabel $(-4,-2,0,2,4) \mathrm{b} 2$ ("Observation Number")
leads to the following plot:

[^26]

While labeling points with observations leads to chaos where there are many points, it effectively highlights and identifies the isolated cases. You can then easily list these cases. For example, the observation 142 stands out and should be examined:

| . list in 142 |  |  |  |  |  |
| :---: | ---: | ---: | ---: | ---: | ---: |
| Observation 142 |  |  |  |  |  |
| lfp | inLF | k5 | 1 | k618 | 2 |
| age | 36 | wc | NoCol | hc | NoCol |
| lwg | -2.054124 | inc | 11.2 | rstd | 3.191524 |
| index | 142 |  |  |  |  |

Alternatively, we can use list to list all observations with large residuals:

```
. list rstd index if rstd>2.5 | rstd<-2.5
```

|  | rstd | index |
| :--- | ---: | ---: |
| 142. | 3.191524 | 142 |
| 345. | 2.873378 | 345 |
| 511. | -2.677243 | 511 |
| 555. | -2.871972 | 555 |
| 752. | 3.192648 | 752 |

We can then check the listed cases to see if there are problems.
Regardless of which method is used, further analyses of the highlighted cases might reveal either incorrectly coded data or some inadequacy in the specification of the model. Cases with large positive or negative residuals should not simply be discarded from the analysis, but rather should be examined to determine why they fit so poorly.

### 4.4.2 Influential cases

As shown in Figure 4.3, large residuals do not necessarily have a strong influence on the estimated parameters, and observations with relatively small residuals can have a large influence. Influential

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points are also sometimes called high-leverage points. These can be determined by examining the change in the estimated $\widehat{\beta}$ that occurs when the $i$ th observation is deleted. While estimating a new logit for each case is usually impractical (although as the speed of computers increases, this may soon no longer be so), Pregibon (1981) derived an approximation that only requires estimating the model once. This measure summarizes the effect of removing the $i$ th observation on the entire vector $\widehat{\beta}$, which is the counterpart to Cook's distance for the linear regression model. The measure is defined as

$$
C_{i}=\frac{r_{i}^{2} h_{i i}}{\left(1-h_{i i}\right)^{2}}
$$

where $h_{i i}$, was defined in Equation 4.4. In Stata, which refers to Cook's distance as dbeta, we can compute and plot Cook's distance as follows:

- predict cook, dbeta
. label var cook "Cook's Statistic"
. graph cook index, s([index]) xscale(0,800) yscale(0,.3) yline(.1,.2) /*
> */ xlabel $(0,200,400,600,800)$ ylabel $(0, .1, .2, .3)$ b2("Observation Number")
These commands produce the following plot, which shows that cases 142,309 , and 752 merit further examination:


Methods for plotting residuals and outliers can be extended in many ways, including plots of different diagnostics against one another. Details of these plots are found in Cook and Weisberg (1999), Hosmer and Lemeshow (2000), and Landwehr et al. (1984).

### 4.5 Scalar measures of fit using fitstat

As discussed in Chapter 3, a scalar measure of fit can be useful in comparing competing models. Within a substantive area, measures of fit provide a rough index of whether a model is adequate. For

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example, if prior models of labor force participation routinely have values of .4 for a given measure of fit, you would expect that new analyses with a different sample or with revised measures of the variables would result in a similar value for that measure of fit. But, it is worth repeating that there is no convincing evidence that selecting a model that maximizes the value of a given measure of fit results in a model that is optimal in any sense other than the model having a larger value of that measure. Details on these measures are presented in Chapter 3.

## Example

To illustrate the use of scalar measures of fit, consider two models. $M_{1}$ contains our original specification of independent variables: $\mathrm{k} 5, \mathrm{k} 618$, age, wc, $\mathrm{hc}, \mathrm{lwg}$, and inc. $M_{2}$ drops the variables k 618 , hc, and lwg, and adds agesq, which is the square of age. These models are estimated and measures of fit are computed:

```
. quietly logit lfp k5 k618 age wc hc lwg inc, nolog
. outreg using 04fit, replace nolabel
. quietly fitstat, save
. gen agesq = age*age
. quietly logit lfp k5 age agesq wc inc, nolog
. outreg using 04fit, append nolabel
```

We used quietly to suppress the output from logit, and then used outreg to combine the results from the two logits:

|  | Model1 | Model2 |
| :---: | :---: | :---: |
| k5 | -1.463 | -1.380 |
|  | (7.43)** | (7.06) ** |
| k618 | -0.065 |  |
|  | (0.95) |  |
| age | -0.063 | 0.057 |
|  | (4.92) ** | (0.50) |
| wc | 0.807 | 1.094 |
|  | (3.51)** | (5.50)** |
| hc | 0.112 |  |
|  | (0.54) |  |
| lwg | 0.605 |  |
|  | (4.01)** |  |
| inc | -0.034 | -0.032 |
|  | (4.20)** | (4.18)** |
| agesq |  | -0.001 |
|  |  | (1.00) |
| Constant | 3.182 | 0.979 |
|  | (4.94)** | (0.40) |
| Observations | 753 | 753 |
| Absolute value of $z$-statistics in paren |  |  |
| * significant at $5 \%$; ** significant at $1 \%$ |  |  |

The output from fitstat for $M_{1}$ was suppressed, but the results were saved to be listed by a second call to fitstat using the dif option:

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| Measures of Fit for logit of lfp |  |  |  |
| :---: | :---: | :---: | :---: |
|  | Current | Saved | Difference |
| Model: | logit | logit |  |
| N: | 753 | 753 | 0 |
| Log-Lik Intercept Only: | -514.873 | -514.873 | 0.000 |
| Log-Lik Full Model: | -461.653 | -452.633 | -9.020 |
| D: | 923.306 (747) | 905.266(745) | 18.040(2) |
| LR: | 106.441(5) | 124.480 (7) | 18.040(2) |
| Prob > LR: | 0.000 | 0.000 | 0.000 |
| McFadden's R2: | 0.103 | 0.121 | -0.018 |
| McFadden's Adj R2: | 0.092 | 0.105 | -0.014 |
| Maximum Likelihood R2: | 0.132 | 0.152 | -0.021 |
| Cragg \& Uhler's R2: | 0.177 | 0.204 | -0.028 |
| McKelvey and Zavoina's R2: | 0.182 | 0.217 | -0.035 |
| Efron's R2: | 0.135 | 0.155 | -0.020 |
| Variance of $\mathrm{y}^{*}$ : | 4.023 | 4.203 | -0.180 |
| Variance of error: | 3.290 | 3.290 | 0.000 |
| Count R2: | 0.677 | 0.693 | -0.016 |
| Adj Count R2: | 0.252 | 0.289 | -0.037 |
| AIC: | 1.242 | 1.223 | 0.019 |
| AIC*n : | 935.306 | 921.266 | 14.040 |
| BIC: | -4024.871 | -4029.663 | 4.791 |
| BIC ${ }^{-}$: | -73.321 | -78.112 | 4.791 |
| Difference of 4.791 in $\mathrm{BIC}^{-}$provides positive support for saved model. |  |  |  |
| Note: p-value for differenc | ce in LR is only | alid if models | are nested. |

These results illustrate the limitations inherent in scalar measures of fit. $M_{2}$ deleted two variables that were not significant and one that was from $M_{1}$. It added a new variable that was not significant in the new model. Since the models are not nested, they cannot be compared using a difference of chi-squared test. ${ }^{5}$ What do the fit statistics show? First, the values of the pseudo- $R^{2}$ s are slightly larger for $M_{2}$ even though a significant variable was dropped and only a nonsignificant variable was added. If you take the pseudo- $R^{2}$ s as evidence for the "best" model, which we do not, there is some evidence preferring $M_{2}$. Second, the BIC statistic is smaller for $M_{1}$, which provides support for that model. Following Raftery's (1996) guidelines, one would say that there is positive (neither weak nor strong) support for $M_{1}$.

### 4.6 Interpretation using predicted values

Since the BRM is nonlinear, no single approach to interpretation can fully describe the relationship between a variable and the outcome. We suggest that you try a variety of methods, with the goal of finding an elegant way to present the results that does justice to the complexities of the nonlinear model.

In general, the estimated parameters from the BRM do not provide directly useful information for understanding the relationship between the independent variables and the outcome. With the exception of the rarely used method of interpreting the latent variable (which we discuss in our treatment of ordinal models in Chapter 5), substantively meaningful interpretations are based on

[^27]predicted probabilities and functions of those probabilities (e.g., ratios, differences). As shown in Figure 4.1, for a given set of values of the independent variables, the predicted probability in BRMs is defined as
$$
\text { Logit: } \widehat{\operatorname{Pr}}(y=1 \mid \mathbf{x})=\Lambda(\mathbf{x} \widehat{\beta}) \quad \text { Probit: } \widehat{\operatorname{Pr}}(y=1 \mid \mathbf{x})=\Phi(\mathbf{x} \widehat{\beta})
$$
where $\Lambda$ is the cdf for the logistic distribution with variance $\pi^{2} / 3$ and $\Phi$ is the cdf for the normal distribution with variance 1 . For any set of values of the independent variables, the predicted probability can be computed. A variety of commands in Stata and our pr* commands make it very simple to work with these predicted probabilities.

### 4.6.1 Predicted probabilities with predict

After running logit or probit,
predict newvarname [if exp] [in range]
can be used to compute the predicted probability of a positive outcome for each observation, given the values on the independent variables for that observation. The predicted probabilities are stored in the new variable newvarname. The predictions are computed for all cases in memory that do not have missing values for the variables in the model, regardless of whether if and in had been used to restrict the estimation sample. For example, if you estimate logit lfp k5 age if wc==1, only 212 cases are used. But predict newvarname computes predictions for the entire dataset, 753 cases. If you only want predictions for the estimation sample, you can use the command predict newvarname if e(sample)==1. ${ }^{6}$
predict can be used to examine the range of predicted probabilities from your model. For example,

```
. predict prlogit
(option p assumed; Pr(lfp))
. summarize prlogit
\begin{tabular}{r|rrrrr} 
Variable & Obs & Mean & Std. Dev. & Min & Max \\
\hline prlogit & 753 & .5683931 & .1944213 & .0139875 & .9621198
\end{tabular}
```

The message (option $p$ assumed; $\operatorname{Pr}(\mathrm{lfp})$ ) reflects that predict can compute many different quantities. Since we did not specify an option indicating which quantity to predict, option p for predicted probabilities was assumed, and the new variable prlogit was given the variable label $\operatorname{Pr}(l f p)$. summarize computes summary statistics for the new variable and shows that the predicted probabilities in the sample range from .014 to .962 , with a mean predicted probability of being in the labor force of .568 .

We can use dotplot to plot the predicted probabilities for our sample, ${ }^{7}$

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```
. label var prlogit "Logit: Pr(lfp)"
. dotplot prlogit, ylabel(0 . 2 to 1) gap(3)
```

which leads to the following plot:


The plot clearly shows that the predicted probabilities for individual observations span almost the entire range from 0 to 1 , but that roughly two-thirds of the observations have predicted probabilities between .40 and .80 .
predict can also be used to demonstrate that the predictions from logit and probit models are essentially identical. Even though the two models make different assumptions about $\operatorname{Var}(\varepsilon)$, these differences are absorbed in the relative magnitudes of the estimated coefficients. To see this, we first estimate the two models and compute their predicted probabilities:

```
. logit lfp k5 k618 age wc hc lwg inc, nolog
    (output omitted)
. predict prlogit
(option p assumed; Pr(lfp))
. label var prlogit "Logit: Pr(lfp)"
. probit lfp k5 k618 age wc hc lwg inc, nolog
(output omitted)
. predict prprobit
(option p assumed; Pr(lfp))
. label var prprobit "Probit: Pr(lfp)"
```

Next, we check the correlation between the two sets of predicted values:

| . pwcorr prlogit prprobit |  |  |
| ---: | :---: | :---: |
|  | prlogit prprobit |  |
| prlogit |  | 1.0000 |
| prprobit | 0.9998 | 1.0000 |

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The extremely high correlation is confirmed by plotting them against one another. The command
. graph prlogit prprobit, /*
$>\quad * / \operatorname{xscale}(0,1)$ yscale $(0,1)$ yline $(.25, .5, .75,1)$ xline (.25,.5,.75,1)
leads to the following plot:


In terms of predictions, there is very little reason to prefer either logit or probit. If your substantive findings turn on whether you used logit or probit, we would not place much confidence in either result. In our own research, we tend to use logit, primarily because of the availability of interpretation in terms of odds and odds ratios (discussed below).

Overall, examining predicted probabilities for the cases in the sample provides an initial check of the model. To better understand and present the substantive findings, it is usually more effective to compute predictions at specific, substantively informative values. Our commands prvalue, prtab, and prgen are designed to make this very simple.

### 4.6.2 Individual predicted probabilities with prvalue

A table of probabilities for ideal types of people (or countries, cows, or whatever you are studying) can quickly summarize the effects of key variables. In our example of labor force participation, we could compute predicted probabilities of labor force participation for women in these three types of families:

- Young, low income and low education families with young children.
- Highly educated, middle aged couples with no children at home.
- An "average family" defined as having the mean on all variables.

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This can be done with a series of calls to prvalue (see Chapter 3 for a discussion of options for this command): ${ }^{8}$

```
. * young, low income, low education families with young children.
. prvalue, x(age=35 k5=2 wc=0 hc=0 inc=15) rest(mean)
logit: Predictions for lfp
\begin{tabular}{llll}
\(\operatorname{Pr}(\mathrm{y}=\operatorname{inLF} \mid \mathrm{x}):\) & 0.1318 & \(95 \% \mathrm{ci}:(0.0723,0.2282)\) \\
\(\operatorname{Pr}(\mathrm{y}=\operatorname{NotInLF} \mid \mathrm{x}):\) & 0.8682 & \(95 \% \mathrm{ci}:(0.7718,0.9277)\)
\end{tabular}
\begin{tabular}{rrrrrr} 
& k 5 & k 618 & age & wc & hc \\
\(\mathrm{x}=\) & 2 & 1.3532537 & 35 & 0 & 0 \\
& & & & 1.0971148 \\
& inc & & & & \\
& 15 & & & & \\
& & & & &
\end{tabular}
```

We have set the values of the independent variables to those that define our first type of family, with other variables held at their mean. The output shows the predicted probability of working (.13), along with the chosen values for each variable. While the values of the independent variables can be suppressed with the brief option, it is safest to look at them to make sure they are correct. This process is repeated for the other ideal types:

```
. * highly educated families with no children at home.
. prvalue, x(age=50 k5=0 k618=0 wc=1 hc=1) rest(mean)
logit: Predictions for lfp
    Pr(y=inLF|x): 
\begin{tabular}{rrrrrr} 
& k 5 & k 618 & age & wc & hc \\
\(\mathrm{x}=\) & 0 & 0 & 50 & 1 & 1 \\
\hline
\end{tabular}
x= inc
. * an average person
. prvalue, rest(mean)
logit: Predictions for lfp
    Pr}(y=inLF|x): 0.5778 95% ci: (0.5388,0.6159
    Pr}(y=NotInLF|x): 0.4222 95% ci: (0.3841,0.4612)
\begin{tabular}{rrrrrr} 
& k 5 & k 618 & age & wc & hc \\
\(\mathrm{x}=\) & .2377158 & 1.3532537 & 42.537849 & .2815405 & .39176627 \\
& inc & & & & 1.0971148 \\
& & & & & \\
\(\mathrm{x}=\) & 20.128965 & & & &
\end{tabular}
```

With predictions in hand, we can summarize the results and get a better general feel for the factors affecting a wife's labor force participation.

[^29]| Ideal Type | Probability <br> of LFP |
| :--- | :---: |
| Young, low income and low education <br> families with young children. | 0.13 |
| Highly educated, middle-aged couples <br> with no children at home. | 0.72 |
| An "average" family | 0.58 |

### 4.6.3 Tables of predicted probabilities with prtab

In some cases, the focus might be on two or three categorical independent variables. Predictions for all combinations of the categories of these variables could be presented in a table. For example,

| Number <br> of Young | Predicted Probability |  |  |
| :---: | :---: | :---: | :---: |
|  | Did Not | Attend | Attended |
| College |  |  |  | Difference |  |  |  |  |
| :---: | :---: | :---: | :---: |
| 0 | 0.61 | 0.78 | 0.17 |
| 1 | 0.26 | 0.44 | 0.18 |
| 2 | 0.08 | 0.16 | 0.08 |
| 3 | 0.02 | 0.04 | 0.02 |

This table shows the strong effect on labor force participation of having young children and how the effect differs according to the wife's education. One way to construct such a table is by a series of calls to prvalue (we use the brief option to limit output):

```
. prvalue, x(k5=0 wc=0) rest(mean) brief
Pr(y=inLF|x): 0.6069 95% ci: (0.5558,0.6558)
Pr(y=NotInLF|x): 0.3931 95% ci: (0.3442,0.4442)
. prvalue, x(k5=1 wc=0) rest(mean) brief
Pr(y=inLF|x): 0.2633 95% ci: (0.1994,0.3391)
Pr(y=NotInLF|x): 0.7367 95% ci: (0.6609,0.8006)
. * and so on, ad nauseam...
```

Even for a simple table, this approach is tedious and error-prone. prtab automates the process by computing a table of predicted probabilities for all combinations of up to four categorical variables. For example,
(Continued on next page)

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```
. prtab k5 wc, rest(mean)
logit: Predicted probabilities of positive outcome for lfp
\begin{tabular}{l|ll}
\hline & \multirow{2}{|c}{\begin{tabular}{l} 
\# kids \(<\) \\
6
\end{tabular}} & \multicolumn{2}{|c}{\begin{tabular}{l} 
Wife College: \\
1=yes, \\
NoColo
\end{tabular}} & College \\
\hline & & \\
\hline & 0.6069 & 0.7758 \\
1 & 0.2633 & 0.4449 \\
2 & 0.0764 & 0.1565 \\
3 & 0.0188 & 0.0412 \\
\hline
\end{tabular}
\begin{tabular}{rrrrrr} 
& k 5 & k 618 & age & wc & hc \\
\(\mathrm{x}=\) & .2377158 & 1.3532537 & 42.537849 & .2815405 & .39176627 \\
& inc & & & & 1.0971148 \\
\(\mathrm{x}=\) & 20.128965 & & & & \\
\end{tabular}
```


### 4.6.4 Graphing predicted probabilities with prgen

When a variable of interest is continuous, you can either select values (e.g., quartiles) and construct a table, or create a graph. For example, to examine the effects of income on labor force participation by age, we can use the estimated parameters to compute predicted probabilities as income changes for fixed values of age. This is shown in Figure 4.4. The command prgen creates data that can be graphed in this way. The first step is to generate the predicted probabilities for those aged 30:

```
. prgen inc, from(0) to(100) generate(p30) x(age=30) rest(mean) n(11)
logit: Predicted values as inc varies from 0 to 100.
```

|  | k 5 | k 618 | age | wc | hc |
| ---: | ---: | ---: | ---: | ---: | ---: |
| $\mathrm{x}=$ | .2377158 | 1.3532537 | 30 | .2815405 | .39176627 |
|  | 1.0971148 |  |  |  |  |

$\mathrm{x}=20.12 \mathrm{inc}$
. label var p30p1 "Age 30"
inc is the independent variable that we want to vary along the $x$-axis. The options that we use are
from (0) and to(100) specify the minimum and maximum values over which inc is to vary. The default is the variable's observed minimum and maximum values.
n (11) indicates that 11 evenly spaced values of inc between 0 and 100 should be used. You should choose the value that corresponds to the number of symbols you want on your graph.
$x($ age $=30)$ indicates that we want to hold the value of age at 30 . By default, other variables will be held at their mean unless rest () is used to specify some other summary statistic.
gen ( p 30 ) indicates the root name used in constructing new variables. prgen creates $p 30 \mathrm{x}$ that contains the values of inc that are used; p30p1 with the values of the probability of a 1 , and p 30 p 0 with values of the probability of a 0 .

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Figure 4.4: Graph of predicted probabilities created using prgen.

Additional calls of prgen are made holding age at different values:

```
. prgen inc, from(0) to(100) generate(p40) x(age=40) rest(mean) n(11)
    (output omitted)
. label var p40p1 "Age 40"
. prgen inc, from(0) to(100) generate(p50) x(age=50) rest(mean) n(11)
    (output omitted)
. label var p50p1 "Age 50"
. prgen inc, from(0) to(100) generate(p60) x(age=60) rest(mean) n(11)
(output omitted)
. label var p60p1 "Age 60"
```

Listing the values for the first eleven observations in the dataset for some of the new variables prgen has created may help you understand better what this command does:

```
. list p30p1 p40p1 p50p1 p60p1 p60x in 1/11
```

|  | p 30 p 1 | p 40 p 1 | p 50 p 1 | p 60 p 1 | p 60 x |
| ---: | ---: | ---: | ---: | ---: | ---: |
| 1. | .8575829 | .7625393 | .6313345 | .4773258 | 0 |
| 2. | .8101358 | .6947005 | .5482202 | .3928797 | 10 |
| 3. | .7514627 | .6172101 | .462326 | .3143872 | 20 |
| 4. | .6817801 | .5332655 | .3786113 | .2452419 | 30 |
| 5. | .6028849 | .4473941 | .3015535 | .187153 | 40 |
| 6. | .5182508 | .36455 | .2342664 | .1402662 | 50 |
| 7. | .4325564 | .289023 | .1781635 | .1036283 | 60 |
| 8. | .3507161 | .2236366 | .1331599 | .0757174 | 70 |
| 9. | .2768067 | .1695158 | .0981662 | .0548639 | 80 |
| 10. | .2133547 | .1263607 | .071609 | .0395082 | 90 |
| 11. | .1612055 | .0929622 | .0518235 | .0283215 | 100 |

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The predicted probabilities of labor force participation for those average on all other variables at ages $30,40,50$, and 60 are in the first four columns. The clear negative effect of age is shown by the increasingly small probabilities as we move across these columns in any row. The last column indicates the value of income for a given row, starting at 0 and ending at 100 . We can see that the probabilities decrease as income increases.

The following graph command generates the plot:
. graph p30p1 p40p1 p50p1 p60p1 p60x, s(pdST) connect(ssss) $/ *$
$>* / \mathrm{b} 2($ "Income") xlabel $(0,20,40,60,80,100)$ xscale $(0,100) / *$
$>* / \mathrm{l} 2($ Pr (In Labor Force)") ylabel $(0, .25, .50, .75,1)$ yscale $(0,1)$
Since we have not used graph much yet, it is worth discussing some points that we find useful (also see the section on Graphics in Chapter 2).

1. Recall that $/ * * /$ is a way of entering long lines in do-files, and that $>$ indicates line wrap in the output.
2. The variables to plot are: p30p1 p40p1 p50p1 p60p1 p60x, where p60x, the last variable in the list, is the variable for the horizontal axis. All variables before the last variable are plotted on the vertical axis.
3. connect (ssss) smooths the four curves. The option s means that you want to smooth the line connecting the points. There are four s's because there are four different lines whose points we are connecting. To see what smoothing does, try running the command using c (llll) to connect the points with straight lines.
4. $s(p d S T)$ are the symbols used to mark the data points on the lines. The options for these symbols are provided in Chapter 2. p is used to print a small plus; d a small diamond; S a large square; and T a large triangle.

### 4.6.5 Changes in predicted probabilities

While graphs are very useful for showing how predicted probabilities are related to an independent variable, for even our simple example it is not practical to plot all possible combinations of the independent variables. And, in some cases, the plots show that a relationship is linear so that a graph is superfluous. In such circumstances, a useful summary measure is the change in the outcome as one variable changes, holding all other variables constant.

## Marginal change

In economics, the marginal effect or change is commonly used:

$$
\text { Marginal Change }=\frac{\partial \operatorname{Pr}(y=1 \mid \mathbf{x})}{\partial x_{k}}
$$

The marginal change is shown by the tangent to the probability curve in Figure 4.5. The value of the marginal effect depends on the level of all variables in the model. It is often computed with all

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variables held at their mean or by computing the marginal for each observation in the sample and then averaging across all values.


Figure 4.5: Marginal change compared to discrete change in the BRM.

Marginal change with prchange command The command prchange computes the marginal at the values of the independent variables specified with $x()$ or rest (). Running prchange without any options computes the marginal change (along with a lot of other things discussed below) with all variables at their mean. Or, we can compute the marginal at specific values of the independent variables, such as when wc $=1$ and age $=40$. Here we request only the results for age:

```
. prchange age, x(wc=1 age=40) help
logit: Changes in Predicted Probabilities for lfp
\begin{tabular}{|c|c|c|c|c|c|c|c|}
\hline \multicolumn{2}{|r|}{min->max} & \(0->1\) & -+1/2 & -+sd/2 & \multicolumn{3}{|l|}{MargEfct} \\
\hline age - & . 3940 & -0.0017 & -0.0121 & -0.0971 & -0.012 & & \\
\hline & NotInLF & inLF & & & & & \\
\hline \multirow[t]{2}{*}{\(\operatorname{Pr}(\mathrm{y} \mid \mathrm{x})\)} & 0.2586 & 0.7414 & & & & & \\
\hline & k5 & k618 & age & wc & hc & lwg & inc \\
\hline \(\mathrm{x}=\) & . 237716 & 1.35325 & 40 & 1 & . 391766 & 1.09711 & 20.129 \\
\hline \(s d(x)=\) & . 523959 & 1.31987 & 8.07257 & . 450049 & . 488469 & . 587556 & 11.6348 \\
\hline
\end{tabular}
Pr(y|x): probability of observing each y for specified x values
AvglChgl: average of absolute value of the change across categories
Min->Max: change in predicted probability as x changes from its minimum to
            its maximum
    0->1: change in predicted probability as x changes from 0 to 1
    -+1/2: change in predicted probability as x changes from 1/2 unit below
            base value to 1/2 unit above
    -+sd/2: change in predicted probability as x changes from 1/2 standard
            dev below base to 1/2 standard dev above
MargEfct: the partial derivative of the predicted probability/rate with
    respect to a given independent variable
```

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In plots that we do not show (but that we encourage you to create them using prgen and graph), we found that the relationship between age and the probability of being in the labor force was essentially linear for those who attend college. Accordingly, we can take the marginal computed by prchange, multiply it by 10 to get the amount of change over 10 years, and report that

For women who attend college, a ten year increase in age decreases the probability of labor force participation by approximately .12 , holding other variables at their mean.

When using the marginal, it is essential to keep two points in mind. First, the amount of change depends on the level of all variables. Second, as shown in Figure 4.5, the marginal is the instantaneous rate of change. In general, it does not equal the actual change for a given finite change in the independent variable unless you are in a region of the probability curve that is approximately linear. Such linearity justifies the interpretation given above.

Marginal change with mfx command The marginal change can also be computed using mfx compute, where the at () option is used to set values of the independent variables. Below we use mfx compute to estimate the marginal change for the same values that we used when calculating the marginal effect for age with prchange above:

| $\begin{aligned} & \text { Marginal effects after logit } \\ & y=\operatorname{Pr}(l \mathrm{fp}) \text { (predict) } \\ &=.74140317 \end{aligned}$ |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| variable | dy/dx | Std. Err. | z | $P>\|z\|$ | [ 95\% | C.I. ] | X |
| k5 | -. 2804763 | . 04221 | -6.64 | 0.000 | -. 363212 | -. 197741 | . 237716 |
| k618 | -. 0123798 | . 01305 | -0.95 | 0.343 | -. 037959 | . 013199 | 1.35325 |
| age | -. 0120538 | . 00245 | -4.92 | 0.000 | -. 016855 | -. 007252 | 40.0000 |
| WC* | . 1802113 | . 04742 | 3.80 | 0.000 | . 087269 | . 273154 | 1.00000 |
| hc* | . 0212952 | . 03988 | 0.53 | 0.593 | -. 056866 | . 099456 | . 391766 |
| $1 \mathrm{l} g$ | . 1159345 | . 03229 | 3.59 | 0.000 | . 052643 | . 179226 | 1.09711 |
| inc | -. 0066042 | . 00163 | -4.05 | 0.000 | -. 009802 | -. 003406 | 20.1290 |

(*) dy/dx is for discrete change of dummy variable from 0 to 1
$m f x$ compute is particularly useful if you need estimates of the standard errors of the marginal effects; however, mfx compute computes the estimates using numerical methods, and for some models the command can take a long time.

## Discrete change

Given the nonlinearity of the model, we prefer the discrete change in the predicted probabilities for a given change in an independent variable. To define discrete change, we need two quantities:
$\operatorname{Pr}\left(y=1 \mid \mathbf{x}, x_{k}\right)$ is the probability of an event given $\mathbf{x}$, noting in particular the value of $x_{k}$.
$\operatorname{Pr}\left(y=1 \mid \mathbf{x}, x_{k}+\delta\right)$ is the probability of the event with only $x_{k}$ increased by some quantity $\delta$.
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Then, the discrete change for a change of $\delta$ in $x_{k}$ equals

$$
\frac{\Delta \operatorname{Pr}(y=1 \mid \mathbf{x})}{\Delta x_{k}}=\operatorname{Pr}\left(y=1 \mid \mathbf{x}, x_{k}+\delta\right)-\operatorname{Pr}\left(y=1 \mid \mathbf{x}, x_{k}\right)
$$

which can be interpreted as
For a change in variable $x_{k}$ from $x_{k}$ to $x_{k}+\delta$, the predicted probability of an event changes by $\frac{\Delta \operatorname{Pr}(y=1 \mid \mathbf{x})}{\Delta x_{k}}$, holding all other variables constant.

As shown in Figure 4.5, in general, the two measures of change are not equal. That is,

$$
\frac{\partial \operatorname{Pr}(y=1 \mid \mathbf{x})}{\partial x_{k}} \neq \frac{\Delta \operatorname{Pr}(y=1 \mid \mathbf{x})}{\Delta x_{k}}
$$

The measures differ because the marginal change is the instantaneous rate of change, while the discrete change is the amount of change in the probability for a given finite change in one independent variable. The two measures are similar, however, when the change occurs over a region of the probability curve that is roughly linear.

The value of the discrete change depends on

1. The start level of the variable that is being changed. For example, do you want to examine the effect of age beginning at 30 ? At 40 ? At 50 ?
2. The amount of change in that variable. Are you interested in the effect of a change of 1 year in age? Of 5 years? Of 10 years?
3. The level of all other variables in the model. Do you want to hold all variables at their mean? Or, do you want to examine the effect for women? Or, to compute changes separately for men and women?

Accordingly, a decision must be made regarding each of these factors. See Chapter 3 for further discussion.

For our example, let's look at the discrete change with all variables held at their mean, which is computed by default by prchange, where the help option is used to get detailed descriptions of what the measures mean:

```
- prchange, help
```

logit: Changes in Predicted Probabilities for lfp

|  | min->max | $0->1$ | $-+1 / 2$ | -+ sd/2 | MargEfct |
| ---: | ---: | ---: | ---: | ---: | ---: |
| k5 | -0.6361 | -0.3499 | -0.3428 | -0.1849 | -0.3569 |
| k618 | -0.1278 | -0.0156 | -0.0158 | -0.0208 | -0.0158 |
| age | -0.4372 | -0.0030 | -0.0153 | -0.1232 | -0.0153 |
| wc | 0.1881 | 0.1881 | 0.1945 | 0.0884 | 0.1969 |
| hc | 0.0272 | 0.0272 | 0.0273 | 0.0133 | 0.0273 |
| lwg | 0.6624 | 0.1499 | 0.1465 | 0.0865 | 0.1475 |
| inc | -0.6415 | -0.0068 | -0.0084 | -0.0975 | -0.0084 |
|  | NotInLF | inLF |  |  |  |
| $\operatorname{Pr}(\mathrm{y} \mid \mathrm{x})$ | 0.4222 | 0.5778 |  |  |  |

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|  | k5 | k618 | age | wc | hc | lwg |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: | inc

First consider the results of changes from the minimum to the maximum. There is little to be learned by analyzing variables whose range of probabilities is small, such as hc, while age, k 5 , wc, lwg, and inc have potentially important effects. For these we can examine the value of the probabilities before and after the change by using the fromto option:
. prchange k5 age wc lwg inc, fromto
logit: Changes in Predicted Probabilities for lfp

|  | from: | to: | dif: | from: | to: | dif: | $\begin{gathered} \text { from: } \\ \text { x-1/2 } \end{gathered}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\mathrm{x}=\mathrm{min}$ | $\mathrm{x}=\mathrm{max}$ | min->max | $\mathrm{x}=0$ | $\mathrm{x}=1$ | $0->1$ |  |
| k5 | 0.6596 | 0.0235 | -0.6361 | 0.6596 | 0.3097 | -0.3499 | 0.7398 |
| age | 0.7506 | 0.3134 | -0.4372 | 0.9520 | 0.9491 | -0.0030 | 0.5854 |
| wc | 0.5216 | 0.7097 | 0.1881 | 0.5216 | 0.7097 | 0.1881 | 0.4775 |
| lwg | 0.1691 | 0.8316 | 0.6624 | 0.4135 | 0.5634 | 0.1499 | 0.5028 |
| inc | 0.7326 | 0.0911 | -0.6415 | 0.7325 | 0.7256 | -0.0068 | 0.5820 |
|  | to: | dif: | from: | to: | dif: |  |  |
|  | $\mathrm{x}+1 / 2$ | -+1/2 | $\mathrm{x}-1 / 2 \mathrm{sd}$ | $\mathrm{x}+1 / 2 \mathrm{sd}$ | -+sd/2 | MargEfct |  |
| k5 | 0.3971 | -0.3428 | 0.6675 | 0.4826 | -0.1849 | -0.3569 |  |
| age | 0.5701 | -0.0153 | 0.6382 | 0.5150 | -0.1232 | -0.0153 |  |
| wc | 0.6720 | 0.1945 | 0.5330 | 0.6214 | 0.0884 | 0.1969 |  |
| lwg | 0.6493 | 0.1465 | 0.5340 | 0.6204 | 0.0865 | 0.1475 |  |
| inc | 0.5736 | -0.0084 | 0.6258 | 0.5283 | -0.0975 | -0.0084 |  |
|  | NotInLF | inLF |  |  |  |  |  |
| $\operatorname{Pr}(\mathrm{y} \mid \mathrm{x})$ | 0.4222 | 0.5778 |  |  |  |  |  |
|  | k5 | k618 | age | wc | hc | 1wg | inc |
| $\mathrm{x}=$ | . 237716 | 1.35325 | 42.5378 | . 281541 | . 3917661 | 1.09711 | 20.129 |
| sd $(x)=$ | . 523959 | 1.31987 | 8.07257 | . 450049 | . 488469 | . 58755611 | 1.6348 |

We learn, for example, that varying age from its minimum of 30 to its maximum of 60 decreases the predicted probability from .75 to .31 , a decrease of .44 . Changing family income (inc) from its minimum to its maximum decreases the probability of a women being in the labor force from .73 to .09. Interpreting other measures of change, the following interpretations can be made:

Using the unit change labeled $-+1 / 2$ : For a woman who is average on all characteristics, an additional young child decreases the probability of employment by .34 .
Using the standard deviation change labeled $-+1 / 2$ sd: A standard deviation change in age centered around the mean will decrease the probability of working by .12 , holding other variables to their means.

Using a change from 0 to 1 labeled $0->1$ : If a woman attends college, her probability of being in the labor force is .18 greater than a woman who does not attend college, holding other variables at their mean.

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What if you need to calculate discrete change for changes in the independent values that are not the default for prchange (e.g., a change of 10 years in age rather than 1 year)? This can be done in two ways:

Nonstandard discrete changes with prvalue command The command prvalue can be used to calculate the change in the probability for a discrete change of any magnitude in an independent variable. Say we want to calculate the effect of a ten-year increase in age for a 30-year old woman who is average on all other characteristics:

```
. prvalue, x(age=30) save brief
Pr(y=inLF|x): 0.7506 95% ci: (0.6771,0.8121)
Pr}(y=NotInLF|x): 0.2494 95% ci: (0.1879,0.3229)
- prvalue, x(age=40) dif brief
\begin{tabular}{lrrr} 
& Current & Saved & Difference \\
\(\operatorname{Pr}(\mathrm{y}=\operatorname{inLF} \mid \mathrm{x}):\) & 0.6162 & 0.7506 & -0.1345 \\
\(\operatorname{Pr}(\mathrm{y}=\operatorname{NotInLF} \mid \mathrm{x}):\) & 0.3838 & 0.2494 & 0.1345
\end{tabular}
```

The save option preserves the results from the first call of prvalue. The second call adds the dif option to compute the differences between the two sets of predictions. We find that an increase in age from 30 to 40 years decreases a woman's probability of being in the labor force by . 13 .

Nonstandard discrete changes with prchange Alternatively, we can use prchange with the delta() and uncentered options. delta(\#) specifies that the discrete change is to be computed for a change of \# units instead of a one-unit change. uncentered specifies that the change should be computed starting at the base value (i.e., values set by the $\mathrm{x}($ ) and rest () options), rather than being centered around the base. In this case, we want an uncentered change of 10 units, starting at age $=30$ :

```
. prchange age, x(age=30) uncentered d(10) rest(mean) brief
\begin{tabular}{lrrrrr} 
& min->max & \(0->1\) & +delta & +sd & MargEfct \\
age & -0.4372 & -0.0030 & -0.1345 & -0.1062 & -0.0118
\end{tabular}
```

The result under the heading +delta is the same as what we just calculated using prvalue.

### 4.7 Interpretation using odds ratios with listcoef

Effects for the logit model, but not probit, can be interpreted in terms of changes in the odds. Recall that for binary outcomes, we typically consider the odds of observing a positive outcome versus a negative one:

$$
\Omega=\frac{\operatorname{Pr}(y=1)}{\operatorname{Pr}(y=0)}=\frac{\operatorname{Pr}(y=1)}{1-\operatorname{Pr}(y=1)}
$$

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Recall also that the log of the odds is called the logit and that the logit model is linear in the logit, meaning that the log odds are a linear combination of the $x$ 's and $\beta$ 's. For example, consider a logit model with three independent variables:

$$
\ln \left[\frac{\operatorname{Pr}(y=1 \mid \mathbf{x})}{1-\operatorname{Pr}(y=1 \mid \mathbf{x})}\right]=\ln \Omega(\mathbf{x})=\beta_{0}+\beta_{1} x_{1}+\beta_{2} x_{2}+\beta_{3} x_{3}
$$

We can interpret the coefficients as

For a unit change in $x_{k}$, we expect the logit to change by $\beta_{k}$, holding all other variables constant.

This interpretation does not depend on the level of the other variables in the model. The problem is that a change of $\beta_{k}$ in the log odds has little substantive meaning for most people (including the authors of this book). Alternatively, by taking the exponential of both sides of this equation, we can create a model that is multiplicative instead of linear, but in which the outcome is the more intuitive measure, the odds:

$$
\Omega\left(\mathbf{x}, x_{2}\right)=e^{\beta_{0}} e^{\beta_{1} x_{1}} e^{\beta_{2} x_{2}} e^{\beta_{3} x_{3}}
$$

where we take particular note of the value of $x_{2}$. If we let $x_{2}$ change by 1 ,

$$
\begin{aligned}
\Omega\left(\mathbf{x}, x_{2}+1\right) & =e^{\beta_{0}} e^{\beta_{1} x_{1}} e^{\beta_{2}\left(x_{2}+1\right)} e^{\beta_{3} x_{3}} \\
& =e^{\beta_{0}} e^{\beta_{0}} e^{\beta_{1} x_{1}} e^{\beta_{2} x_{2}} e^{\beta_{2}} e^{\beta_{3} x_{3}}
\end{aligned}
$$

which leads to the odds ratio:

$$
\frac{\Omega\left(\mathbf{x}, x_{2}+1\right)}{\Omega\left(\mathbf{x}, x_{2}\right)}=\frac{e^{\beta_{0}} e^{\beta_{1} x_{1}} e^{\beta_{2} x_{2}} e^{\beta_{2}} e^{\beta_{3} x_{3}}}{e^{\beta_{0}} e^{\beta_{1} x_{1}} e^{\beta_{2} x_{2}} e^{\beta_{3} x_{3}}}=e^{\beta_{2}}
$$

Accordingly, we can interpret the exponential of the coefficient as

For a unit change in $x_{k}$, the $o d d s$ are expected to change by a factor of $\exp \left(\beta_{k}\right)$, holding all other variables constant.

For $\exp \left(\beta_{k}\right)>1$, you could say that the odds are " $\exp \left(\beta_{k}\right)$ times larger". For $\exp \left(\beta_{k}\right)<1$, you could say that the odds are "exp $\left(\beta_{k}\right)$ times smaller". We can evaluate the effect of a standard deviation change in $x_{k}$ instead of a unit change:

For a standard deviation change in $x_{k}$, the odds are expected to change by a factor of $\exp \left(\beta_{k} \times s_{k}\right)$, holding all other variables constant.

The odds ratios for both a unit and a standard deviation change of the independent variables can be obtained with listcoef:

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```
. listcoef, help
logit (N=753): Factor Change in Odds
    Odds of: inLF vs NotInLF
```

| lfp | b | z | $\mathrm{P}>\|\mathrm{z}\|$ | $\mathrm{e}^{\wedge} \mathrm{b}$ | $\mathrm{e}^{\wedge} \mathrm{bStdX}$ | SDofX |
| ---: | ---: | ---: | :---: | :---: | ---: | ---: |
| k 5 | -1.46291 | -7.426 | 0.000 | 0.2316 | 0.4646 | 0.5240 |
| k 618 | -0.06457 | -0.950 | 0.342 | 0.9375 | 0.9183 | 1.3199 |
| age | -0.06287 | -4.918 | 0.000 | 0.9391 | 0.6020 | 8.0726 |
| wc | 0.80727 | 3.510 | 0.000 | 2.2418 | 1.4381 | 0.4500 |
| hc | 0.11173 | 0.542 | 0.588 | 1.1182 | 1.0561 | 0.4885 |
| lwg | 0.60469 | 4.009 | 0.000 | 1.8307 | 1.4266 | 0.5876 |
| inc | -0.03445 | -4.196 | 0.000 | 0.9661 | 0.6698 | 11.6348 |

```
        b = raw coefficient
        z = z-score for test of b=0
    P>|z| = p-value for z-test
        e^b = exp(b) = factor change in odds for unit increase in X
e^bStdX = exp(b*SD of X) = change in odds for SD increase in X
    SDofX = standard deviation of X
```

Examples of interpretations are

For each additional young child, the odds of being employed decrease by a factor of .23 , holding all other variables constant.
For a standard deviation increase in the log of the wife's expected wages, the odds of being employed are 1.43 times greater, holding all other variables constant.
Being ten years older decreases the odds by a factor of $.53\left(=e^{[-.063] \times 10}\right)$, holding all other variables constant.

Other ways of computing odds ratios Odds ratios can also be computed with the or option for logit. This approach does not, however, report the odds ratios for a standard deviation change in the independent variables.

## Multiplicative coefficients

When interpreting the odds ratios, remember that they are multiplicative. This means that positive effects are greater than one and negative effects are between zero and one. Magnitudes of positive and negative effects should be compared by taking the inverse of the negative effect (or vice versa). For example, a positive factor change of 2 has the same magnitude as a negative factor change of $.5=1 / 2$. Thus, a coefficient of $.1=1 / 10$ indicates a stronger effect than a coefficient of 2 . Another consequence of the multiplicative scale is that to determine the effect on the odds of the event not occurring, you simply take the inverse of the effect on the odds of the event occurring. listcoef will automatically calculate this for you if you specify the reverse option:

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. listcoef, reverse
logit (N=753) : Factor Change in Odds
Odds of: NotInLF vs inLF

|  |  |  |  |  |  |  |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| lfp | b | z | P>\|z| | $e^{\wedge} b$ | $e^{\wedge} b S t d X$ | SDofX |
| k5 | -1.46291 | -7.426 | 0.000 | 4.3185 | 2.1522 | 0.5240 |
| k618 | -0.06457 | -0.950 | 0.342 | 1.0667 | 1.0890 | 1.3199 |
| wc | -0.06287 | -4.918 | 0.000 | 1.0649 | 1.6612 | 8.0726 |
| hc | 0.80727 | 3.510 | 0.000 | 0.4461 | 0.6954 | 0.4500 |
| lwg | 0.11173 | 0.542 | 0.588 | 0.8943 | 0.9469 | 0.4885 |
| inc | -0.03445 | -4.196 | 0.000 | 1.0350 | 1.4930 | 11.6348 |

Note that the header indicates that these are now the factor changes in the odds of NotInLF versus inLF, whereas before we computed the factor change in the odds of inLF versus NotInLF. We can interpret the result for k 5 as follows:

For each additional child, the odds of not being employed are increased by a factor of 4.3 ( $=1 / .23$ ), holding other variables constant.

## Effect of the base probability

The interpretation of the odds ratio assumes that the other variables have been held constant, but it does not require that they be held at any specific values. While the odds ratio seems to resolve the problem of nonlinearity, it is essential to keep the following in mind: A constant factor change in the odds does not correspond to a constant change or constant factor change in the probability. For example, if the odds are $1 / 100$, the corresponding probability is $.01 .{ }^{9}$ If the odds double to $2 / 100$, the probability increases only by approximately .01 . Depending on one's substantive purposes, this small change may be trivial or quite important (such as when one identifies a risk factor that makes it twice as likely that a subject will contract a fatal disease). Meanwhile, if the odds are $1 / 1$ and double to $2 / 1$, the probability increases by .167 . Accordingly, the meaning of a given factor change in the odds depends on the predicted probability, which in turn depends on the levels of all variables in the model.

## Percent change in the odds

Instead of a multiplicative or factor change in the outcome, some people prefer the percent change,

$$
100\left[\exp \left(\beta_{k} \times \delta\right)-1\right]
$$

which is listed by listcoef with the percent option.

[^30]```
. listcoef, percent
logit (N=753): Percentage Change in Odds
    Odds of: inLF vs NotInLF
```

| lfp | b | z | $\mathrm{P}>\|\mathrm{z}\|$ | $\%$ | $\%$ StdX | SDofX |
| ---: | :---: | :---: | :---: | :---: | ---: | ---: |
| k5 | -1.46291 | -7.426 | 0.000 | -76.8 | -53.5 | 0.5240 |
| k618 | -0.06457 | -0.950 | 0.342 | -6.3 | -8.2 | 1.3199 |
| age | -0.06287 | -4.918 | 0.000 | -6.1 | -39.8 | 8.0726 |
| wc | 0.80727 | 3.510 | 0.000 | 124.2 | 43.8 | 0.4500 |
| hc | 0.11173 | 0.542 | 0.588 | 11.8 | 5.6 | 0.4885 |
| lwg | 0.60469 | 4.009 | 0.000 | 83.1 | 42.7 | 0.5876 |
| inc | -0.03445 | -4.196 | 0.000 | -3.4 | -33.0 | 11.6348 |

With this option, the interpretations would be

For each additional young child, the odds of being employed decrease by $77 \%$, holding all other variables constant.
A standard deviation increase in the log of the wife's expected wages increases the odds of being employed by $83 \%$, holding all other variables constant.

Percentage and factor change provide the same information; which you use for the binary model is a matter of preference. While we both tend to prefer percentage change, methods for the graphical interpretation of the multinomial logit model (Chapter 6) only work with factor change coefficients.

### 4.8 Other commands for binary outcomes

Logit and probit models are the most commonly used models for binary outcomes and are the only ones that we consider in this book, but other models exist that can be estimated in Stata. Among them, cloglog assumes a complementary log-log distribution for the errors instead of a logistic or normal distribution. scobit estimates a logit model that relaxes the assumption that the marginal change in the probability is greatest when $\operatorname{Pr}(y=1)=.5$. hetprob allows the assumed variance of the errors in the probit model to vary as a function of the independent variables. blogit and bprobit estimate logit and probit models on grouped ("blocked") data. Further details on all of these models can be found in the appropriate entries in the Stata manuals.

[^31] either electronically or in printed form, to others.

# REGRESSION MODELS FOR CATEGORICAL DEPENDENT VARIABLES USING STATA 

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## 5 Models for Ordinal Outcomes

The categories of an ordinal variable can be ranked, but the distances between the categories are unknown. For example, in survey research, opinions are often ranked as strongly agree, agree, disagree, and strongly disagree, without an assumption that the distance from strongly agreeing and agreeing is the same as the distance from agree to disagree. Educational attainments can be ordered as elementary education, high school diploma, college diploma, and graduate or professional degree. Ordinal variables also commonly result from limitations of data availability that require a coarse categorization of a variable that could, in principle, have been measured on an interval scale. For example, we might have a measure of income that is simply low, medium, or high.

Ordinal variables are often coded as consecutive integers from 1 to the number of categories. Perhaps as a consequence of this coding, it is tempting to analyze ordinal outcomes with the linear regression model. However, an ordinal dependent variable violates the assumptions of the LRM, which can lead to incorrect conclusions, as demonstrated strikingly by McKelvey and Zavoina $(1975,117)$ and Winship and Mare (1984, 521-523). Accordingly, with ordinal outcomes it is much better to use models that avoid the assumption that the distances between categories are equal. While many different models have been designed for ordinal outcomes, in this chapter we focus on the logit and probit versions of the ordinal regression model (ORM), introduced by McKelvey and Zavoina (1975) in terms of an underlying latent variable and in biostatistics by McCullagh (1980) who referred to the logit version as the proportional odds model.

As with the binary regression model, the ORM is nonlinear and the magnitude of the change in the outcome probability for a given change in one of the independent variables depends on the levels of all of the independent variables. And, as with the BRM, the challenge is to summarize the effects of the independent variables in a way that fully reflects key substantive processes without overwhelming and distracting detail. For ordinal outcomes, as well as for the models for nominal outcomes in Chapter 6, the difficulty of this task is increased by having more than two outcomes to explain.

Before proceeding, we caution that researchers should think carefully before concluding that their outcome is indeed ordinal. Simply because the values of a variable can be ordered does not imply that the variable should be analyzed as ordinal. A variable that can be ordered when considered for one purpose could be unordered or ordered differently when used for another purpose. Miller and Volker (1985) show how different assumptions about the ordering of occupations resulted in different conclusions. A variable might also reflect ordering on more than one dimension, such as attitude scales that reflect both the intensity of opinion and the direction of opinion. Moreover, surveys commonly include the category "don't know", which probably does not correspond to the

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middle category in a scale, even though analysts might be tempted to treat it this way. Overall, when the proper ordering is ambiguous, the models for nominal outcomes discussed in Chapter 6 should be considered.

We begin by reviewing the statistical model, followed by an examination of testing, fit, and methods of interpretation. These discussions are intended as a review for those who are familiar with the models. For a complete discussion, see Long (1997). We end the chapter by considering several less common models for ordinal outcomes, which can be estimated using ado-files that others have developed. As always, you can obtain sample do-files and data files by downloading the spostst4 package (see Chapter 1 for details).

### 5.1 The statistical model

The ORM can be developed in different ways, each of which leads to the same form of the model. These approaches to the model parallel those for the BRM. Indeed, the BRM can be viewed as a special case of the ordinal model in which the ordinal outcome has only two categories.

### 5.1.1 A latent variable model

The ordinal regression model is commonly presented as a latent variable model. Defining $y^{*}$ as a latent variable ranging from $-\infty$ to $\infty$, the structural model is

$$
y_{i}^{*}=\mathbf{x}_{i} \beta+\varepsilon_{i}
$$

Or, for the case of a single independent variable,

$$
y_{i}^{*}=\alpha+\beta x_{i}+\varepsilon_{i}
$$

where $i$ is the observation and $\varepsilon$ is a random error, discussed further below.
The measurement model for binary outcomes is expanded to divide $y^{*}$ into $J$ ordinal categories:

$$
y_{i}=m \quad \text { if } \tau_{m-1} \leq y_{i}^{*}<\tau_{m} \quad \text { for } m=1 \text { to } J
$$

where the cutpoints $\tau_{1}$ through $\tau_{J-1}$ are estimated. (Some authors refer to these as thresholds.) We assume $\tau_{0}=-\infty$ and $\tau_{J}=\infty$ for reasons that will be clear shortly.

To illustrate the measurement model, consider the example that is used in this chapter. People are asked to respond to the following statement:

A working mother can establish just as warm and secure of a relationship with her child as a mother who does not work.

Possible responses are: $1=$ Strongly Disagree (SD), 2=Disagree (D), 3=Agree (A), and $4=$ Strongly Agree (SA). The continuous latent variable can be thought of as the propensity to agree that working

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mothers can be good mothers. The observed response categories are tied to the latent variable by the measurement model:

$$
y_{i}= \begin{cases}1 \Rightarrow \mathrm{SD} & \text { if } \tau_{0}=-\infty \leq y_{i}^{*}<\tau_{1} \\ 2 \Rightarrow \mathrm{D} & \text { if } \tau_{1} \leq y_{i}^{*}<\tau_{2} \\ 3 \Rightarrow \mathrm{~A} & \text { if } \tau_{2} \leq y_{i}^{*}<\tau_{3} \\ 4 \Rightarrow \mathrm{SA} & \text { if } \tau_{3} \leq y_{i}^{*}<\tau_{4}=\infty\end{cases}
$$

Thus, when the latent $y^{*}$ crosses a cutpoint, the observed category changes.


Figure 5.1: Relationship between observed $y$ and latent $y^{*}$ in ordinal regression model with a single independent variable.

For a single independent variable, the structural model is $y^{*}=\alpha+\beta x+\varepsilon$, which is plotted in Figure 5.1 along with the cutpoints for the measurement model. This figure is similar to that for the binary regression model, except that there are now three horizontal lines representing the cutpoints $\tau_{1}, \tau_{2}$, and $\tau_{3}$. The three cutpoints lead to four levels of $y$ that are labeled on the right-hand side of the graph.

The probability of an observed outcome for a given value of $x$ is the area under the curve between a pair of cutpoints. For example, the probability of observing $y=m$ for given values of the $x$ 's corresponds to the region of the distribution where $y^{*}$ falls between $\tau_{m-1}$ and $\tau_{m}$ :

$$
\operatorname{Pr}(y=m \mid \mathbf{x})=\operatorname{Pr}\left(\tau_{m-1} \leq y^{*}<\tau_{m} \mid \mathbf{x}\right)
$$

Substituting $\mathrm{x} \beta+\varepsilon$ for $y^{*}$ and using some algebra leads to the standard formula for the predicted probability in the ORM,

$$
\begin{equation*}
\operatorname{Pr}(y=m \mid \mathbf{x})=F\left(\tau_{m}-\mathbf{x} \beta\right)-F\left(\tau_{m-1}-\mathbf{x} \beta\right) \tag{5.1}
\end{equation*}
$$

where $F$ is the cdf for $\varepsilon$. In ordinal probit, $F$ is normal with $\operatorname{Var}(\varepsilon)=1$; in ordinal logit, $F$ is logistic with $\operatorname{Var}(\varepsilon)=\pi^{2} / 3$. Note that for $y=1$, the second term on the right drops out since $F(-\infty-\mathbf{x} \beta)=0$, and for $y=J$, the first term equals $F(\infty-\mathbf{x} \beta)=1$.

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Comparing these equations to those for the BRM shows that the ORM is identical to the binary regression model, with one exception. To show this, we estimate Chapter 4's binary model for labor force participation using both logit and ologit (the command for ordinal logit):

```
. use binlfp2, clear
(Data from 1976 PSID-T Mroz)
. logit lfp k5 k618 age wc hc lwg inc, nolog
    (output omitted)
. outreg using 05lgtolgt, xstats replace
. ologit lfp k5 k618 age wc hc lwg inc, nolog
    (output omitted)
. outreg using 05lgtolgt, xstats append
```

To compare the coefficients, we combine them using outreg; this leads to the following table, which has been slightly edited:

|  | logit results | ologit results |
| :---: | :---: | :---: |
| \# kids < 6 | -1.463 | -1.463 |
|  | (7.43)** | (7.43)** |
| \# kids 6-18 | -0.065 | -0.065 |
|  | (0.95) | (0.95) |
| Wife's age in years | -0.063 | -0.063 |
|  | (4.92)** | (4.92)** |
| Wife College: 1=yes 0=no | 0.807 | 0.807 |
|  | (3.51)** | (3.51)** |
| Husband College: 1=yes $0=$ no | 0.112 | 0.112 |
|  | (0.54) | (0.54) |
| Log of wife's estimated wages | 0.605 | 0.605 |
|  | (4.01)** | (4.01)** |
| Family income excluding wife's | -0.034 | -0.034 |
|  | (4.20)** | (4.20)** |
| Constant | 3.182 |  |
|  | (4.94)** |  |
| _cut1 |  | -3.182 |
|  |  | (4.94)** |
| Observations | 753 | 753 |

Absolute value of $z$-statistics in parentheses

* significant at $5 \%$ level; ** significant at $1 \%$ level

The slope coefficients and their standard errors are identical, but for logit an intercept is reported (i.e., the coefficient associated with _cons), while for ologit the constant is replaced by the cutpoint labeled _cut1, which is equal but of opposite sign.

This difference is due to how the two models are identified. As the ORM has been presented, there are "too many" free parameters; that is, you can't estimate $J-1$ thresholds and the constant too. For a unique set of ML estimates to exist, an identifying assumption needs to be made about either the intercept or one of the cutpoints. In Stata, the ORM is identified by assuming that the intercept is 0 and the values of all cutpoints are estimated. Some statistics packages for the ORM instead fix one of the cutpoints to 0 and estimate the intercept. And, in presenting the BRM, we immediately assumed that the value that divided $y^{*}$ into observed 0 s and 1 s was 0 . In effect, we identified the model by assuming a threshold of 0 . While different parameterizations can be confusing, keep in mind that the slope coefficients and predicted probabilities are the same under either parameterization (see Long 1997, 122-23 for further details).

[^32] either electronically or in printed form, to others.

### 5.1.2 A nonlinear probability model

The ordinal regression model can also be developed as a nonlinear probability model without appealing to the idea of a latent variable. Here we show how this can be done for the ordinal logit model. First, define the odds that an outcome is less than or equal to $m$ versus greater than $m$ given x :

$$
\Omega_{\leq m \mid>m}(\mathbf{x}) \equiv \frac{\operatorname{Pr}(y \leq m \mid \mathbf{x})}{\operatorname{Pr}(y>m \mid \mathbf{x})} \quad \text { for } m=1, J-1
$$

For example, we could compute the odds of disagreeing or strongly disagreeing (i.e., $m \leq 2$ ) versus agreeing or strongly agreeing $(m>2)$. The log of the odds is assumed to equal

$$
\begin{equation*}
\ln \Omega_{\leq m \mid>m}(\mathbf{x})=\tau_{m}-\mathbf{x} \beta \tag{5.2}
\end{equation*}
$$

For a single independent variable and three categories (where we are fixing the intercept to equal 0 ),

$$
\begin{aligned}
& \ln \frac{\operatorname{Pr}(y \leq 1 \mid \mathbf{x})}{\operatorname{Pr}(y>1 \mid \mathbf{x})}=\tau_{1}-\beta_{1} x_{1} \\
& \ln \frac{\operatorname{Pr}(y \leq 2 \mid \mathbf{x})}{\operatorname{Pr}(y>2 \mid \mathbf{x})}=\tau_{2}-\beta_{1} x_{1}
\end{aligned}
$$

While it may seem confusing that the model subtracts $\beta x$ rather than adding it, this is a consequence of computing the logit of $y \leq m$ versus $y>m$. While we agree that it would be simpler to stick with $\tau_{m}+\beta x$, this is not the way the model is normally presented.

### 5.2 Estimation using ologit and oprobit

The ordered logit and probit models can be estimated with the following commands:

```
ologit depvar [indepvars] [weight] [if exp] [in range] [, level(#)
    nolog table cluster(varname) robust ]
oprobit depvar [indepvars] [weight] [if exp] [in range] [, level(#)
    nolog table cluster(varname) robust ]
```

In our experience, these models take more steps to converge than either the models for binary or nominal outcomes.

## Variable Lists

depvar is the dependent variable. The specific values assigned to the outcome categories are irrelevant except that larger values are assumed to correspond to "higher" outcomes. For example, if you had three outcomes, you could use the values 1,2 , and 3 or $-1.23,2.3$, and 999 . Up to 50 outcomes are allowed in Intercooled Stata; 20 outcomes are allowed in Small Stata.
indepvars is a list of independent variables. If indepvars is not included, Stata estimates a model with only cutpoints.

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## Specifying the estimation sample

if and in qualifiers can be used to restrict the estimation sample. For example, if you want to estimate an ordered logit model for only those in the 1989 sample, you could specify ologit warm age ed prst male white if $\mathrm{yr} 89==1$.

Listwise deletion Stata excludes cases in which there are missing values for any of the variables in the model. Accordingly, if two models are estimated using the same dataset but have different sets of independent variables, it is possible to have different samples. We recommend that you use mark and markout (discussed in Chapter 3) to explicitly remove cases with missing data.

Weights Both ologit and oprobit can be used with fweights, pweights, and iweights. See Chapter 3 for further details.

## Options

nolog suppresses the iteration history.
table lists the equations for predicted probabilities and reports the observed percent of cases for each category in the estimation sample. For example,

| warm | Probability | Observed |
| ---: | :--- | ---: |
| SD | $\operatorname{Pr}(\quad$ xb+u<_cut1) | 0.1295 |
| D | $\operatorname{Pr}\left(\_\right.$cut1<xb+u<_cut2) | 0.3153 |
| A | $\operatorname{Pr}\left(\_c u t 2<x b+u<\right.$ _cut3 $)$ | 0.3733 |
| SA | $\operatorname{Pr}\left(\_\right.$cut3<xb+u) | 0.1819 |

level (\#) specifies the level of the confidence interval for estimated parameters. By default, Stata uses a $95 \%$ interval. You can also change the default level, say, to a $90 \%$ interval, with the command set level 90.
cluster (varname) specifies that the observations are independent across the groups specified by unique values of varname but not necessarily within the groups. See Chapter 3 for further details.
robust indicates that robust variance estimates are to be used. When cluster () is specified, robust standard errors are automatically used. See Chapter 3 for further details.

### 5.2.1 Example of attitudes toward working mothers

Our example is based on a question from the 1977 and 1989 General Social Survey. As we have already described, respondents were asked to evaluate the following statement: "A working mother can establish just as warm and secure of a relationship with her child as a mother who does not work". Responses were coded as: $1=$ Strongly Disagree (SD), $2=$ Disagree (D), 3=Agree (A), and $4=$ Strongly Agree (SA). A complete description of the data can be obtained by using describe, summarize, and tabulate:

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. sum warm yr89 male white age ed prst

| Variable | Obs | Mean | Std. Dev. | Min | Max |
| ---: | ---: | ---: | ---: | ---: | ---: |
| warm | 2293 | 2.607501 | .9282156 | 1 | 4 |
| yr89 | 2293 | .3986044 | .4897178 | 0 | 1 |
| male | 2293 | .4648932 | .4988748 | 0 | 1 |
| white | 2293 | .8765809 | .3289894 | 0 | 1 |
| age | 2293 | 44.93546 | 16.77903 | 18 | 89 |
| ed | 2293 | 12.21805 | 3.160827 | 0 | 20 |
| prst | 2293 | 39.58526 | 14.49226 | 12 | 82 |

. tab warm

| Mom can <br> have warm <br> relations <br> with child | Freq. | Percent | Cum. |
| ---: | ---: | ---: | ---: |
| SD | 297 | 12.95 | 12.95 |
| D | 723 | 31.53 | 44.48 |
| A | 856 | 37.33 | 81.81 |
| SA | 417 | 18.19 | 100.00 |
| Total | 2293 | 100.00 |  |
|  |  |  |  |

Using these data, we estimated the model

$$
\operatorname{Pr}\left(\text { warm }=m \mid \mathbf{x}_{i}\right)=F\left(\tau_{m}-\mathbf{x} \beta\right)-F\left(\tau_{m-1}-\mathbf{x} \beta\right)
$$

where

$$
\mathbf{x} \beta=\beta_{\mathrm{yr} 89} \mathrm{yr} 89+\beta_{\text {male }} \text { male }+\beta_{\text {white }} \text { white }+\beta_{\text {age }} \text { age }+\beta_{\text {prst }} \text { prst }
$$

Here is the output from ologit and oprobit, which we combine using outreg:

| . ologit warm yr89 male white age ed prst, nolog |  |  |  |
| :--- | :--- | :--- | :--- |
| Ordered logit estimates | Number of obs | $=$ | 2293 |
|  | LR chi2 $(6)$ | $=$ | 301.72 |
| Log likelihood $=-2844.9123$ |  | Prob $>$ chi2 | $=$ |
|  | Pseudo R2 | $=$ | 0.0000 |
|  |  |  |  |


| warm | Coef. | Std. Err. | z | P>\|z| | [95\% Conf. Interval] |  |
| ---: | ---: | ---: | :---: | ---: | ---: | ---: |
| yr89 | .5239025 | .0798988 | 6.56 | 0.000 | .3673037 | .6805013 |
| male | -.7332997 | .0784827 | -9.34 | 0.000 | -.8871229 | -.5794766 |

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| white | -.3911595 | .1183808 | -3.30 | 0.001 | -.6231815 | -.1591374 |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| age | -.0216655 | .0024683 | -8.78 | 0.000 | -.0265032 | -.0168278 |
| ed | .0671728 | .015975 | 4.20 | 0.000 | .0358624 | .0984831 |
| prst | .0060727 | .0032929 | 1.84 | 0.065 | -.0003813 | .0125267 |
| _cut1 | -2.465362 | .2389126 |  | (Ancillary parameters) |  |  |
| _cut2 | -.630904 | .2333155 |  |  |  |  |
| _cut3 | 1.261854 | .2340179 |  |  |  |  |


| Ordered probit estimates | Number of obs | $=$ | 2293 |
| :---: | :---: | :---: | :---: |
|  | LR chi2(6) |  | 294.32 |
|  | Prob > chi2 | = | 0.0000 |
| Log likelihood = -2848.611 | Pseudo R2 | = | 0.0491 |


| warm | Coef. | Std. Err. | z | P>\|z| | [95\% Conf. Interval] |  |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| yr89 | .3188147 | .0468519 | 6.80 | 0.000 | .2269867 | .4106427 |
| male | -.4170287 | .0455459 | -9.16 | 0.000 | -.5062971 | -.3277603 |
| white | -.2265002 | .0694773 | -3.26 | 0.001 | -.3626733 | -.0903272 |
| age | -.0122213 | .0014427 | -8.47 | 0.000 | -.0150489 | -.0093937 |
| ed | .0387234 | .0093241 | 4.15 | 0.000 | .0204485 | .0569982 |
| prst | .003283 | .001925 | 1.71 | 0.088 | -.0004899 | .0070559 |
| _cut1 | -1.428578 | .1387742 |  | (Ancillary parameters) |  |  |
| _cut2 | -.3605589 | .1369219 |  |  |  |  |
| _cut3 | .7681637 | .1370564 |  |  |  |  |

. outreg using 05lgtpbt, append
The information in the header and the table of coefficients is in the same form as discussed in Chapter 3.

The estimated coefficients have been combined using outreg. The first call of the program saves the coefficients from ologit to the file 05lgtpbt. out, while the second call using append adds the coefficients from oprobit. After making a few edits to the file, we get

|  | Ordered <br> Logit | Ordered <br> Probit |
| :---: | :---: | :---: |
| Year of survey | 0.524 | 0.319 |
|  | (6.56) | (6.80) |
| Sex | -0.733 | -0.417 |
|  | (9.34) | (9.16) |
| Race | -0.391 | -0.227 |
|  | (3.30) | (3.26) |
| Age in years | -0.022 | -0.012 |
|  | (8.78) | (8.47) |
| Years of education | 0.067 | 0.039 |
|  | (4.20) | (4.15) |
| Occupational prestige | 0.006 | 0.003 |
|  | (1.84) | (1.71) |
| Observations | 2293 | 2293 |
| Absolute value of $z$-st | tistics in | arenthes |

As with the BRM, the estimated coefficients differ from logit to probit by a factor of about 1.7, reflecting the differing scaling of the ordered logit and ordered probit models. Values of the $z$-tests are very similar since they are not affected by the scaling.

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### 5.2.2 Predicting perfectly

If the dependent variable does not vary within one of the categories of an independent variable, there will be a problem with estimation. To see what happens, let's transform the prestige variable prst into a dummy variable:

| dumprst | SD Mother has warm relationship ${ }_{\text {d }}$ |  |  |  | Total |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 0 | 257 | 723 | 856 | 417 | 2253 |
| 1 | 40 | 0 | 0 | 0 | 40 |
| Total | 297 | 723 | 856 | 417 | 2293 |

In all cases where dumprst is 1 , respondents have values of SD for warm. That is, if you know dumprst is 1 you can predict perfectly that warm is 1 (i.e., SD ). While we purposely constructed dumprst so this would happen, perfect prediction can also occur in real data. If we estimate the ORM using dumprst rather than prst,
. ologit warm yr89 male white age ed dumprst, nolog

| Ordered logit estimates | Number of obs | $=$ | 2293 |
| :--- | :--- | :--- | :--- |
|  | LR chi2 $(6)$ | $=$ | 447.02 |
|  | Prob $>$ chi2 | $=$ | 0.0000 |
| Log likelihood $=-2772.2621$ | Pseudo R2 | $=$ | 0.0746 |


| warm | Coef. | Std. Err. | z | P>\|z| | [95\% Conf. Interval] |  |  |  |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: | :---: | :---: |
| yr89 | .5268578 | .0805997 | 6.54 | 0.000 | .3688853 | .6848303 |  |  |
| male | -.7251825 | .0792896 | -9.15 | 0.000 | -.8805872 | -.5697778 |  |  |
| white | -.4240687 | .1197416 | -3.54 | 0.000 | -.658758 | -.1893795 |  |  |
| age | -.0210592 | .0024462 | -8.61 | 0.000 | -.0258536 | -.0162648 |  |  |
| ed | .072143 | .0133133 | 5.42 | 0.000 | .0460494 | .0982366 |  |  |
| dumprst | -37.58373 | 8670897 | -0.00 | 1.000 | $-1.70 \mathrm{e}+07$ | $1.70 \mathrm{e}+07$ |  |  |
| _cut1 | -2.776233 | .243582 |  | (Ancillary parameters) |  |  |  |  |
| _cut2 | -.8422903 | .2363736 |  |  |  |  |  |  |
| _cut3 | 1.06148 | .236561 |  |  |  |  |  |  |

note: 40 observations completely determined. Standard errors questionable.
The note: 40 observations completely determined. Standard errors questionable indicates the problem. In practice, the next step would be to delete the 40 cases in which dumprst equals 1 (you could use the command drop if dumprst==1 to do this) and re-estimate the model without dumprst. This corresponds to what is done automatically for binary models estimated by logit and probit.

### 5.3 Hypothesis testing with test and Irtest

Hypothesis tests of regression coefficients can be evaluated with the $z$-statistics in the estimation output, with test for Wald tests of simple and complex hypotheses, and with lrtest for the corresponding likelihood-ratio tests. We briefly review each.

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### 5.3.1 Testing individual coefficients

If the assumptions of the model hold, the ML estimators from ologit and oprobit are distributed asymptotically normally. The hypothesis $H_{0}: \beta_{k}=\beta^{*}$ can be tested with $z=\left(\widehat{\beta}_{k}-\beta^{*}\right) / \widehat{\sigma}_{\widehat{\beta}_{k}}$. Under the assumptions justifying ML, if $H_{0}$ is true, then $z$ is distributed approximately normally with a mean of 0 and a variance of 1 for large samples. For example, consider the results for the variable male from the ologit output above:

```
. ologit warm male yr89 white age ed prst, nolog
    (output omitted)
```

| warm | Coef. | Std. Err. | $z$ | P>\|z| | [95\% Conf. Interval] |  |
| :---: | ---: | :---: | :---: | :---: | :---: | :---: |
| male | -.7332997 | .0784827 | -9.34 | 0.000 | -.8871229 | -.5794766 |
| (output omitted) |  |  |  |  |  |  |

We conclude that
Gender significantly affects attitudes toward working mothers ( $z=-9.34, p<0.01$ for a two-tailed test).

Either a one-tailed and two-tailed test can be used as discussed in Chapter 4.
The $z$-test in the output of estimation commands is a Wald test, which can also be computed using test. For example, to test $H_{0}: \beta_{\text {male }}=0$,
. test male
( 1) male $=0.0$

$$
\begin{array}{rlr}
\operatorname{chi2}(1) & = & 87.30 \\
\text { Prob }>\text { chi2 } & = & 0.0000
\end{array}
$$

We conclude that
Gender significantly affects attitudes toward working mothers $\left(X^{2}=87.30, d f=\right.$ $1, p<0.01$ ).

The value of a chi-squared test with 1 degree of freedom is identical to the square of the corresponding $z$-test, which can be demonstrated with the display command:

```
. display "z*z=" -9.343*-9.343
z*z=87.291649
```

An LR test is computed by comparing the log likelihood from a full model to that of a restricted model. To test a single coefficient, we begin by estimating the full model:
. ologit warm yr89 male white age ed prst, nolog

| Ordered logit estimates | Number of obs | $=$ | 2293 |
| :--- | :--- | :--- | :--- |
|  | LR chi2 $(6)$ | $=$ | 301.72 |
|  | Prob $>$ chi2 | $=$ | 0.0000 |
| Log likelihood $=-2844.9123$ | Pseudo R2 | $=$ | 0.0504 |

    (output omitted)
    . Irtest, saving(0)

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Then we estimate the model excluding male:

| Ordered logit estimates | Number of obs | = | 2293 |
| :---: | :---: | :---: | :---: |
|  | LR chi2(5) | = | 212.98 |
|  | Prob > chi2 | = | 0.0000 |
| ```Log likelihood = -2889.278 (output omitted)``` | Pseudo R2 | = | 0.0355 |
| . lrtest |  |  |  |
| Ologit: likelihood-ratio test | chi2(1) | = | 88.73 |
|  | Prob > chi2 | $=$ | 0.0000 |

The resulting LR test can be interpreted as
The effect of being male is significant at the . 01 level $\left(L R X^{2}=88.73, d f=1, p<\right.$ .01).

### 5.3.2 Testing multiple coefficients

We can also test a complex hypothesis that involves more than one coefficient. For example, our model has three demographic variables: age, white, and male. To test that all of the demographic factors are simultaneously equal to zero, $H_{0}: \beta_{\text {age }}=\beta_{\text {white }}=\beta_{\text {male }}=0$, we can use either a Wald or a LR test. For the Wald test, we estimate the full model as before and then type

```
. test age white male
(1) age = 0.0
(2) white = 0.0
( 3) male = 0.0
    chi2( 3) = 166.62
    Prob > chi2 = 0.0000
```

We conclude that
The hypothesis that the demographic effects of age, race, and gender are simultaneously equal to zero can be rejected at the .01 level $\left(X^{2}=166.62, d f=3, p<.01\right)$.
test can also be used to test the equality of effects as shown in Chapter 4.
To compute an LR test of multiple coefficients, we first estimate the full model and save the results with lrtest, $\operatorname{saving}(0)$. Then, to test $H_{0}: \beta_{\text {age }}=\beta_{\text {white }}=\beta_{\text {male }}=0$, we estimate the model that excludes these three variables and run lrtest:

```
* estimate full model
. ologit warm yr89 male white age ed prst, nolog
    (output omitted)
. lrtest, saving(0)
* estimate constrained model
. ologit warm yr89 ed prst, nolog
    (output omitted)
. lrtest
```

Ologit: likelihood-ratio test $\quad$ chi2(3) $=\quad 171.58$

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We conclude that

The hypothesis that the demographic effects of age, race, and gender are simultaneously equal to zero can be rejected at the .01 level ( $X^{2}=171.58, d f=3, p<.01$ ).

In our experience, the Wald and LR tests usually lead to the same decisions. When there are differences, they generally occur when the tests are near the cutoff for statistical significance. Given that the LR test is invariant to reparameterization, we prefer the LR test.

### 5.4 Scalar measures of fit using fitstat

As we discuss at greater length in Chapter 3, scalar measures of fit can be useful in comparing competing models (see also Long 1997, 85-113). Several different measures can be computed after either ologit or oprobit with the SPost command fitstat:

```
. ologit warm yr89 male white age ed prst, nolog
    (output omitted)
. fitstat
Measures of Fit for ologit of warm
Log-Lik Intercept Only: -2995.770
D(2284): 5689.825
McFadden's R2:
Ma.050
Maximum Likelihood R2: 0.123
McKelvey and Zavoina's R2: 0.127
Variance of y*: 3.768
lrlr
lrlr
BIC: -11982.891 BIC`: -255.291
Log-Lik Full Model:
LR(6):
-2844.912
    301.716
Prob > LR:
    301.716
McFadden's Adj R2:
        0.047
Cragg & Uhler's R2:
        0.133
Variance of y*: 
```

Using simulations, both Hagle and Mitchell (1992) and Windmeijer (1995) find that, for ordinal outcomes, McKelvey and Zavonia's $R^{2}$ most closely approximates the $R^{2}$ obtained by estimating the linear regression model on the underlying latent variable.

### 5.5 Converting to a different parameterization*

Earlier we noted that different software packages use different parameterizations to identify the model. Stata sets $\beta_{0}=0$ and estimates $\tau_{1}$, while some programs fix $\tau_{1}=0$ and estimate $\beta_{0}$. While all quantities of interest for purpose of interpretation (e.g., predicted probabilities) are the same under both parameterizations, it is useful to see how Stata can estimate the model under either parameterization. The key to understanding how this is done is the equation:

$$
\operatorname{Pr}(y=m \mid \mathbf{x})=F\left(\left[\tau_{m}-\delta\right]-\left[\beta_{0}-\delta\right]-\mathbf{x} \beta\right)-F\left(\left[\tau_{m-1}-\delta\right]-\left[\beta_{0}-\delta\right]-\mathbf{x} \beta\right)
$$

Without further constraints, it is only possible to estimate the differences $\tau_{m}-\delta$ and $\beta_{0}-\delta$. Stata assumes $\delta=\beta_{0}$, which forces the estimate of $\beta_{0}$ to be 0 , while some other programs assume $\delta=\tau_{1}$, which forces the estimate of $\tau_{1}$ to be 0 . For example,

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| Model <br> Parameter | Stata's <br> Estimate | Alternative <br> Parameterization |
| :---: | :---: | :---: |
| $\beta_{0}$ | $\beta_{0}-\beta_{0}=0$ | $\beta_{0}-\tau_{1}$ |
| $\tau_{1}$ | $\tau_{1}-\beta_{0}$ | $\tau_{1}-\tau_{1}=0$ |
| $\tau_{2}$ | $\tau_{2}-\beta_{0}$ | $\tau_{2}-\tau_{1}$ |
| $\tau_{3}$ | $\tau_{3}-\beta_{0}$ | $\tau_{3}-\tau_{1}$ |

While you would only need to compute the alternative parameterization if you wanted to compare your results to those produced by another statistics package, seeing how this is done illustrates why the intercept and thresholds are arbitrary. To estimate the alternative parameterization, we use lincom to estimate the difference between Stata's estimates (see page 143) and the estimated value of the first cutpoint:

```
. ologit warm yr89 male white age ed prst, nolog
    (output omitted)
. * intercept
. lincom O - _b[_cut1]
( 1) - _cut1 + _cut2 = 0.0
\begin{tabular}{r|rccccc}
\hline warm & Coef. & Std. Err. & \(z\) & \(P>|z|\) & [95\% Conf. Interval] \\
\hline\((1)\) & 2.777878 & .2437943 & 11.39 & 0.000 & 2.30005 & 3.255706 \\
\hline
\end{tabular}
```

Here we are computing the alternative parameterization of the intercept. ologit assumes that $\beta_{0}=$ 0 , so we simply estimate $0-\tau_{1}$; that is, $0-\_$[_cut1]. The trick is that the cutpoints are contained in the vector _b [ ], with the index for these scalars specified as _cut1, _cut2, and _cut3. For the thresholds, we are estimating $\tau_{2}-\tau_{1}$ and $\tau_{3}-\tau_{1}$, which correspond to _b [_cut2]-_b [_cut1] and _b[_cut3]-_b[_cut1]:
. * cutpoint 2
. lincom _b[_cut2] - _b[_cut1]
( 1) - _cut1 + _cut2 $=0.0$

| warm | Coef. | Std. Err. | z | P>\|z| | [95\% Conf. Interval] |  |
| ---: | ---: | ---: | :---: | :---: | :---: | :---: |
| $(1)$ | 1.834458 | .0630432 | 29.10 | 0.000 | 1.710895 | 1.95802 |

. * cutpoint 3
. lincom _b[_cut3] - _b[_cut1]
( 1) - _cut1 + _cut3 $=0.0$

| warm | Coef. | Std. Err. | z | P>\|z| | [95\% Conf. Interval] |  |
| ---: | ---: | :---: | :---: | :---: | :---: | ---: |
| $(1)$ | 3.727216 | .0826215 | 45.11 | 0.000 | 3.565281 | 3.889151 |

The estimate of $\tau_{1}-\tau_{1}$ is, of course, 0 .
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### 5.6 The parallel regression assumption

Before discussing interpretation, it is important to understand an assumption that is implicit in the ORM, known both as the parallel regression assumption and, for the ordinal logit model, the proportional odds assumption. Using Equation 5.1, the ORM can be written as

$$
\begin{aligned}
\operatorname{Pr}(y=1 \mid \mathbf{x}) & =F\left(\tau_{m}-\mathbf{x} \beta\right) \\
\operatorname{Pr}(y=m \mid \mathbf{x}) & =F\left(\tau_{m}-\mathbf{x} \beta\right)-F\left(\tau_{m-1}-\mathbf{x} \beta\right) \text { for } m=2 \text { to } J-1 \\
\operatorname{Pr}(y=J \mid \mathbf{x}) & =1-F\left(\tau_{m-1}-\mathbf{x} \beta\right)
\end{aligned}
$$

These equations can be used to compute the cumulative probabilities, which have the simple form

$$
\begin{equation*}
\operatorname{Pr}(y \leq m \mid \mathbf{x})=F\left(\tau_{m}-\mathbf{x} \beta\right) \quad \text { for } m=1 \text { to } J-1 \tag{5.3}
\end{equation*}
$$

This equation shows that the ORM is equivalent to $J-1$ binary regressions with the critical assumption that the slope coefficients are identical across each regression.

For example, with four outcomes and a single independent variable, the equations are

$$
\begin{aligned}
& \operatorname{Pr}(y \leq 1 \mid \mathbf{x})=F\left(\tau_{1}-\beta x\right) \\
& \operatorname{Pr}(y \leq 2 \mid \mathbf{x})=F\left(\tau_{2}-\beta x\right) \\
& \operatorname{Pr}(y \leq 3 \mid \mathbf{x})=F\left(\tau_{3}-\beta x\right)
\end{aligned}
$$

The intercept $\alpha$ is not in the equation since it has been assumed to equal 0 to identify the model. These equations lead to the following figure: ${ }^{1}$


[^33]Each probability curve differs only in being shifted to the left or right. That is, they are parallel as a consequence of the assumption that the $\beta$ 's are equal for each equation.

This figure suggests that the parallel regression assumption can be tested by comparing the estimate from the $J-1$ binary regressions,

$$
\operatorname{Pr}(y \leq m \mid \mathbf{x})=F\left(\tau_{m}-\mathbf{x} \beta_{m}\right) \quad \text { for } m=1, J-1
$$

where the $\beta$ 's are allowed to differ across the equations. The parallel regression assumption implies that $\beta_{1}=\beta_{2}=\cdots=\beta_{J-1}$. To the degree that the parallel regression assumption holds, the coefficients $\widehat{\beta}_{1}, \widehat{\beta}_{2}, \ldots, \widehat{\beta}_{J-1}$ should be "close". There are two commands in Stata that perform this test:

An approximate LR test The command omodel Wolfe and Gould (1998) is not part of official Stata, but can be obtained by typing net search omodel and following the prompts. omodel computes an approximate LR test. Essentially, this method compares the log likelihood from ologit (or oprobit) to that obtained from pooling $J-1$ binary models estimated with logit (or probit), making an adjustment for the correlation between the binary outcomes defined by $y \leq m$. The syntax is

```
omodel [logit|probit] depvar [varlist] [weight] [if exp] [in range]
```

where the options logit or probit indicates whether ordered logit or ordered probit is to be used. For example,

```
. omodel logit warm yr89 male white age ed prst
    (same output as for ologit warm yr89 male white age ed prst)
Approximate likelihood-ratio test of proportionality of odds
across response categories:
    chi2(12) = 48.91
    Prob > chi2 = 0.0000
```

In this case, the parallel regression assumption can be rejected at the .01 level.

A Wald test The LR test is an omnibus test that the coefficients for all variables are simultaneously equal. Accordingly, you cannot determine whether the coefficients for some variables are identical across the binary equations while coefficients for other variables differ. To this end, a Wald test by Brant (1990) is useful since it tests the parallel regression assumption for each variable individually. The messy details of computing this test are found in Brant (1990) or Long (1997, 143-144). In Stata the test is computed quickly with brant, which is part of SPost. After running ologit (brant does not work with oprobit), you run brant with the syntax:

```
brant [, detail]
```

The detail option provides a table of coefficients from each of the binary models. For example,

[^34]```
. brant, detail
Estimated coefficients from j-1 binary regressions
\begin{tabular}{rrrr} 
& \(y>1\) & \(y>2\) & \(y>3\) \\
yr89 & .9647422 & .56540626 & .31907316 \\
male & -.30536425 & -.69054232 & -1.0837888 \\
white & -.55265759 & -.31427081 & -.39299842 \\
age & -.0164704 & -.02533448 & -.01859051 \\
ed & .10479624 & .05285265 & .05755466 \\
prst & -.00141118 & .00953216 & .00553043 \\
cons & 1.8584045 & .73032873 & -1.0245168
\end{tabular}
Brant Test of Parallel Regression Assumption
```

| Variable | chi2 | p>chi2 | df |
| ---: | ---: | ---: | ---: |
| All | 49.18 | 0.000 | 12 |
| yr89 | 13.01 | 0.001 | 2 |
| male | 22.24 | 0.000 | 2 |
| white | 1.27 | 0.531 | 2 |
| age | 7.38 | 0.025 | 2 |
| ed | 4.31 | 0.116 | 2 |
| prst | 4.33 | 0.115 | 2 |

A significant test statistic provides evidence that the parallel regression assumption has been violated.

The chi-squared of 49.18 for the Brant test is very close to the value of 48.91 from the LR test. However, the Brant test shows that the largest violations are for yr89 and male, which suggests that there may be problems related to these variables.

Caveat regarding the parallel regression assumption In our experience, the parallel regression assumption is frequently violated. When the assumption of parallel regressions is rejected, alternative models should be considered that do not impose the constraint of parallel regressions. Alternative models that can be considered include models for nominal outcomes discussed in Chapter 6 or other models for ordinal outcomes discussed in Section 5.9

### 5.7 Residuals and outliers using predict

While no methods for detecting influential observations and outliers have been developed specifically for the ORM, Hosmer and Lemeshow $(2000,305)$ suggest applying the methods for binary models to the $J-1$ cumulative probabilities that were discussed in the last section. As noted by Hosmer and Lemeshow, the disadvantage of this approach is that you are only evaluating an approximation to the model you have estimated, since the coefficients of the binary models differ from those estimated in the ordinal model. But, if the parallel regression assumption is not rejected, you can be more confident in the results of your residual analysis.

To illustrate this approach, we start by generating three binary variables corresponding to warm $<2$, warm $<3$, and warm $<4$ :

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```
. gen warmlt2 = (warm<2) if warm ~=.
. gen warmlt3 = (warm<3) if warm ~=.
. gen warmlt4 = (warm<4) if warm ~=.
```

For example, warmlt3 is 1 if warm equals 1 or 2 , else 0 . Next, we estimate binary logits for warmlt2, warmlt3, and warmlt4 using the same independent variables as in our original ologit model. After estimating each logit, we generate standardized residuals using predict (for a detailed discussion of generating and inspecting these residuals, see Chapter 4):

* warm < 2
. logit warmlt2 yr89 male white age ed prst (output omitted)
. predict rstd_lt2, rs
* warm < 3
. logit warmlt3 yr89 male white age ed prst (output omitted)
. predict rstd_lt2, rs
. predict rstd_lt3, rs
* warm < 4
. logit warmlt4 yr89 male white age ed prst (output omitted)
. predict rstd_lt4, rs
Next we create an index plot for each of the three binary equations. For example, using the results from the logit of warmlt3,
. sort prst
- gen index = _n
- graph rstd_lt3 index, s(o) xscale(0,800) yscale(-4,4) yline(0) /*
> */ ylabel ( $-4,-2,0,2,4$ ) b2("Observation Number") gap(3)


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Given the size of the dataset, no residual stands out as being especially large. However, since the parallel regression assumption was violated, alternative models should still be considered.

### 5.8 Interpretation

If the idea of a latent variable makes substantive sense, simple interpretations are possible by rescaling $y^{*}$ to compute standardized coefficients that can be used just like coefficients for the linear regression model. If the focus is on the categories of the ordinal variable (e.g., what affects the likelihood of strongly agreeing), the methods illustrated for the BRM can be extended to multiple outcomes. Since the ORM is nonlinear in the outcome probabilities, no single approach can fully describe the relationship between a variable and the outcome probabilities. Consequently, you should consider each of these methods before deciding which approach is most effective in a your application. For purposes of illustration, we continue to use the example of attitudes toward working mothers. Keep in mind, however, that the test of the parallel regression assumption suggests that this model is not appropriate for these data.

### 5.8.1 Marginal change in $y^{*}$

In the ORM, $y^{*}=\mathbf{x} \beta+\varepsilon$ and the marginal change in $y^{*}$ with respect to $x_{k}$ is

$$
\frac{\partial y^{*}}{\partial x_{k}}=\beta_{k}
$$

Since $y^{*}$ is latent (and hence its metric is unknown), the marginal change cannot be interpreted without standardizing by the estimated standard deviation of $y^{*}$,

$$
\widehat{\sigma}_{y^{*}}^{2}=\widehat{\beta}^{\prime} \widehat{\operatorname{Var}}(\mathbf{x}) \widehat{\beta}+\operatorname{Var}(\varepsilon)
$$

where $\widehat{\operatorname{Var}}(\mathbf{x})$ is the covariance matrix for the observed $x$ 's, $\widehat{\beta}$ contains ML estimates, and $\operatorname{Var}(\varepsilon)=$ 1 for ordered probit and $\pi^{2} / 3$ for ordered logit. Then, the $y^{*}$-standardized coefficient for $x_{k}$ is

$$
\beta_{k}^{S y^{*}}=\frac{\beta_{k}}{\sigma_{y^{*}}}
$$

which can be interpreted as
For a unit increase in $x_{k}, y^{*}$ is expected to increase by $\beta_{k}^{S y^{*}}$ standard deviations, holding all other variables constant.

The fully standardized coefficient is

$$
\beta_{k}^{S}=\frac{\sigma_{k} \beta_{k}}{\sigma_{y^{*}}}=\sigma_{k} \beta_{k}^{S y^{*}}
$$

which can be interpreted as
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For a standard deviation increase in $x_{k}, y^{*}$ is expected to increase by $\beta_{k}^{S}$ standard deviations, holding all other variables constant.

These coefficients can be computed with listcoef using the std option. For example, after estimating the ordered logit model,

```
. listcoef, std help
ologit ( \(\mathrm{N}=2293\) ) : Unstandardized and Standardized Estimates
Observed SD: . 9282156
    Latent SD: 1.9410634
```

| warm | b | z | P>\|z| | bStdX | bStdY | bStdXY | SDofX |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| yr89 | 0.52390 | 6.557 | 0.000 | 0.2566 | 0.2699 | 0.1322 | 0.4897 |
| male | -0.73330 | -9.343 | 0.000 | -0.3658 | -0.3778 | -0.1885 | 0.4989 |
| white | -0.39116 | -3.304 | 0.001 | -0.1287 | -0.2015 | -0.0663 | 0.3290 |
| age | -0.02167 | -8.778 | 0.000 | -0.3635 | -0.0112 | -0.1873 | 16.7790 |
| ed | 0.06717 | 4.205 | 0.000 | 0.2123 | 0.0346 | 0.1094 | 3.1608 |
| prst | 0.00607 | 1.844 | 0.065 | 0.0880 | 0.0031 | 0.0453 | 14.4923 |

b = raw coefficient
$z=z$-score for test of $b=0$
$P>|z|=p$-value for $z$-test
bStdX $=\mathrm{x}$-standardized coefficient
bStdY $=$ y-standardized coefficient
bStdXY = fully standardized coefficient
SDof $X=$ standard deviation of $X$
If we think of the dependent variable as measuring "support" for mothers in the workplace, then the effect of the year of the interview can be interpreted as

In 1989 support was .27 standard deviations higher than in 1977, holding all other variables constant.

To consider the effect of education,
Each standard deviation increase in education increases support by .11 standard deviations, holding all other variables constant.

### 5.8.2 Predicted probabilities

For the most part, we prefer interpretations based in one way or another on predicted probabilities. These probabilities can be estimated with the formula

$$
\widehat{\operatorname{Pr}}(y=m \mid \mathbf{x})=F\left(\widehat{\tau}_{m}-\mathbf{x} \widehat{\beta}\right)-F\left(\widehat{\tau}_{m-1}-\mathbf{x} \widehat{\beta}\right)
$$

with cumulative probabilities computed as

$$
\widehat{\operatorname{Pr}}(y \leq m \mid \mathbf{x})=F\left(\tau_{m}-\mathbf{x} \widehat{\beta}\right)
$$

The values of $\mathbf{x}$ can be based on observations in the sample or can be hypothetical values of interest. The most basic command for computing probabilities is predict, but our SPost commands can be used to compute predicted probabilities in particularly useful ways.

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### 5.8.3 Predicted probabilities with predict

After estimating a model with ologit or oprobit, a useful first step is to compute the in sample predictions with the command

```
predict newvarl [newvar2[newvar3...]] [if exp] [in range]
```

where you indicate one new variable name for each category of the dependent variable. For instance, in the following example predict specifies that the variables SDwarm, Dwarm, Awarm, and SAwarm should be created with predicted values for the four outcome categories:

```
. ologit warm yr89 male white age ed prst, nolog
    (output omitted)
. predict SDlogit Dlogit Alogit SAlogit
(option p assumed; predicted probabilities)
```

The message (option p assumed; predicted probabilities) reflects that predict can compute many different quantities. Since we did not specify an option indicating which quantity to predict, option p for predicted probabilities was assumed.

An easy way to see the range of the predictions is with dotplot, one of our favorite commands for quickly checking data:

- label var SDwarm $" \operatorname{Pr}(S D) "$
- label var Dwarm $" \operatorname{Pr}(D) "$
. label var Awarm "Pr(A)"
. label var SAwarm "Pr(SA)"
. dotplot SDwarm Dwarm Awarm SAwarm, ylabel (0,.25, .5,.75)
which leads to the following plot:


The predicted probabilities for the extreme categories tend to be less than .25 , with the majority of predictions for the middle categories falling between .25 and .5. In only a few cases is the probability of any outcome greater than .5.

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Examining predicted probabilities within the sample provides a first, quick check of the model. To understand and present the substantive findings, however, it is usually more effective to compute predictions at specific, substantively informative values. Our commands prvalue, prtab, prgen, and prchange are designed to make this simple.

### 5.8.4 Individual predicted probabilities with prvalue

Predicted probabilities for individuals with a particular set of characteristics can be computed with prvalue. For example, we might want to examine the predicted probabilities for individuals with the following characteristics:

- Working class men in 1977 who are near retirement.
- Young, highly educated women with prestigious jobs.
- An "average individual" in 1977.
- An "average individual" in 1989.

Each of these can be easily computed with prvalue (see Chapter 3 for a discussion of options for this command). The predicted probabilities for older, working class men are

```
. prvalue, x(yr89=0 male=1 prst=20 age=64 ed=16) rest(mean)
ologit: Predictions for warm
    Pr(y=SD|x): 0.2317
    Pr}(y=D|x): 0.422
    Pr}(y=A|x): 0.272
    Pr}(y=SA|x): 0.073
```


or, for young, highly educated women with prestigious jobs

| $\operatorname{Pr}(\mathrm{y}=$ SD $\mid \mathrm{x})$ : | 0.0164 |
| :---: | :---: |
| $\operatorname{Pr}(\mathrm{y}=\mathrm{D} \mid \mathrm{x})$ : | 0.0781 |
| $\operatorname{Pr}(\mathrm{y}=\mathrm{A} \mid \mathrm{x})$ : | 0.3147 |
| $\operatorname{Pr}(\mathrm{y}=$ SA $\mid x)$ : | 0.5908 |

and so on, for other sets of values.
There are several points about using prvalue that are worth emphasizing. First, we have set the values of the independent variables that define our hypothetical person using the x () and rest () options. The output from the first call of prvalue lists the values that have been set for all independent variables. This allows you to verify that x() and rest () did what you intended. For the second call, we added the brief option. This suppresses the output showing the levels of the independent variables. If you use this option, be certain that you have correctly specified the levels of all variables. Second, the output of prvalue labels the categories according to the value labels assigned to the dependent variable. For example, $\operatorname{Pr}(y=S D \mid x): 0.2317$. Since it is very easy

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to be confused about the outcome categories when using these models, it is prudent to assign clear value labels to your dependent variable (we describe how to assign value labels in Chapter 2).

We can summarize the results in a table that lists the ideal types and provides a clear indication of which variables are important:

|  | Outcome Category |  |  |  |
| :--- | :---: | :---: | :---: | :---: |
| Ideal Type | $S D$ | $D$ | $A$ | $S A$ |
| Working class men in 1977 who are near retirement. | 0.23 | 0.42 | 0.27 | 0.07 |
| Young, highly educated women in 1989 with <br> prestigious jobs. |  |  |  |  |
| An "average individual" in 1977. | 0.02 | 0.08 | 0.32 | 0.59 |
| An "average individual" in 1989. | 0.13 | 0.36 | 0.37 | 0.14 |

### 5.8.5 Tables of predicted probabilities with prtab

In other cases, it can be useful to compute predicted probabilities for all combinations of a set of categorical independent variables. For example, the ideal types illustrate the importance of gender and the year when the question was asked. Using prtab, we can easily show the degree to which these variables affect opinions for those average on other characteristics.

```
. prtab yr89 male, novarlbl
ologit: Predicted probabilities for warm
Predicted probability of outcome 1 (SD)
\begin{tabular}{c|cc}
\hline & \multicolumn{2}{|c}{ male } \\
yr89 & Women & Men \\
\hline 1977 & 0.0989 & 0.1859 \\
1989 & 0.0610 & 0.1191 \\
\hline
\end{tabular}
Predicted probability of outcome 2 (D)
\begin{tabular}{c|cc}
\hline & \multicolumn{2}{|c}{ male } \\
yr89 & Women & Men \\
\hline 1977 & 0.3083 & 0.4026 \\
1989 & 0.2282 & 0.3394 \\
\hline
\end{tabular}
(tables for other outcomes omitted)
```

Tip Sometimes the output of prtab is clearer without the variable labels. These can be suppressed with the novarlbl option.

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The output from prtab can be rearranged into a table that clearly shows that men are more likely than women to strongly disagree or disagree with the proposition that working mothers can have as warm of relationships with their children as mothers who do not work. The table also shows that between 1977 and 1989 there was a movement for both men and women toward more positive attitudes about working mothers.

| $\mathbf{1 9 7 7}$ | SD | D | A | SA |
| ---: | :--- | :--- | ---: | ---: |
| Men | 0.19 | 0.40 | 0.32 | 0.10 |
| Women | 0.10 | 0.31 | 0.41 | 0.18 |
| Difference | 0.09 | 0.09 | -0.09 | -0.08 |
|  |  |  |  |  |
| $\mathbf{1 9 8 9}$ | SD | D | A | SA |
| Men | 0.12 | 0.34 | 0.39 | 0.15 |
| Women | 0.06 | 0.23 | 0.44 | 0.27 |
| Difference | 0.06 | 0.11 | -0.05 | -0.12 |


| Change from $\mathbf{1 9 7 7}$ to $\mathbf{1 9 8 9}$ |  |  |  |  |
| ---: | :---: | :---: | :---: | :---: |
|  | SD | D | A | SA |
| Men | -0.07 | -0.06 | 0.07 | 0.05 |
| Women | -0.04 | -0.08 | 0.03 | 0.09 |

### 5.8.6 Graphing predicted probabilities with prgen

Graphing predicted probabilities for each outcome can also be useful for the ORM. In this example, we consider women in 1989 and show how predicted probabilities are affected by age. Of course, the plot could also be constructed for other sets of characteristics. The predicted probabilities as age ranges from 20 to 80 are generated by prgen:

```
. prgen age, from(20) to(80) generate(w89) x(male=0 yr89=1) ncases(13)
ologit: Predicted values as age varies from 20 to 80.
\begin{tabular}{rrrrrr} 
& yr89 & male & white & age & ed \\
\(\mathrm{x}=\) & 1 & 0 & .8765809 & 44.935456 & 12.218055 \\
39.585259
\end{tabular}
```

You should be familiar with how x() operates, but it is useful to review the other options:
from (20) and to (80) specify the minimum and maximum values over which inc is to vary. The default is the variable's minimum and maximum values.
ncases (13) indicates that 13 evenly spaced values of age between 20 and 80 should be generated. gen(w89) is the root name for the new variables.

In our example, w89x contains values of age ranging from 20 to 80 . The $\mathrm{p} \#$ variables contain the predicted probability for outcome \# (e.g., w89p2 is the predicted probability of outcome 2 ). With

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ordinal outcomes, prgen also computes cumulative probabilities (i.e., summed) that are indicated by s (e.g., w89s2 is the sum of the predicted probability of outcomes 1 and 2 ). A list of the variables that are created should make this clear:
. desc w89*

| variable name | storage <br> type | display <br> format | value <br> label | variable label |
| :--- | ---: | ---: | :--- | :--- | | w89x | float | $\% 9.0 \mathrm{~g}$ |
| :--- | :--- | :--- |
| w89p1 | float | $\% 9.0 \mathrm{~g}$ |
| w89s1 | float | $\% 9.0 \mathrm{~g}$ |
| w89p2 | float | $\% 9.0 \mathrm{~g}$ |
| w89s2 | float | $\% 9.0 \mathrm{~g}$ |
| w89p3 | float | $\% 9.0 \mathrm{~g}$ |
| w89s3 | float | $\% 9.0 \mathrm{~g}$ |
| w89p4 | float | $\% 9.0 \mathrm{~g}$ |
| w89s4 | float | $\% 9.0 \mathrm{~g}$ |

While prgen assigns variable labels to the variables it creates, we can change these to improve the look of the plot that we are creating. Specifically,

- label var w89p1 "SD"
- label var w89p2 "D"
. label var w89p3 "A"
. label var w89p4 "SA"
. label var w89s1 "SD"
- label var w89s2 "SD \& D"
. label var w89s3 "SD, D \& A"

First we plot the probabilities of individual outcomes using graph. Since the graph command is long, we use $/ * * /$ to allow the commands to be longer than one line in our do-file. In the output, the $>$ 's indicate that the same command is continuing across lines:

```
. * step 1: graph predicted probabilities
. graph w89p1 w89p2 w89p3 w89p4 w89x, /*
> */ title("Panel A: Predicted Probabilities") b2("Age") /*
> */ xlabel(20,30,40,50,60,70,80) ylabel(0,.25,.50) xscale(20,80) yscale(0,.5) /*
>// s(OdST) connect(ssss) noaxis yline(0,.5) gap(4) xline(44.93) /*
> */ saving(tmp1.gph, replace)
```

This graph command plots the four predicted probabilities against generated values for age contained in w89x. Standard options for graph are used to specify the axes and labels. The vertical line specified by xline (44.93) marks the average age in the sample. This line is used to illustrate the marginal effect discussed in Section 5.8.7. Option saving (tmp1.gph, replace) saves the graph to the temporary file tmp1.gph so that we can combine it with the next graph, which plots the cumulative probabilities:

```
. * step 2: graph cumulative probabilities
. graph w89s1 w89s2 w89s3 w89x, /*
> */ title("Panel B: Cumulative Probabilities") b2("Age") /*
>*/ xlabel(20,30,40,50,60,70,80) ylabel(0,.25,.50,.75,1.0) xscale(20,80) /*
>*/ yscale(0,1) s(0dST) connect(sss) yline(0,1) gap(4) noaxis /*
> */ saving(tmp2.gph, replace)
```

Next we combine these two graphs along with a null graph (see Chapter 2 for details on combining graphs):

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. * step 3: create an empty graph

- graph using, saving(null, replace)
. * step 4: combine graphs
- graph using tmp1 null tmp2

This leads to Figure 5.2. Panel A plots the predicted probabilities and shows that with age the probability of SA decreases rapidly while the probability of $D$ (and to a lesser degree SD) increases. Panel B plots the cumulative probabilities. Since both panels present the same information, which one you use is largely a matter of personal preference.


Figure 5.2: Plot of predicted probabilities for the ordered logit model.

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### 5.8.7 Changes in predicted probabilities

When there are many variables in the model, it is impractical to plot them all. In such cases, measures of change in the outcome probabilities are a useful way to summarize the effects of each variable. Before proceeding, however, we hasten to note that values of both discrete and marginal change depend on the levels of all variables in the model. We return to this point shortly.

## Marginal change with prchange

The marginal change in the probability is computed as

$$
\frac{\partial \operatorname{Pr}(y=m \mid \mathbf{x})}{\partial x_{k}}=\frac{\partial F\left(\tau_{m}-\mathbf{x} \beta\right)}{\partial x_{k}}-\frac{\partial F\left(\tau_{m-1}-\mathbf{x} \beta\right)}{\partial x_{k}}
$$

which is the slope of the curve relating $x_{k}$ to $\operatorname{Pr}(y=m \mid \mathbf{x})$, holding all other variables constant. In our example, we consider the marginal effect of age $(\partial \operatorname{Pr}(y=m \mid \mathbf{x}) / \partial$ age $)$ for women in 1989 who are average on all other variables. This corresponds to the slope of the curves in Panel A of Figure 5.2 evaluated at the vertical line (recall that this line is drawn at the average age in the sample). The marginal is computed with prchange, where we specify that only the coefficients for age should be computed:

```
. prchange age, x(male=0 yr89=1) rest(mean)
ologit: Changes in Predicted Probabilities for warm
age
\begin{tabular}{rrrrrr} 
& Avg|Chg| & SD & D & A & SA \\
Min->Max & .16441458 & .10941909 & .21941006 & -.05462247 & -.27420671 \\
\(-+1 / 2\) & .00222661 & .00124099 & .00321223 & -.0001803 & -.00427291 \\
-+ sd/2 & .0373125 & .0208976 & .05372739 & -.00300205 & -.07162295 \\
MargEfct & .00890647 & .00124098 & .00321226 & -.00018032 & -.00427292
\end{tabular}
\begin{tabular}{rrrrrrrr} 
& & SD & \multicolumn{7}{r}{ D } & A & SA \\
\(\operatorname{Pr}(\mathrm{y} \mid \mathrm{x})\) & .06099996 & .22815652 & .44057754 & .27026597 & \\
& & yr89 & male & white & age & ed & prst \\
\(\mathrm{x}=\) & 1 & 0 & .876581 & 44.9355 & 12.2181 & 39.5853 \\
\(\operatorname{sd}(\mathrm{x})=\) & .489718 & .498875 & .328989 & 16.779 & 3.16083 & 14.4923
\end{tabular}
```

The first thing to notice is the row labeled $\operatorname{Pr}(\mathrm{y} \mid \mathrm{x})$, which is the predicted probabilities at the values set by $x$ () and rest (). In Panel A, these probabilities correspond to the intersection of the vertical line and the probability curves. The row MargEfct lists the slopes of the probability curves at the point of intersection with the vertical line in the figure. For example, the slope for SD (shown with circles) is .00124 , while the slope for A (shown with squares) is negative and small. As with the BRM, the size of the slope indicates the instantaneous rate of change, but does not correspond exactly to the amount of change in the probability for a change of one unit in the independent variable. However, when the probability curve is approximately linear, the marginal effect can be used to summarize the effect of a unit change in the variable on the probability of an outcome.

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## Marginal change with mfx compute

Marginal change can also be computed using mfx compute, where at () is used to set values of the independent variables. Unlike prchange, mfx does not allow you to compute effects for a subset of the independent variables. And, it only estimates the marginal effects for one outcome category at a time, where the category is specified with the option predict (outcome (\#)). Using the same values for the independent variables as in the example above, we obtain the following results:
. mfx compute, at(male=0 yr89=1) predict(outcome(1))
Marginal effects after ologit
$\mathrm{y}=\operatorname{Pr}($ warm==1) (predict, outcome(1))
$=.06099996$

| variable | $\mathrm{dy} / \mathrm{dx}$ | Std. Err. | z | $\mathrm{P}>\mid \mathrm{zl}$ | [ | 95\% C.I. | ] | X |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| yr89* | -.0378526 | .00601 | -6.30 | 0.000 | -.049633 | -.026072 | 1.00000 |  |
| male* | .0581355 | .00731 | 7.95 | 0.000 | .043803 | .072468 | 0.00000 |  |
| white* | .0197511 | .0055 | 3.59 | 0.000 | .008972 | .03053 | .876581 |  |
| age | .001241 | .00016 | 7.69 | 0.000 | .000925 | .001557 | 44.9355 |  |
| ed | -.0038476 | .00097 | -3.96 | 0.000 | -.005754 | -.001941 | 12.2181 |  |
| prst | -.0003478 | .00019 | -1.83 | 0.068 | -.000721 | .000025 | 39.5853 |  |

(*) dy/dx is for discrete change of dummy variable from 0 to 1
The marginal for age is .001241 , which matches the result obtained from prchange. The advantage of mfx is that it computes standard errors.

## Discrete change with prchange

Since the marginal change can be misleading when the probability curve is changing rapidly or when an independent variable is a dummy variable, we prefer using discrete change ( mfx computes discrete change for independent variables that are binary, but not for other independent variables). The discrete change is the change in the predicted probability for a change in $x_{k}$ from the start value $x_{\mathrm{S}}$ to the end value $x_{\mathrm{E}}$ (e.g., a change from $x_{k}=0$ to $x_{k}=1$ ). Formally,

$$
\frac{\Delta \operatorname{Pr}(y=m \mid \mathbf{x})}{\Delta x_{k}}=\operatorname{Pr}\left(y=m \mid \mathbf{x}, x_{k}=x_{\mathrm{E}}\right)-\operatorname{Pr}\left(y=m \mid \mathbf{x}, x_{k}=x_{\mathrm{S}}\right)
$$

where $\operatorname{Pr}\left(y=m \mid \mathbf{x}, x_{k}\right)$ is the probability that $y=m$ given $\mathbf{x}$, noting a specific value for $x_{k}$. The change is interpreted as

When $x_{k}$ changes from $x_{\mathrm{S}}$ to $x_{\mathrm{E}}$, the predicted probability of outcome $m$ changes by $\frac{\Delta \operatorname{Pr}(y=m \mid \mathbf{x})}{\Delta x_{k}}$, holding all other variables at $\mathbf{x}$.

The value of the discrete change depends on: (1) the value at which $x_{k}$ starts; (2) the amount of change in $x_{k}$; and (3) the values of all other variables. Most frequently, each continuous variable except $x_{k}$ is held at its mean. For dummy independent variables, the change could be computed for both values of the variable. For example, we could compute the discrete change for age separately for men and women.

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In our example, the discrete change coefficients for male, age, and prst for women in 1989, with other variables at their mean, are computed as follows:

| ologit: Changes in Predicted Probabilities for warm |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| male |  |  |  |  |  |
|  | Avgl Chg 1 | SD | D | A | SA |
| $0->1$ | . 08469636 | . 05813552 | . 11125721 | -. 05015317 | -. 11923955 |
| age |  |  |  |  |  |
|  | AvglChg | SD | D | A | SA |
| Min->Max | . 16441458 | . 10941909 | . 21941006 | -. 05462247 | -. 27420671 |
| -+1/2 | . 00222661 | . 00124099 | . 00321223 | -. 0001803 | -. 00427291 |
| -+sd/2 | . 0373125 | . 0208976 | . 05372739 | -. 00300205 | -. 07162295 |
| MargEfct | . 00890647 | . 00124098 | . 00321226 | -. 00018032 | -. 00427292 |
| prst |  |  |  |  |  |
|  | Avg 1 Chg 1 | SD | D | A | SA |
| Min->Max | . 04278038 | -. 02352008 | -. 06204067 | . 00013945 | . 08542132 |
| -+1/2 | . 00062411 | -. 00034784 | -. 00090037 | . 00005054 | . 00119767 |
| -+sd/2 | . 00904405 | -. 00504204 | -. 01304607 | . 00073212 | . 01735598 |
| MargEfct | . 00249643 | -. 00034784 | -. 00090038 | . 00005054 | . 00119767 |
|  | SD | D | A | SA |  |
| $\operatorname{Pr}(\mathrm{y} \mid \mathrm{x})$ | . 06099996 | . 22815652.44 | 4057754.27 | 7026597 |  |
|  | yr89 m | male white | age | ed | rst |
| $\mathrm{x}=$ | 1 | 0.876581 | 44.9355 | 12.218139 .5 |  |
| $s d(x)=$ | . 489718.498 | 8875 . 328989 | 16.779 | 3.1608314 .4 | 923 |

For variables that are not binary, the discrete change can be interpreted for a unit change centered around the mean, for a standard deviation change centered around the mean, or as the variable changes from its minimum to its maximum value. For example,

For a standard deviation increase in age, the probability of disagreeing increases by .05 , holding other variables constant at their means.
Moving from the minimum prestige to the maximum prestige changes the predicted probability of strongly agreeing by .06 , holding all other variables constant at their means.

The $J$ discrete change coefficients for a variable can be summarized by computing the average of the absolute values of the changes across all of the outcome categories:

$$
\bar{\Delta}=\frac{1}{J} \sum_{j=1}^{J}\left|\frac{\Delta \operatorname{Pr}(y=j \mid \overline{\mathbf{x}})}{\Delta x_{k}}\right|
$$

The absolute value must be used since the sum of the changes without taking the absolute value is necessarily zero. These are labeled as Avg | Chg |. For example, the effect of being a male is on average 0.08 , which is larger than the average effect of a standard deviation change in either age or occupational prestige.

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## Computing discrete change for a 10 year increase in age

In the example above, age was measured in years. Not surprisingly, the change in the predicted probability for a one-year increase in age is trivially small. But, to characterize the effect of age, we could report the effect of a ten-year change in age.

Warning It is tempting to compute the discrete change for a ten-year change in age by simply multiplying the one-year discrete change by 10 . This will give you approximately the right answer if the probability curve is nearly linear over the range of change. But, when the curve is not linear, simply multiplying can give very misleading results and even the wrong sign. To be safe, don't do it!

The delta(\#) option for prchange computes the discrete change as an independent value changes from \#/2 units below the base value to \#/2 above. In our example, we use delta (10) and set the base value of age to its mean:

| ologit: Changes in Predicted Probabilities for warm |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| age |  |  |  |  |  |  |  |  |
|  | Avg 1 |  | SD | D | D | A |  | SA |
| Min->Max | . 16441 | 458.10 | 0941909 | . 21941006 | -. 0546 | 2247 |  | . 27420671 |
| -+d/2 | . 02225 | 603.01 | 1242571 | . 03208634 | -. 0017 | 9818 |  | . 04271388 |
| -+sd/2 | . 0373 | 125 . 02 | 0208976 | . 05372739 | -. 0030 | 0205 |  | . 07162295 |
| MargEfct | . 00890 | 647 . 00 | 0124098 | . 00321226 | -. 0001 | 8032 |  | . 00427292 |
|  | S | D | D | A | SA |  |  |  |
| $\operatorname{Pr}(\mathrm{y} \mid \mathrm{x})$ | . 0609999 | 6 . 22815 | 5652.44 | 057754.2 | 27026597 |  |  |  |
|  | yr89 | male | white | age | ed |  | prs |  |
| $\mathrm{x}=$ | 1 | 0 | . 876581 | 44.9355 | 12.2181 |  |  |  |
| $s d(x)=$ | . 489718 | . 498875 | . 328989 | 16.779 | 3.16083 | 14. | 492 |  |

For females interviewed in 1989, the results in the $-+\mathrm{d} / 2$ row show the changes in the predicted probabilities associated with a ten-year increase in age centered on the mean.

### 5.8.8 Odds ratios using listcoef

For ologit, but not oprobit, we can interpret the results using odds ratios. Earlier, Equation 5.2 defined the ordered logit model as

$$
\Omega_{\leq m \mid>m}(\mathbf{x})=\exp \left(\tau_{m}-\mathbf{x} \beta\right)
$$

For example, with four outcomes we would simultaneously estimate three equations:

$$
\begin{aligned}
& \Omega_{\leq 1 \mid>1}(\mathbf{x})=\exp \left(\tau_{1}-\mathbf{x} \beta\right) \\
& \Omega_{\leq 2 \mid>2}(\mathbf{x})=\exp \left(\tau_{2}-\mathbf{x} \beta\right) \\
& \Omega_{\leq 3 \mid>3}(\mathbf{x})=\exp \left(\tau_{3}-\mathbf{x} \beta\right)
\end{aligned}
$$

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Using the same approach as shown for binary logit, the effect of a change in $x_{k}$ of 1 equals

$$
\frac{\Omega_{\leq m \mid>m}\left(\mathbf{x}, x_{k}+1\right)}{\Omega_{\leq m \mid>m}\left(\mathbf{x}, x_{k}\right)}=e^{-\beta_{k}}=\frac{1}{e^{\beta_{k}}}
$$

which can be interpreted as

For a unit increase in $x_{k}$, the odds of an outcome being less than or equal to $m$ is changed by the factor $\exp \left(-\beta_{k}\right)$, holding all other variables constant.

The value of the odds ratio does not depend on the value of $m$, which is why the parallel regression assumption is also known as the proportional odds assumption. That is to say, we could interpret the odds ratio as

For a unit increase in $x_{k}$, the odds of a lower outcome compared to a higher outcome are changed by the factor $\exp \left(-\beta_{k}\right)$, holding all other variables constant.
or, for a change in $x_{k}$ of $\delta$,

$$
\frac{\Omega_{\leq m \mid>m}\left(\mathbf{x}, x_{k}+\delta\right)}{\Omega_{\leq m \mid>m}\left(\mathbf{x}, x_{k}\right)}=\exp \left(-\delta \times \beta_{k}\right)=\frac{1}{\exp \left(\delta \times \beta_{k}\right)}
$$

so that

For an increase of $\delta$ in $x_{k}$, the odds of lower outcome compared to a higher outcome change by the factor $\exp \left(-\delta \times \beta_{k}\right)$, holding all other variables constant.

In these results, we are discussing factor changes in the odds of lower outcomes compared to higher outcomes. This is done since the model is traditionally written as $\ln \Omega_{\leq m \mid>m}(\mathbf{x})=\tau_{m}-\mathbf{x} \beta$, which leads to the factor change coefficient of $\exp \left(-\beta_{k}\right)$. For purposes of interpretation, we could just as well consider the factor change in the odds of higher versus lower values; that is, changes in the odds $\Omega_{>m \mid \leq m}(\mathbf{x})$. This would equal $\exp \left(\beta_{k}\right)$.

The odds ratios for both a unit and a standard deviation change of the independent variables can be computed with listcoef, which lists the factor changes in the odds of higher versus lower outcomes. Here, we request coefficients for only male and age:

```
. ologit warm yr89 male white age ed prst, nolog
    (output omitted)
. listcoef male age, help
ologit (N=2293): Factor Change in Odds
    Odds of: >m vs <=m
```

| warm | b | z | $\mathrm{P}>\|\mathrm{z}\|$ | $\mathrm{e}^{\wedge} \mathrm{b}$ | $\mathrm{e}^{\wedge} \mathrm{bStdX}$ | SDofX |
| ---: | :---: | :---: | :---: | :---: | :---: | ---: |
| male | -0.73330 | -9.343 | 0.000 | 0.4803 | 0.6936 | 0.4989 |
| age | -0.02167 | -8.778 | 0.000 | 0.9786 | 0.6952 | 16.7790 |

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```
        b = raw coefficient
        z = z-score for test of b=0
    p>|z| = p-value for z-test
    e^b = exp(b) = factor change in odds for unit increase in X
e^bStdX = exp(b*SD of X) = change in odds for SD increase in X
    SDofX = standard deviation of X
```

or to compute percent changes in the odds,

```
. listcoef male age, help percent
ologit (N=2293): Percentage Change in Odds
    Odds of: >m vs <=m
```

| warm | b | z | $\mathrm{P}>\|\mathrm{z}\|$ | $\%$ | $\%$ StdX | SDofX |
| ---: | :---: | :---: | :---: | :---: | :---: | ---: |
| male | -0.73330 | -9.343 | 0.000 | -52.0 | -30.6 | 0.4989 |
| age | -0.02167 | -8.778 | 0.000 | -2.1 | -30.5 | 16.7790 |

b = raw coefficient
$z=z$-score for test of $b=0$
$P>|z|=p$-value for $z$-test
$\%=$ percent change in odds for unit increase in X
$\%$ StdX $=$ percent change in odds for $S D$ increase in $X$
SDof $X=$ standard deviation of $X$
These results can be interpreted as
The odds of having more positive attitudes towards working mothers are .48 times smaller for men than women, holding all other variables constant. Equivalently, the odds of having more positive values are 52 percent smaller for men than women, holding other variables constant.
For a standard deviation increase in age, the odds of having more positive attitudes decrease by a factor of .69 , holding all other variables constant.

When presenting odds ratios, our experience is that people find it easier to understand the results if you talk about increases in the odds rather than decreases. That is, it is clearer to say, "The odds increased by a factor of 2 " than to say, "The odds decreased by a factor of . 5 ". If you agree, then you can reverse the order when presenting odds. For example, we could say

The odds of having more negative attitudes towards working mothers are 2.08 times larger for men than women, holding all other variables constant.

This new factor change, 2.08 , is just the inverse of the old value .48 (that is, $1 / .48$ ). listcoef computes the odds of a lower category versus a higher category if you specify the reverse option:
. listcoef male, reverse
ologit ( $\mathrm{N}=2293$ ) : Factor Change in Odds
Odds of: <=m vs >m

| warm | b | z | $\mathrm{P}>\|\mathrm{z}\|$ | $\mathrm{e}^{\wedge} \mathrm{b}$ | $\mathrm{e}^{\wedge} \mathrm{bStdX}$ | SDofX |
| :---: | :---: | :---: | :---: | :---: | :---: | ---: |
| male | -0.73330 | -9.343 | 0.000 | 2.0819 | 1.4417 | 0.4989 |

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Notice that the output now says Odds of: $<=\mathrm{m}$ vs $>\mathrm{m}$ instead of Odds of: $>\mathrm{m}$ vs $<=\mathrm{m}$ as it did earlier.

When interpreting the odds ratios, it is important to keep in mind two points that are discussed in detail in Chapter 4. First, since odds ratios are multiplicative coefficients, positive and negative effects should be compared by taking the inverse of the negative effect (or vice versa). For example, a negative factor change of .5 has the same magnitude as a positive factor change of $2=1 / .5$. Second, the interpretation only assumes that the other variables have been held constant, not held at any specific values (as was required for discrete change). But, a constant factor change in the odds does not correspond to a constant change or constant factor change in the probability.

### 5.9 Less common models for ordinal outcomes

Stata can also be used to estimate several less commonly used models for ordinal outcomes. In concluding this chapter, we describe these models briefly and note their commands for estimation. Our SPost commands do not work with these models. For gologit and ocratio, this is mainly because these commands do not fully incorporate the new methods of returning information that were introduced with Stata 6.

### 5.9.1 Generalized ordered logit model

The parallel regression assumption results from assuming the same coefficient vector $\beta$ for all comparisons in the $J-1$ equations

$$
\ln \Omega_{\leq m \mid>m}(\mathbf{x})=\tau_{m}-\mathbf{x} \beta
$$

where $\Omega_{\leq m \mid>m}(\mathbf{x})=\frac{\operatorname{Pr}(y \leq m \mid \mathbf{x})}{\operatorname{Pr}(y>m \mid \mathbf{x})}$. The generalized ordered logit model (GOLM) allows $\beta$ to differ for each of the $J-1$ comparisons. That is,

$$
\ln \Omega_{\leq m \mid>m}(\mathbf{x})=\tau_{m}-\mathbf{x} \beta_{m} \quad \text { for } j=1 \text { to } J-1
$$

where predicted probabilities are computed as

$$
\begin{aligned}
& \operatorname{Pr}(y=1 \mid \mathbf{x})=\frac{\exp \left(\tau_{1}-\mathbf{x} \beta_{1}\right)}{1+\exp \left(\tau_{1}-\mathbf{x} \beta_{1}\right)} \\
& \operatorname{Pr}(y=j \mid \mathbf{x})=\frac{\exp \left(\tau_{j}-\mathbf{x} \beta_{j}\right)}{1+\exp \left(\tau_{j}-\mathbf{x} \beta_{j}\right)}-\frac{\exp \left(\tau_{j-1}-\mathbf{x} \beta_{j-1}\right)}{1+\exp \left(\tau_{j-1}-\mathbf{x} \beta_{j-1}\right)} \quad \text { for } j=2 \text { to } J-1 \\
& \operatorname{Pr}(y=J \mid \mathbf{x})=1-\frac{\exp \left(\tau_{J-1}-\mathbf{x} \beta_{J-1}\right)}{1+\exp \left(\tau_{J-1}-\mathbf{x} \beta_{J-1}\right)}
\end{aligned}
$$

To insure that the $\operatorname{Pr}(y=j \mid \mathbf{x})$ is between 0 and 1 , the condition

$$
\left(\tau_{j}-\mathbf{x} \beta_{j}\right) \geq\left(\tau_{j-1}-\mathbf{x} \beta_{j-1}\right)
$$

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must hold. Once predicted probabilities are computed, all of the approaches used to interpret the ORM results can be readily applied. This model has been discussed by Clogg and Shihadeh (1994, 146-147), Fahrmeir and Tutz (1994, 91), and McCullagh and Nelder $(1989,155)$. It can be estimated in Stata with the add-on command gologit (Fu 1998). To obtain this command, enter net search gologit and follow the prompts to download.

### 5.9.2 The stereotype model

The stereotype ordered regression model (SORM) was proposed by Anderson (1984) in response to the restrictive assumption of parallel regressions in the ordered regression model. The model, which can be estimated with mclest written by Hendrickx (2000) (type net search mclest to download), is a compromise between allowing the coefficients for each independent variable to vary by outcome category and restricting them to be identical across all outcomes. The SORM is defined $\mathrm{as}^{2}$

$$
\begin{equation*}
\ln \frac{\operatorname{Pr}(y=q \mid \mathbf{x})}{\operatorname{Pr}(y=r \mid \mathbf{x})}=\left(\alpha_{q}-\alpha_{r}\right) \beta_{0}+\left(\phi_{q}-\phi_{r}\right)(\mathbf{x} \beta) \tag{5.4}
\end{equation*}
$$

where $\beta_{0}$ is the intercept and $\beta$ is a vector of coefficients associated with the independent variables; since $\beta_{0}$ is included in the equation, it is not included in $\beta$. The $\alpha$ 's and $\phi$ 's are scale factors associated with the outcome categories. The model allows the coefficients associated with each independent variable to differ by a scalar factor that depends on the pair of outcomes on the lefthand side of the equation. Similarly, the $\alpha$ 's allow different intercepts for each pair of outcomes. As the model stands, there are too many unconstrained $\alpha$ 's and $\phi$ 's for the parameters to be uniquely determined. The model can be identified in a variety of ways. For example, we can assume that $\phi_{1}=1, \phi_{J}=0, \alpha_{1}=1$, and $\alpha_{J}=0$. Or, using the approach from loglinear models for ordinal outcomes, the model is identified by the constraints $\sum_{j=1}^{J} \phi_{j}=0$ and $\sum_{j=1}^{J} \phi_{j}^{2}=1$. See DiPrete (1990) for further discussion. To insure ordinality of the outcomes, $\phi_{1}=1>\phi_{2}>\cdots>\phi_{J-1}>$ $\phi_{J}=0$ must hold. Note that mclest does not impose this inequality constraint during estimation.

Equation 5.4 can be used to compute the predicted probabilities:

$$
\operatorname{Pr}(y=m \mid \mathbf{x})=\frac{\exp \left(\alpha_{m} \beta_{0}+\phi_{m} \mathbf{x} \beta\right)}{\sum_{j=1}^{J} \exp \left(\alpha_{j} \beta_{0}+\phi_{j} \mathbf{x} \beta\right)}
$$

This formula can be used for interpreting the model using methods discussed above. The model can also be interpreted in terms of the effect of a change in $x_{k}$ on the odds of outcome $q$ versus $r$. After rewriting Equation 5.4 in terms of odds,

$$
\Omega_{q \mid r}\left(\mathbf{x}, x_{k}\right)=\frac{\operatorname{Pr}\left(y=q \mid \mathbf{x}, x_{k}\right)}{\operatorname{Pr}\left(y=r \mid \mathbf{x}, x_{k}\right)}=\exp \left[\left(\alpha_{q}-\alpha_{r}\right) \beta_{0}+\left(\phi_{q}-\phi_{r}\right)(\mathbf{x} \beta)\right]
$$

It is easy to show that

$$
\frac{\Omega_{q \mid r}\left(\mathbf{x}, x_{k}+1\right)}{\Omega_{q \mid r}\left(\mathbf{x}, x_{k}\right)}=e^{\left(\phi_{q}-\phi_{r}\right) \beta_{k}}=\left(\frac{e^{\phi_{q}}}{e^{\phi_{r}}}\right)^{\beta_{k}}
$$

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Thus, the effect of $x_{k}$ on the odds of $q$ versus $r$ differs across outcome comparisons according to the scaling coefficients $\phi$.

### 5.9.3 The continuation ratio model

The continuation ratio model was proposed by Fienberg $(1980,110)$ and was designed for ordinal outcomes in which the categories represent the progression of events or stages in some process though which an individual can advance. For example, the outcome could be faculty rank, where the stages are assistant professor, associate professor, and full professor. A key characteristic of the process is that an individual must pass through each stage. For example, to become an associate professor you must be an assistant professor; to be a full professor, an associate professor. While there are versions of this model based on other binary models (e.g., probit), here we consider the logit version.

If $\operatorname{Pr}(y=m \mid \mathbf{x})$ is the probability of being in stage $m$ given $\mathbf{x}$ and $\operatorname{Pr}(y>m \mid \mathbf{x})$ is the probability of being in a stage later than $m$, the continuation ratio model for the $\log$ odds is

$$
\ln \left[\frac{\operatorname{Pr}(y=m \mid \mathbf{x})}{\operatorname{Pr}(y>m \mid \mathbf{x})}\right]=\tau_{m}-\mathbf{x} \beta \quad \text { for } m=1 \text { to } J-1
$$

where the $\beta$ 's are constrained to be equal across outcome categories, while the constant term $\tau_{m}$ differs by stage. As with other logit models, we can also express the model in terms of the odds:

$$
\frac{\operatorname{Pr}(y=m \mid \mathbf{x})}{\operatorname{Pr}(y>m \mid \mathbf{x})}=\exp \left(\tau_{m}-\mathbf{x} \beta\right)
$$

Accordingly, $\exp \left(-\beta_{k}\right)$ can be interpreted as the effect of a unit increase in $x_{k}$ on the odds of being in $m$ compared to being in a higher category given that an individual is in category $m$ or higher, holding all other variables constant. From this equation, the predicted probabilities can be computed as

$$
\begin{aligned}
& \operatorname{Pr}(y=m \mid \mathbf{x})=\frac{\exp \left(\tau_{m}-\mathbf{x} \beta\right)}{\prod_{j=1}^{m}\left[1+\exp \left(\tau_{j}-\mathbf{x} \beta\right)\right]} \text { for } m=1 \text { to } J-1 \\
& \operatorname{Pr}(y=J \mid \mathbf{x})=1-\sum_{j=1}^{J-1} \operatorname{Pr}(y=j \mid \mathbf{x})
\end{aligned}
$$

These predicted probabilities can be used for interpreting the model. In Stata, this model can be estimated using ocratio by Wolfe (1998); type net search ocratio and follow the prompts to download.

[^36] either electronically or in printed form, to others.

# REGRESSION MODELS FOR CATEGORICAL DEPENDENT VARIABLES USING STATA 

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## 6 Models for Nominal Outcomes

An outcome is nominal when the categories are assumed to be unordered. For example, marital status can be grouped nominally into the categories of divorced, never married, married, or widowed. Occupations might be organized as professional, white collar, blue collar, craft, and menial, which is the example we use in this chapter. Other examples include reasons for leaving the parents' home, the organizational context of scientific work (e.g., industry, government, and academia), and the choice of language in a multilingual society. Further, in some cases a researcher might prefer to treat an outcome as nominal even though it is ordered or partially ordered. For example, if the response categories are strongly agree, agree, disagree, strongly disagree, and don't know, the category "don't know" invalidates models for ordinal outcomes. Or, you might decide to use a nominal regression model when the assumption of parallel regressions is rejected. In general, if there are concerns about the ordinality of the dependent variable, the potential loss of efficiency in using models for nominal outcomes is outweighed by avoiding potential bias.

This chapter focuses on two closely related models for nominal outcomes. The multinomial logit model (MNLM) is the most frequently used nominal regression model. In this model, the effects of the independent variables are allowed to differ for each outcome, and are similar to the generalized ordered logit model discussed in the last chapter. In the conditional logit model (CLM), characteristics of the outcomes are used to predict which choice is made. While probit versions of these models are theoretically possible, issues of computation and identification limit their use (Keane 1992) .

The biggest challenge in using the MNLM is that the model includes a lot of parameters, and it is easy to be overwhelmed by the complexity of the results. This complexity is compounded by the nonlinearity of the model, which leads to the same difficulties of interpretation found for models in prior chapters. While estimation of the model is straight-forward, interpretation involves many challenges that are the focus of this chapter. We begin by reviewing the statistical model, followed by a discussion of testing, fit, and finally methods of interpretation. These discussions are intended as a review for those who are familiar with the models. For a complete discussion, see Long (1997). As always, you can obtain sample do-files and data files by downloading the spostst4 package (see Chapter 1 for details).

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### 6.1 The multinomial logit model

The MNLM can be thought of as simultaneously estimating binary logits for all comparisons among the dependent categories. For example, let occ3 be a nominal outcome with the categories $M$ for manual jobs, $W$ for white collar jobs, and $P$ for professional jobs. Assume that there is a single independent variable ed measuring years of education. We can examine the effect of ed on occ3 by estimating three binary logits,

$$
\begin{aligned}
& \ln \left[\frac{\operatorname{Pr}(P \mid \mathbf{x})}{\operatorname{Pr}(M \mid \mathbf{x})}\right]=\beta_{0, P \mid M}+\beta_{1, P \mid M} \mathrm{ed} \\
& \ln \left[\frac{\operatorname{Pr}(W \mid \mathbf{x})}{\operatorname{Pr}(M \mid \mathbf{x})}\right]=\beta_{0, W \mid M}+\beta_{1, W \mid M} \mathrm{ed} \\
& \ln \left[\frac{\operatorname{Pr}(P \mid \mathbf{x})}{\operatorname{Pr}(W \mid \mathbf{x})}\right]=\beta_{0, P \mid W}+\beta_{1, P \mid W} \text { ed }
\end{aligned}
$$

where the subscripts to the $\beta$ 's indicate which comparison is being made (e.g., $\beta_{1, P \mid M}$ is the coefficient for the first independent variable for the comparison of $P$ and $M$ ).

The three binary logits include redundant information. Since $\ln \frac{a}{b}=\ln a-\ln b$, the following equality must hold:

$$
\ln \left[\frac{\operatorname{Pr}(P \mid \mathbf{x})}{\operatorname{Pr}(M \mid \mathbf{x})}\right]-\ln \left[\frac{\operatorname{Pr}(W \mid \mathbf{x})}{\operatorname{Pr}(M \mid \mathbf{x})}\right]=\ln \left[\frac{\operatorname{Pr}(P \mid \mathbf{x})}{\operatorname{Pr}(W \mid \mathbf{x})}\right]
$$

This implies that

$$
\begin{align*}
& \beta_{0, P \mid M}-\beta_{0, W \mid M}=\beta_{0, P \mid W}  \tag{6.1}\\
& \beta_{1, P \mid M}-\beta_{1, W \mid M}=\beta_{1, P \mid W}
\end{align*}
$$

In general, with $J$ outcomes, only $J-1$ binary logits need to be estimated. Estimates for the remaining coefficients can be computed using equalities of the sort shown in Equation 6.1.

The problem with estimating the MNLM by estimating a series of binary logits is that each binary logit is based on a different sample. For example, in the logit comparing $P$ to $M$, those in $W$ are dropped. To see this, we can look at the output from a series of binary logits. First, we estimate a binary logit comparing manual and professional workers:

| .use nomintro2, clear |
| :--- |
| (1982 General Social Survey) |
| . tab prof_man, miss |
| prof_man |
|  |
| Manual |
| Prof |

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6.1 The multinomial logit model

| Logit estimates |  |  |  | Number of obs LR chi2(1) |  |  | $\begin{array}{r} 296 \\ 139.78 \end{array}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  |  |  |
|  |  |  |  | Prob > chi2 |  | = | 0.0000 |
| Log likelihood $=-126.43879$ |  |  |  | Pseudo R2 |  |  | 0.3560 |
| prof_man | Coef. | Std. Err. | z | $\mathrm{P}>\|\mathrm{z}\|$ | [95\% | onf | Interval] |
| ed | . 7184599 | . 0858735 | 8.37 | 0.000 | . 550 |  | . 8867688 |
| _cons | -10.19854 | 1.177457 | -8.66 | 0.000 | -12.50 |  | -7.89077 |

Notice that 41 cases are missing for prof_man and have been deleted. These correspond to respondents who have white collar occupations. In the same way, the next two binary logits also exclude cases corresponding to the excluded category:
. tab wc_man, miss

| wc_man | Freq. | Percent | Cum. |
| ---: | ---: | ---: | ---: |
| Manual | 184 | 54.60 | 54.60 |
| WhiteCol | 41 | 12.17 | 66.77 |
| $\cdot$ | 112 | 33.23 | 100.00 |
| Total | 337 | 100.00 |  |

. logit wc_man ed, nolog

. tab prof_wc, miss

| prof_wc | Freq. | Percent | Cum. |
| ---: | ---: | ---: | ---: |
| WhiteCol | 41 | 12.17 | 12.17 |
| Prof | 112 | 33.23 | 45.40 |
| $\cdot$ | 184 | 54.60 | 100.00 |
| Total | 337 | 100.00 |  |

. logit prof_wc ed, nolog


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The results from the binary logits can be compared to the output from mlogit, the command that estimates the MNLM:

| . tab occ3, miss |  |  |  |
| ---: | ---: | ---: | ---: |
| occ3 | Freq. | Percent | Cum. |
| Manual | 184 | 54.60 | 54.60 |
| WhiteCol | 41 | 12.17 | 66.77 |
| Prof | 112 | 33.23 | 100.00 |
| Total | 337 | 100.00 |  |

. mlogit occ3 ed, nolog

| Multinomial regression | Number of obs | $=$ | 337 |
| :--- | :--- | :--- | :--- |
|  | LR chi2 (2) | $=$ | 145.89 |
|  | Prob > chi2 | $=$ | 0.0000 |
| Log likelihood $=-248.14786$ |  | Pseudo R2 | $=$ |


| occ3 |  | Coef. | Std. Err. | z | P>\|z| | [95\% Conf. Interval] |  |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| WhiteCol |  |  |  |  |  |  |  |
|  | ed | .3000735 | .0841358 | 3.57 | 0.000 | .1351703 | .4649767 |
|  | _cons | -5.232602 | 1.096086 | -4.77 | 0.000 | -7.380892 | -3.084312 |
| Prof |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |
|  | ed | .7195673 | .0805117 | 8.94 | 0.000 | .5617671 | .8773674 |
|  | _cons | -10.21121 | 1.106913 | -9.22 | 0.000 | -12.38072 | -8.041698 |

(Outcome occ3==Manual is the comparison group)
The output from mlogit is divided into two panels. The top panel is labeled WhiteCol, which is the value label for the second category of the dependent variable; the second panel is labeled Prof, which corresponds to the third outcome category. The key to understanding the two panels is the last line of output: Outcome occ3==Manual is the comparison group. This means that the panel WhiteCol presents coefficients from the comparison of $W$ to $M$; the second panel labeled Prof holds the comparison of $P$ to $M$. Accordingly, the top panel should be compared to the coefficients from the binary logit for $W$ and $M$ (outcome variable wc_man) listed above. For example, the coefficient for the comparison of $W$ to $M$ from mlogit is $\widehat{\beta}_{1, W \mid M}=.3000735$ with $z=3.567$, while the logit estimate is $\widehat{\beta}_{1, W \mid M}=.3418255$ with $z=3.658$. Overall, the estimates from the binary model are close to those from the MNLM, but not exactly the same.

Next, notice that while theoretically $\beta_{1, P \mid M}-\beta_{1, W \mid M}=\beta_{1, P \mid W}$, the estimates from the binary logits are $\widehat{\beta}_{1, P \mid M}-\widehat{\beta}_{1, W \mid M}=.7184599-.3418255=.3766344$, which does not equal the binary logit estimate $\widehat{\beta}_{1, P \mid W}=.3735466$. The general point is that a series of binary logits using logit does not impose the constraints among coefficients that are implicit in the definition of the model. When estimating the model with mlogit, the constraints are imposed. Indeed, the output from mlogit only presents two of the three comparisons from our example, namely, $W$ versus $M$ and $P$ versus $M$. The remaining comparison, $W$ versus $P$, is the difference between the two sets of estimated coefficients. Details on using listcoef to automatically compute the remaining comparisons are given below.

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### 6.1.1 Formal statement of the model

Formally, the MNLM can be written as

$$
\ln \Omega_{m \mid b}(\mathbf{x})=\ln \frac{\operatorname{Pr}(y=m \mid \mathbf{x})}{\operatorname{Pr}(y=b \mid \mathbf{x})}=\mathbf{x} \beta_{m \mid b} \quad \text { for } m=1 \text { to } J
$$

where $b$ is the base category, which is also referred to as the comparison group. Since $\ln \Omega_{b \mid b}(\mathbf{x})=$ $\ln 1=0$, it must hold that $\beta_{b \mid b}=\mathbf{0}$. That is, the log odds of an outcome compared to itself is always 0 , and thus the effects of any independent variables must also be 0 . These $J$ equations can be solved to compute the predicted probabilities:

$$
\operatorname{Pr}(y=m \mid \mathbf{x})=\frac{\exp \left(\mathbf{x} \beta_{m \mid b}\right)}{\sum_{j=1}^{J} \exp \left(\mathbf{x} \beta_{j \mid b}\right)}
$$

While the predicted probability will be the same regardless of the base category $b$, changing the base category can be confusing since the resulting output from mlogit appears to be quite different. For example, suppose you have three outcomes and estimate the model with outcome 1 as the base category. Your probability equations would be

$$
\operatorname{Pr}(y=m \mid \mathbf{x})=\frac{\exp \left(\mathbf{x} \beta_{m \mid 1}\right)}{\sum_{j=1}^{J} \exp \left(\mathbf{x} \beta_{j \mid 1}\right)}
$$

and you would obtain estimates $\widehat{\beta}_{2 \mid 1}$ and $\widehat{\beta}_{3 \mid 1}$, where $\beta_{1 \mid 1}=\mathbf{0}$. If someone else set up the model with base category 2 , their equations would be

$$
\operatorname{Pr}(y=m \mid \mathbf{x})=\frac{\exp \left(\mathbf{x} \beta_{m \mid 2}\right)}{\sum_{j=1}^{J} \exp \left(\mathbf{x} \beta_{j \mid 2}\right)}
$$

and they would obtain $\widehat{\beta}_{1 \mid 2}$ and $\widehat{\beta}_{3 \mid 2}$, where $\beta_{2 \mid 2}=\mathbf{0}$. While the estimated parameters are different, they are only different parameterizations that provide the same predicted probabilities. The confusion arises only if you are not clear about which parameterization you are using. Unfortunately, some software packages, but not Stata, make it very difficult to tell which set of parameters is being estimated. We return to this issue when we discuss how Stata's mlogit parameterizes the model in the next section.

### 6.2 Estimation using mlogit

The multinomial logit model is estimated with the following command:

```
mlogit depvar [indepvars] [weight] [if exp] [in range] [, level(#) nolog
    cluster(varname) robust blasecategory(#) constraints(clist) \underline{rrr}
    noconstant ]
```

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In our experience, the model converges very quickly, even when there are many outcome categories and independent variables.

## Variable lists

depvar is the dependent variable. The actual values taken on by the dependent variable are irrelevant. For example, if you had three outcomes, you could use the values 1,2 , and 3 or $-1,0$, and 999. Up to 50 outcomes are allowed in Intercooled Stata, and 20 outcomes are allowed in Small Stata.
indepvars is a list of independent variables. If indepvars is not included, Stata estimates a model with only constants.

## Specifying the estimation sample

if and in qualifiers can be used to restrict the estimation sample. For example, if you want to estimate the model with only white respondents, use the command mlogit occ ed exper if white==1.

Listwise deletion Stata excludes cases in which there are missing values for any of the variables. Accordingly, if two models are estimated using the same dataset but have different sets of independent variables, it is possible to have different samples. We recommend that you use mark and markout (discussed in Chapter 3) to explicitly remove cases with missing data.

## Weights

mlogit can be used with fweights, pweights, and iweights. In Chapter 3, we provide a brief discussion of the different types of weights and how weights are specified in Stata's syntax.

## Options

basecategory (\#) specifies the value of depvar that is the base category (i.e., reference group) for the coefficients that are listed. This determines how the model is parameterized. If the basecategory option is not specified, the most frequent category in the estimation sample is chosen as the base. The base category is always reported immediately below the estimates; for example, Outcome occ3==Manual is the comparison group.
constraints (clist) specifies the linear constraints to be applied during estimation. The default is to perform unconstrained estimation. Constraints are defined with the constraint command. This option is illustrated in Section 6.3.3 when we discuss an LR test for combining outcome categories.
rrr reports the estimated coefficients transformed to relative risk ratios, defined as $\exp (b)$ rather than $b$. We do not consider this option further, because the same information is available through listcoef.

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noconstant excludes the constant terms from the model.
nolog suppresses the iteration history.
level(\#) specifies the level of the confidence interval for estimated parameters. By default, Stata uses $95 \%$ intervals. You can also change the default level, say, to a $90 \%$ interval, with the command set level 90.
cluster (varname) specifies that the observations are independent across the groups specified by unique values of varname but not necessarily independent within the groups. See Chapter 3 for further details.
robust indicates that robust variance estimates are to be used. When cluster () is specified, robust standard errors are automatically used. See Chapter 3 for further details.

### 6.2.1 Example of occupational attainment

The 1982 General Social Survey asked respondents their occupation, which we recoded into five broad categories: menial jobs $(M)$, blue collar jobs $(B)$, craft jobs $(C)$, white collar jobs $(W)$, and professional jobs $(P)$. Three independent variables are considered: white indicating the race of the respondent, ed measuring years of education, and exper measuring years of work experience.
. sum white ed exper

| Variable | Obs | Mean | Std. Dev. | Min | Max |
| ---: | ---: | ---: | ---: | ---: | ---: |
| white | 337 | .9169139 | .2764227 | 0 | 1 |
| ed | 337 | 13.09496 | 2.946427 | 3 | 20 |
| exper | 337 | 20.50148 | 13.95936 | 2 | 66 |

The distribution among outcome categories is

| tab occ, missing |  |  |  |
| ---: | ---: | ---: | ---: |
| Occupation | Freq. | Percent | Cum. |
| Menial | 31 | 9.20 | 9.20 |
| BlueCol | 69 | 20.47 | 29.67 |
| Craft | 84 | 24.93 | 54.60 |
| WhiteCol | 41 | 12.17 | 66.77 |
| Prof | 112 | 33.23 | 100.00 |
| Total | 337 | 100.00 |  |

Using these variables the following MNLM was estimated:

$$
\begin{aligned}
\ln \Omega_{M \mid P}\left(\mathbf{x}_{i}\right) & =\beta_{0, M \mid P}+\beta_{1, M \mid P} \text { white }+\beta_{2, M \mid P} \text { ed }+\beta_{3, M \mid P} \text { exper } \\
\ln \Omega_{B \mid P}\left(\mathbf{x}_{i}\right) & =\beta_{0, B \mid P}+\beta_{1, B \mid P} \text { white }+\beta_{2, B \mid P} \text { ed }+\beta_{3, B \mid P} \text { exper } \\
\ln \Omega_{C \mid P}\left(\mathbf{x}_{i}\right) & =\beta_{0, C \mid P}+\beta_{1, C \mid P} \text { white }+\beta_{2, C \mid P} \text { ed }+\beta_{3, C \mid P} \text { exper } \\
\ln \Omega_{W \mid P}\left(\mathbf{x}_{i}\right) & =\beta_{0, W \mid P}+\beta_{1, W \mid P \text { white }+\beta_{2, W \mid P} \text { ed }+\beta_{3, W \mid P} \text { exper }}
\end{aligned}
$$

where we specify the fifth category $P$ as the base category:

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| Multinomial regression |  |  |  | Number of obs LR chi2(12) <br> Prob > chi2 <br> Pseudo R2 |  | $\begin{array}{r} 337 \\ 166.09 \\ 0.0000 \\ 0.1629 \end{array}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| occ | Coef. | Std. Err. | z | $P>\|z\|$ | [95\% Con | Interval] |
| Menial |  |  |  |  |  |  |
| white | -1.774306 | . 7550543 | -2.35 | 0.019 | -3. 254186 | -. 2944273 |
| ed | -. 7788519 | . 1146293 | -6.79 | 0.000 | -1.003521 | -. 5541826 |
| exper | -. 0356509 | . 018037 | -1.98 | 0.048 | -. 0710028 | -. 000299 |
| _cons | 11.51833 | 1.849356 | 6.23 | 0.000 | 7.893659 | 15.143 |
| BlueCol |  |  |  |  |  |  |
| white | -. 5378027 | . 7996033 | -0.67 | 0.501 | -2.104996 | 1.029391 |
| ed | -. 8782767 | . 1005446 | -8.74 | 0.000 | -1.07534 | -. 6812128 |
| exper | -. 0309296 | . 0144086 | -2.15 | 0.032 | -. 05917 | -. 0026893 |
| _cons | 12.25956 | 1.668144 | 7.35 | 0.000 | 8.990061 | 15.52907 |
| Craft |  |  |  |  |  |  |
| white | -1.301963 | . 647416 | -2.01 | 0.044 | -2.570875 | -. 0330509 |
| ed | -. 6850365 | . 0892996 | -7.67 | 0.000 | -. 8600605 | -. 5100126 |
| exper | -. 0079671 | . 0127055 | -0.63 | 0.531 | -. 0328693 | . 0169351 |
| _cons | 10.42698 | 1.517943 | 6.87 | 0.000 | 7.451864 | 13.40209 |
| WhiteCol |  |  |  |  |  |  |
| white | -. 2029212 | . 8693072 | -0.23 | 0.815 | -1.906732 | 1.50089 |
| ed | -. 4256943 | . 0922192 | -4.62 | 0.000 | -. 6064407 | -. 2449479 |
| exper | -. 001055 | . 0143582 | -0.07 | 0.941 | -. 0291967 | . 0270866 |
| _cons | 5.279722 | 1.684006 | 3.14 | 0.002 | 1.979132 | 8.580313 |

(Outcome occ==Prof is the comparison group)
Methods of testing coefficients and interpretation of the estimates are considered after we discuss the effects of using different base categories.

### 6.2.2 Using different base categories

By default, mlogit sets the base category to the outcome with the most observations. Alternatively, as illustrated in the last example, you can select the base category with basecategory ().mlogit then reports coefficients for the effect of each independent variable on each category relative to the base category. However, you should also examine the effects on other pairs of outcome categories. For example, you might be interested in how race affects the allocation of workers between Craft and BlueCol (e.g., $\beta_{1, B \mid C}$ ), which was not estimated in the output listed above. While this coefficient can be estimated by re-running mlogit with a different base category (e.g., mlogit occ white ed exper, basecategory (3)), it is easier to use listcoef, which presents estimates for all combinations of outcome categories. Since listcoef can generate a lot of output, we illustrate two options that limit which coefficients are listed. First, you can include a list of variables, and only coefficients for those variables will be listed. For example,

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| mlogit ( $\mathrm{N}=337$ ) : Factor Change in the Odds of occ |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Variable: white (sd | .276423) |  |  |  |  |
| Odds comparing Group 1 vs Group 2 | b | z | $P>\|z\|$ | e^b | e^bStdX |
| Menial -BlueCol | -1.23650 | -1.707 | 0.088 | 0.2904 | 0.7105 |
| Menial -Craft | -0.47234 | -0.782 | 0.434 | 0.6235 | 0.8776 |
| Menial -WhiteCol | -1.57139 | -1.741 | 0.082 | 0.2078 | 0.6477 |
| Menial -Prof | -1.77431 | -2.350 | 0.019 | 0.1696 | 0.6123 |
| BlueCol -Menial | 1.23650 | 1.707 | 0.088 | 3.4436 | 1.4075 |
| BlueCol -Craft | 0.76416 | 1.208 | 0.227 | 2.1472 | 1.2352 |
| BlueCol -WhiteCol | -0.33488 | -0.359 | 0.720 | 0.7154 | 0.9116 |
| BlueCol -Prof | -0.53780 | -0.673 | 0.501 | 0.5840 | 0.8619 |
| Craft -Menial | 0.47234 | 0.782 | 0.434 | 1.6037 | 1.1395 |
| Craft -BlueCol | -0.76416 | -1.208 | 0.227 | 0.4657 | 0.8096 |
| Craft -WhiteCol | -1.09904 | -1.343 | 0.179 | 0.3332 | 0.7380 |
| Craft -Prof | -1.30196 | -2.011 | 0.044 | 0.2720 | 0.6978 |
| WhiteCol-Menial | 1.57139 | 1.741 | 0.082 | 4.8133 | 1.5440 |
| WhiteCol-BlueCol | 0.33488 | 0.359 | 0.720 | 1.3978 | 1.0970 |
| WhiteCol-Craft | 1.09904 | 1.343 | 0.179 | 3.0013 | 1.3550 |
| WhiteCol-Prof | -0.20292 | -0.233 | 0.815 | 0.8163 | 0.9455 |
| Prof -Menial | 1.77431 | 2.350 | 0.019 | 5.8962 | 1.6331 |
| Prof -BlueCol | 0.53780 | 0.673 | 0.501 | 1.7122 | 1.1603 |
| Prof -Craft | 1.30196 | 2.011 | 0.044 | 3.6765 | 1.4332 |
| Prof -WhiteCol | 0.20292 | 0.233 | 0.815 | 1.2250 | 1.0577 |
| $\mathrm{b}=$ raw coefficient |  |  |  |  |  |
| $z=z$-score for test of $b=0$ |  |  |  |  |  |
| $\mathrm{P}>\|z\|=\mathrm{p}$-value for z-test |  |  |  |  |  |
| $e^{\wedge} \mathrm{bStdX}=\exp (\mathrm{b} *$ SD of X$)=$ change in odds for $S D$ increase in X |  |  |  |  |  |

Or, you can limit the output to those coefficients that are significant at a given level using the pvalue(\#) option, which specifies that only coefficients significant at the \# significance level or smaller will be printed. For example,
. listcoef, pvalue(.05)
mlogit ( $N=337$ ): Factor Change in the Odds of occ when $P>|z|<0.05$
Variable: white ( $\mathrm{sd}=.276423$ )

Variable: white (sd= .276423)

| Odds comparing |  |  |  |  |  |  |
| :--- | :--- | :---: | :---: | :---: | :---: | ---: |
| Group 1 vs Group 2 | b | z | P>\|z| | $e^{\wedge} b$ | $e^{\wedge} b S t d X$ |  |
| Menial | -Prof | -1.77431 | -2.350 | 0.019 | 0.1696 | 0.6123 |
| Craft | -Prof | -1.30196 | -2.011 | 0.044 | 0.2720 | 0.6978 |
| Prof | -Menial | 1.77431 | 2.350 | 0.019 | 5.8962 | 1.6331 |
| Prof | -Craft | 1.30196 | 2.011 | 0.044 | 3.6765 | 1.4332 |


| Odds comparing Group 1 vs Group 2 | b | z | $\mathrm{P}>\|\mathrm{z}\|$ | e^b | $e^{\wedge} \mathrm{bStdX}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Menial -WhiteCol | -0.35316 | -3.011 | 0.003 | 0.7025 | 0.3533 |
| Menial -Prof | -0.77885 | -6.795 | 0.000 | 0.4589 | 0.1008 |
| BlueCol -Craft | -0.19324 | -2.494 | 0.013 | 0.8243 | 0.5659 |
| BlueCol -WhiteCol | -0.45258 | -4.425 | 0.000 | 0.6360 | 0.2636 |
| BlueCol -Prof | -0.87828 | -8.735 | 0.000 | 0.4155 | 0.0752 |
| Craft -BlueCol | 0.19324 | 2.494 | 0.013 | 1.2132 | 1.7671 |

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|  | -0.25934 | -2.773 | 0.006 | 0.7716 | 0.4657 |  |
| :--- | :--- | ---: | ---: | ---: | ---: | ---: |
| Craft | -WhiteCol | -0.68504 | -7.671 | 0.000 | 0.5041 | 0.1329 |
| Craft | Prof | -0.351 |  |  |  |  |
| WhiteCol-Menial | 0.35316 | 3.011 | 0.003 | 1.4236 | 2.8308 |  |
| WhiteCol-BlueCol | 0.45258 | 4.425 | 0.000 | 1.5724 | 3.7943 |  |
| WhiteCol-Craft | 0.25934 | 2.773 | 0.006 | 1.2961 | 2.1471 |  |
| WhiteCol-Prof | -0.42569 | -4.616 | 0.000 | 0.6533 | 0.2853 |  |
| Prof | -Menial | 0.77885 | 6.795 | 0.000 | 2.1790 | 9.9228 |
| Prof | -BlueCol | 0.87828 | 8.735 | 0.000 | 2.4067 | 13.3002 |
| Prof | -Craft | 0.68504 | 7.671 | 0.000 | 1.9838 | 7.5264 |
| Prof | -WhiteCol | 0.42569 | 4.616 | 0.000 | 1.5307 | 3.5053 |



### 6.2.3 Predicting perfectly

mlogit handles perfect prediction somewhat differently than the estimations commands for binary and ordinal models that we have discussed. logit and probit automatically remove the observations that imply perfect prediction and compute estimates accordingly. ologit and oprobit keep these observations in the model, estimate the $z$ for the problem variable as 0 , and provide an incorrect LR chi-squared, but also warn that a given number of observations are completely determined. You should delete these observations and re-estimate the model. mlogit is just like ologit and oprobit except that you do not receive a warning message. You will see, however, that all coefficients associated with the variable causing the problem have $z=0$ (and $p>|z|=1$ ). You should re-estimate the model, excluding the problem variable and deleting the observations that imply the perfect predictions. Using the tabulate command to generate a cross-tabulation of the problem variable and the dependent variable should reveal the combination that results in perfect prediction.

### 6.3 Hypothesis testing of coefficients

In the MNLM you can test individual coefficients with the reported $z$-statistics, with a Wald test using test, or with an LR test using lrtest. Since the methods of testing a single coefficient that were discussed in Chapters 4 and 5 still apply fully, they are not considered further here. However, in the MNLM there are new reasons for testing groups of coefficients. First, testing that a variable has no effect requires a test that $J-1$ coefficients are simultaneously equal to zero. Second, testing whether the independent variables as a group differentiate between two outcomes requires a test of $K$ coefficients. This section focuses on these two kinds of tests.

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Caution Regarding Specification Searches Given the difficulties of interpretation that are associated with the MNLM, it is tempting to search for a more parsimonious model by excluding variables or combining outcome categories based on a sequence of tests. Such a search requires great care. First, these tests involve multiple coefficients. While the overall test might indicate that as a group the coefficients are not significantly different from zero, an individual coefficient can still be substantively and statistically significant. Accordingly, you should examine the individual coefficients involved in each test before deciding to revise your model. Second, as with all searches that use repeated, sequential tests, there is a danger of over-fitting the data. When models are constructed based on prior testing using the same data, significance levels should only be used as rough guidelines.

### 6.3.1 mlogtest for tests of the MNLM

While the tests in this section can be computed using test or lrtest, in practice this is tedious. The mlogtest command (Freese and Long 2000) makes the computation of these tests very simple. The syntax is

```
mlogtest [, lr wald combine lrcomb set(varlist[\ varlist[\...]]) all iia
    hausman smhsiao detail blase ]
```


## Options

Ir requests LR tests for each independent variable.
wald requests Wald tests for each independent variable.
combine requests Wald tests of whether dependent categories can be combined.
lrcomb requests LR tests of whether dependent categories can be combined. These tests use constrained estimation and overwrite constraint \#999 if it is already defined.
set (varlist $[\backslash$ varlist $[\backslash \ldots]]$ ) specifies that a set of variables is to be considered together for the LR test or Wald test. $\backslash$ is used to specify multiple sets of variables. For example, mlogtest, lr set (age age $2 \backslash$ iscat1 iscat2) computes one LR test for the hypothesis that the effects of age and age 2 are jointly 0 and a second LR test that the effects of iscat1 and iscat2 are jointly 0 .

Other options for mlogtest are discussed later in the chapter.

### 6.3.2 Testing the effects of the independent variables

With $J$ dependent categories, there are $J-1$ nonredundant coefficients associated with each independent variable $x_{k}$. For example, in our logit on occupation there are four coefficients associated

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with ed: $\beta_{2, M \mid P}, \beta_{2, B \mid P}, \beta_{2, C \mid P}$, and $\beta_{2, W \mid P}$. The hypothesis that $x_{k}$ does not affect the dependent variable can be written as

$$
H_{0}: \beta_{k, 1 \mid b}=\cdots=\beta_{k, J \mid b}=0
$$

where $b$ is the base category. Since $\beta_{k, b \mid b}$ is necessarily 0 , the hypothesis imposes constraints on $J-1$ parameters. This hypothesis can be tested with either a Wald or an LR test.

## A likelihood-ratio test

The LR test involves: 1) estimating the full model including all of the variables, resulting in the likelihood-ratio statistic $L R_{F}^{2} ; 2$ ) estimating the restricted model that excludes variable $x_{k}$, resulting in $L R_{R}^{2}$; and 3) computing the difference $L R_{R v s F}^{2}=L R_{F}^{2}-L R_{R}^{2}$, which is distributed as chi-squared with $J-1$ degrees of freedom if the null hypothesis is true. This can be done using lrtest:

```
. mlogit occ white ed exper, basecategory(5) nolog
(output omitted)
. lrtest, saving(0)
. mlogit occ ed exper, basecategory(5) nolog
    (output omitted)
. lrtest
Mlogit: likelihood-ratio test }\quad\begin{array}{l}{\mathrm{ chi2(4) }}
. mlogit occ white exper, basecategory(5) nolog
    (and so on)
```

While using lrtest is straightforward, the command mlogtest, 1 r is even simpler since it automatically computes the tests for all variables by making repeated calls to lrtest:

```
. mlogit occ white ed exper, basecategory(5) nolog
    (output omitted)
. mlogtest, lr
**** Likelihood-ratio tests for independent variables
    Ho: All coefficients associated with given variable(s) are 0.
\begin{tabular}{r|rrr} 
occ & chi2 & df & P>chi2 \\
\hline white & 8.095 & 4 & 0.088 \\
ed & 156.937 & 4 & 0.000 \\
exper & 8.561 & 4 & 0.073 \\
\hline
\end{tabular}
```

The results of the LR test, regardless of how they are computed, can be interpreted as follows:
The effect of race on occupation is significant at the .10 level but not at the .05 level ( $X^{2}=8.10, d f=4, p=.09$ ). The effect of education is significant at the .01 level ( $X^{2}=156.94, d f=4, p<.01$ ).
or, more formally,
The hypothesis that all of the coefficients associated with education are simultaneously equal to 0 can be rejected at the .01 level $\left(X^{2}=156.94, d f=4, p<.01\right)$.

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## A Wald test

Although the LR test is generally considered superior, if the model is complex or the sample is very large, the computational costs of the LR test can be prohibitive. Alternatively, $K$ Wald tests can be computed using test without estimating additional models. For example,

```
. mlogit occ white ed exper, basecategory(5) nolog
    (output omitted)
. test white
    ( 1) [Menial]white = 0.0
    ( 2) [BlueCol] white = 0.0
    ( 3) [Craft]white = 0.0
    (4) [WhiteCol]white = 0.0
            chi2( 4) = 8.15
    Prob > chi2 = 0.0863
. test ed
    ( 1) [Menial]ed = 0.0
    ( 2) [BlueCol] ed = 0.0
    ( 3) [Craft]ed = 0.0
    ( 4) [WhiteCol]ed = 0.0
            chi2( 4) = 84.97
            Prob > chi2 = 0.0000
. test exper
    ( 1) [Menial] exper = 0.0
    ( 2) [BlueCol] exper = 0.0
    (3) [Craft] exper = 0.0
    (4) [WhiteCol] exper = 0.0
    chi2( 4) = 7.99
    Prob > chi2 = 0.0918
```

The output from test makes explicit which coefficients are being tested. Here we see the way in which Stata labels parameters in models with multiple equations. For example, [Menial]white is the coefficient for the effect of white in the equation comparing the outcome Menial to the base category Prof; [BlueCol] white is the coefficient for the effect of white in the equation comparing the outcome BlueCol to the base category Prof.

As with the LR test, mlogtest, wald automates this process:
. mlogtest, wald
**** Wald tests for independent variables
Ho: All coefficients associated with given variable(s) are 0.

| occ | chi2 | df | P>chi2 |
| ---: | ---: | ---: | ---: |
| white | 8.149 | 4 | 0.086 |
| ed | 84.968 | 4 | 0.000 |
| exper | 7.995 | 4 | 0.092 |

These tests can be interpreted in the same way as illustrated for the LR test above.
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## Testing multiple independent variables

The logic of the Wald or LR tests can be extended to test that the effects of two or more independent variables are simultaneously zero. For example, the hypothesis to test that $x_{k}$ and $x_{\ell}$ have no effect is

$$
H_{0}: \beta_{k, 1 \mid b}=\cdots=\beta_{k, J \mid b}=\beta_{\ell, 1 \mid b}=\cdots=\beta_{\ell, J \mid b}=0
$$

The set (varlist $[\backslash \operatorname{varlist}[\backslash \ldots]]$ ) option in mlogtest specifies which variables are to be simultaneously tested. For example, to test the hypothesis that the effects of ed and exper are simultaneously equal to 0 , we could use lrtest as follows:


### 6.3.3 Tests for combining dependent categories

If none of the independent variables significantly affect the odds of outcome $m$ versus outcome $n$, we say that $m$ and $n$ are indistinguishable with respect to the variables in the model (Anderson 1984). Outcomes $m$ and $n$ being indistinguishable corresponds to the hypothesis

$$
H_{0}: \beta_{1, m \mid n}=\cdots \beta_{K, m \mid n}=0
$$

which can be tested with either a Wald or an LR test. In our experience, the two tests provide very similar results. If two outcomes are indistinguishable with respect to the variables in the model, then you can obtain more efficient estimates by combining them. To test whether categories are indistinguishable, you can use mlogtest.

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6.3 Hypothesis testing of coefficients

## A Wald test for combining outcomes

The command mlogtest, combine computes Wald tests of the null hypothesis that two categories can be combined for all combinations of outcome categories. For example,

| ```. mlogit occ white ed exper, basecategory(5) (output omitted) . mlogtest, combine``` |  |  |  |
| :---: | :---: | :---: | :---: |
| **** Wald tests for combining outcome categories |  |  |  |
| Ho: All coefficients except intercepts associated with given pair of outcomes are 0 (i.e., categories can be collapsed). |  |  |  |
| Categories tested | chi2 | df | $\mathrm{P}>\mathrm{chi2}$ |
| Menial- BlueCol | 3.994 | 3 | 0.262 |
| Menial- Craft | 3.203 | 3 | 0.361 |
| Menial-WhiteCol | 11.951 | 3 | 0.008 |
| Menial- Prof | 48.190 | 3 | 0.000 |
| BlueCol- Craft | 8.441 | 3 | 0.038 |
| BlueCol-WhiteCol | 20.055 | 3 | 0.000 |
| BlueCol- Prof | 76.393 | 3 | 0.000 |
| Craft-WhiteCol | 8.892 | 3 | 0.031 |
| Craft- Prof | 60.583 | 3 | 0.000 |
| WhiteCol- Prof | 22.203 | 3 | 0.000 |

For example, we can reject the hypothesis that outcomes Menial and Prof are indistinguishable, while we cannot reject that Menial and BlueCol are indistinguishable.

## Using test [category]*

The mlogtest command computes the tests for combining categories with the test command. For example, to test that Menial is indistinguishable from the base category Prof, type

```
. test [Menial]
( 1) [Menial]white = 0.0
( 2) [Menial]ed = 0.0
( 3) [Menial] exper = 0.0
    chi2( 3) = 48.19
    Prob > chi2 = 0.0000
```

which matches the results from mlogtest in row Menial-Prof. [category] in test is used to indicate which equation is being referenced in multiple equation commands. mlogit is a multiple equation command since it is in effect estimating $J-1$ binary logit equations.

The test is more complicated when neither category is the base category. For example, to test that $m$ and $n$ are indistinguishable when the base category $b$ is neither $m$ nor $n$, the hypothesis you want to test is

$$
H_{0}:\left(\beta_{1, m \mid b}-\beta_{1, n \mid b}\right)=\cdots=\left(\beta_{K, m \mid b}-\beta_{K, n \mid b}\right)=0
$$

That is, you want to test the difference between two sets of coefficients. This can be done with test [categoryl=category2]. For example, to test if Menial and Craft can be combined, type

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```
. test [Menial=Craft]
( 1) [Menial]white - [Craft]white = 0.0
( 2) [Menial]ed - [Craft]ed = 0.0
( 3) [Menial] exper - [Craft] exper = 0.0
    chi2( 3) = 3.20
    Prob > chi2 = 0.3614
```

Again, the results are identical to those from mlogtest.

## An LR test for combining outcomes

An LR test of combining $m$ and $n$ can be computed by first estimating the full model with no constraints, with the resulting LR statistic $L R_{F}^{2}$. Then estimate a restricted model $M_{R}$ in which category $m$ is used as the base category and all the coefficients except the constant in the equation for category $n$ are constrained to 0 , with the resulting test statistic $L R_{R}^{2}$. The test statistic is the difference $L R_{R \mathrm{vs} F}^{2}=L R_{F}^{2}-L R_{R}^{2}$, which is distributed as chi-squared with $K$ degrees of freedom. The command mlogtest, lrcomb computes $J \times(J-1)$ tests for all pairs of outcome categories. For example,

| mlogit occ white (output omitted) <br> . mlogtest, lrcomb |  |  | egory (5) |
| :---: | :---: | :---: | :---: |
|  |  |  |  |
| **** LR tests for combining outcome categories |  |  |  |
| Ho: All coefficients except intercepts associated with given pair of outcomes are 0 (i.e., categories can be collapsed). |  |  |  |
| Categories tested | chi2 | df | $\mathrm{P}>$ chi2 |
| Menial- BlueCol | 4.095 | 3 | 0.251 |
| Menial- Craft | 3.376 | 3 | 0.337 |
| Menial-WhiteCol | 13.223 | 3 | 0.004 |
| Menial- Prof | 64.607 | 3 | 0.000 |
| BlueCol- Craft | 9.176 | 3 | 0.027 |
| BlueCol-WhiteCol | 22.803 | 3 | 0.000 |
| BlueCol- Prof | 125.699 | 3 | 0.000 |
| Craft-WhiteCol | 9.992 | 3 | 0.019 |
| Craft- Prof | 95.889 | 3 | 0.000 |
| WhiteCol- Prof | 26.736 | 3 | 0.000 |

## Using constraint with Irtest*

The command mlogtest, lrcomb computes the test by using the powerful constraint command. To illustrate this, we use the test comparing Menial and BlueCol reported by mlogtest, lrcomb above. First, we estimate the full model and save the results of lrtest:

```
. mlogit occ exper ed white, nolog
    (output omitted)
. lrtest, saving(lrf)
```

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Second, we define a constraint using the command:
. constraint define 999 [Menial]
This defines constraint number 999, where the number is arbitrary. The option [Menial] indicates that all of the coefficients except the constant from the Menial equation should be constrained to 0 . Third, we re-estimate the model with this constraint. The base category must be BlueCol, so that the coefficients indicated by [Menial] are comparisons of BlueCol and Menial:

```
. mlogit occ exper ed white, base(2) constraint(999) nolog
    ( 1) [Menial] exper = 0.0
    ( 2) [Menial] ed = 0.0
    ( 3) [Menial]white = 0.0
```

| Multinomial regression | Number of obs | $=$ |
| :--- | :--- | :--- |
|  | LR chi2 $(9)$ | 161.99 |
| Log likelihood $=-428.84791$ | Prob $>$ chi2 | $=$ |
|  | Pseudo R2 | $=0.0000$ |
|  |  | 0.1589 |


| Occ | Coef. | Std. Err. | z | $P>\|z\|$ | [95\% Conf. | Interval] |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Menial |  |  |  |  |  |  |
| exper | (dropped) |  |  |  |  |  |
|  | (dropped) |  |  |  |  |  |
| white | (dropped) |  |  |  |  |  |
| _cons | $-.8001193$ | . 2162194 | $-3.70$ | 0.000 | -1.223901 | -. 3763371 |
| Craft |  |  |  |  |  |  |
| exper | . 0242824 | . 0113959 | 2.13 | 0.033 | . 0019469 | . 0466179 |
| ed | . 1599345 | . 0693853 | 2.31 | 0.021 | . 0239418 | . 2959273 |
| white | -. 2381783 | . 4978563 | -0.48 | 0.632 | -1.213959 | . 7376021 |
| _cons | -1.969087 | 1.054935 | -1.87 | 0.062 | -4.036721 | . 098547 |
| WhiteCol |  |  |  |  |  |  |
| exper | . 0312007 | . 0143598 | 2.17 | 0.030 | . 0030561 | . 0593454 |
| ed | .4195709 | . 0958978 | 4.38 | 0.000 | . 2316147 | . 607527 |
| white | $.8829927$ | . 843371 | $1.05$ | 0.295 | $-.7699841$ | $2.535969$ |
| _cons | $-7.140306$ | 1.623401 | -4.40 | 0.000 | -10.32211 | -3.958498 |
| Prof |  |  |  |  |  |  |
| exper | . 032303 | . 0133779 | 2.41 | 0.016 | . 0060827 | . 0585233 |
| ed | . 8445092 | . 093709 | 9.01 | 0.000 | . 6608429 | 1.028176 |
| white | $1.097459$ | $.6877939$ | $1.60$ | $0.111$ | -. 2505923 | $2.44551$ |
| _cons | -12.42143 | 1.569897 | -7.91 | 0.000 | -15.49837 | -9.344489 |

(Outcome occ==BlueCol is the comparison group)
mlogit requires the option constraint (999) to indicate that estimation should impose this constraint. The output clearly indicates which constraints have been imposed. Finally, we use lrtest to compute the test:

```
. lrtest, using(lrf)
Mlogit: likelihood-ratio test chi2(3) = 4.09
Prob > chi2 = 0.2514
```

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### 6.4 Independence of irrelevant alternatives

Both the MNLM and the conditional logit model (discussed below) make the assumption known as the independence of irrelevant alternatives (IIA). Here we describe the assumption in terms of the MNLM. In this model,

$$
\frac{\operatorname{Pr}(y=m \mid \mathbf{x})}{\operatorname{Pr}(y=n \mid \mathbf{x})}=\exp \left(\mathbf{x}\left[\beta_{m \mid b}-\beta_{n \mid b}\right]\right)
$$

where the odds do not depend on other outcomes that are available. In this sense, these alternative outcomes are "irrelevant." What this means is that adding or deleting outcomes does not affect the odds among the remaining outcomes. This point is often made with the red bus/blue bus example. Suppose that you have the choice of a red bus or a car to get to work and that the odds of taking a red bus compared to a car are $1: 1$. IIA implies that the odds will remain $1: 1$ between these two alternatives even if a new blue bus company comes to town that is identical to the red bus company except for the color of the bus. Thus, the probability of driving a car can be made arbitrarily small by adding enough different colors of buses! More reasonably, we might expect that the odds of a red bus compared to a car would be reduced to $1: 2$ since half of those riding the red bus would be expected to ride the blue bus.

There are two tests of the IIA assumption. Hausman and McFadden (1984) proposed a Hausmantype test and McFadden, Tye, and Train (1976) proposed an approximate likelihood ratio test that was improved by Small and Hsiao (1985). For both the Hausman and the Small-Hsiao tests, multiple tests of IIA are possible. Assuming that the MNLM is estimated with base category $b, J-1$ tests can be computed by excluding each of the remaining categories to form the restricted model. By changing the base category, a test can also be computed that excludes $b$. The results of the test differ depending on which base category was used to estimate the model. See Zhang and Hoffman (1993) or Long (1997, Chapter 6) for further information.

## Hausman test of IIA

The Hausman test of IIA involves the following steps:

1. Estimate the full model with all $J$ outcomes included, with estimates in $\widehat{\beta}_{F}$.
2. Estimate a restricted model by eliminating one or more outcome categories, with estimates in $\widehat{\beta}_{R}$.
3. Let $\widehat{\beta}_{F}^{*}$ be a subset of $\widehat{\beta}_{F}$ after eliminating coefficients not estimated in the restricted model. The test statistic is

$$
H=\left(\widehat{\beta}_{R}-\widehat{\beta}_{F}^{*}\right)^{\prime}\left[\widehat{\operatorname{Var}}\left(\widehat{\beta}_{R}\right)-\widehat{\operatorname{Var}}\left(\widehat{\beta}_{F}^{*}\right)\right]^{-1}\left(\widehat{\beta}_{R}-\widehat{\beta}_{F}^{*}\right)
$$

where $H$ is asymptotically distributed as chi-squared with degrees of freedom equal to the rows in $\widehat{\beta}_{R}$ if IIA is true. Significant values of $H$ indicate that the IIA assumption has been violated.

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The Hausman test of IIA can be computed with mlogtest. In our example, the results are
. mlogit occ white ed exper, basecategory(5) nolog
(output omitted)

- mlogtest, hausman base
**** Hausman tests of IIA assumption
Ho: Odds (Outcome-J vs Outcome-K) are independent of other alternatives.
Omitted

| Menial | chi2 | df | P>chi2 | evidence |
| ---: | :--- | ---: | :--- | :--- | :--- |
| BlueCol | 7.324 | 12 | 0.835 | for Ho |
| Craft | 0.320 | 12 | 1.000 | for Ho |
| WhiteCol | -14.436 | 12 | --- | for Ho |
| Prof | -5.541 | 11 | --- | for Ho |

Note: If chi2<0, the estimated model does not meet asymptotic assumptions of the test.

Five tests of IIA are reported. The first four correspond to excluding one of the four non-base categories. The fifth test, in row Prof, is computed by re-estimating the model using the largest remaining category as the base category. ${ }^{1}$ While none of the tests reject the $H_{0}$ that IIA holds, the results differ considerably depending on the category considered. Further, three of the test statistics are negative, which we find to be very common. Hausman and McFadden $(1984,1226)$ note this possibility and conclude that a negative result is evidence that IIA has not been violated. A further sense of the variability of the results can be seen by re-running mlogit with a different base category and then running mlogtest, hausman base.

## Small and Hsiao test of IIA

To compute Small and Hsiao's test, the sample is divided randomly into two subsamples of about equal size. The unrestricted MNLM is estimated on both subsamples, where $\widehat{\beta}_{u}^{S_{1}}$ contains estimates from the unrestricted model on the first subsample and $\widehat{\beta}_{u}^{S_{2}}$ is its counterpart for the second subsample. A weighted average of the coefficients is computed as

$$
\widehat{\beta}_{u}^{S_{1} S_{2}}=\left(\frac{1}{\sqrt{2}}\right) \widehat{\beta}_{u}^{S_{1}}+\left[1-\left(\frac{1}{\sqrt{2}}\right)\right] \widehat{\beta}_{u}^{S_{2}}
$$

Next, a restricted sample is created from the second subsample by eliminating all cases with a chosen value of the dependent variable. The MNLM is estimated using the restricted sample yielding the estimates $\widehat{\beta}_{r}^{S_{2}}$ and the likelihood $L\left(\widehat{\beta}_{r}^{S_{2}}\right)$. The Small-Hsiao statistic is

$$
S H=-2\left[L\left(\widehat{\beta}_{u}^{S_{1} S_{2}}\right)-L\left(\widehat{\beta}_{r}^{S_{2}}\right)\right]
$$

which is asymptotically distributed as a chi-squared with the degrees of freedom equal to $K+1$, where $K$ is the number of independent variables.

[^37]To compute the Small-Hsiao test, you use the command mlogtest, smhsiao (our program uses code from smhsiao by Nick Winter, available at the SSC-IDEAS archive). For example,
. mlogtest, smhsiao
**** Small-Hsiao tests of IIA assumption
Ho: Odds(Outcome-J vs Outcome-K) are independent of other alternatives.

| Omitted | $\operatorname{lnL}$ (full) | $\operatorname{lnL}$ (omit) | chi2 | df | $\mathrm{P}>$ chi2 | evidence |
| ---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Menial | -182.140 | -169.907 | 24.466 | 4 | 0.000 | against Ho |
| BlueCol | -148.711 | -140.054 | 17.315 | 4 | 0.002 | against Ho |
| Craft | -131.801 | -119.286 | 25.030 | 4 | 0.000 | against Ho |
| WhiteCol | -161.436 | -148.550 | 25.772 | 4 | 0.000 | against Ho |

The results vary considerably from those of the Hausman tests. In this case, each test indicates that IIA has been violated.

Since the Small-Hsiao test requires randomly dividing the data into subsamples, the results will differ with successive calls of the command since the sample will be divided differently. To obtain test results that can be replicated, you must explicitly set the seed used by the random number generator. For example,

| **** Small-Hsiao tests of IIA assumption |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Ho: Odds (Outcome-J vs Outcome-K) are independent of other alternatives. |  |  |  |  |  |  |
| Omitted | $\operatorname{lnL}(f u l l)$ | lnL(omit | chi2 | df | $\mathrm{P}>\mathrm{chi2}$ | evidence |
| Menial | -169.785 | -161.523 | 16.523 | 4 | 0.002 | against Ho |
| BlueCol | -131.900 | -125.871 | 12.058 | 4 | 0.017 | against Ho |
| Craft | -136.934 | -129.905 | 14.058 | 4 | 0.007 | against Ho |
| WhiteCol | -155.364 | -150.239 | 10.250 | 4 | 0.036 | against Ho |

Advanced: setting the random seed The random numbers that divide the sample for the SmallHsiao test are based on Stata's uniform() function, which uses a pseudo-random number generator. This generator creates a sequence of numbers based on a seed number. While these numbers appear to be random, exactly the same sequence will be generated each time you start with the same seed number. In this sense (and some others), these numbers are pseudorandom rather than random. If you specify the seed with set seed \#, you can ensure that you will be able to replicate your results later. See the User's Guide for further details.

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## Conclusions regarding tests of IIA

Our experience with these tests is that they often give inconsistent results and provide little guidance to violations of the IIA assumption. Unfortunately, there do not appear to be simulation studies that examine their small sample properties. Perhaps as a result of the practical limitations of these tests, McFadden (1973) suggested that IIA implies that the multinomial and conditional logit models should only be used in cases where the outcome categories "can plausibly be assumed to be distinct and weighed independently in the eyes of each decision maker." Similarly, Amemiya (1981, 1517) suggests that the MNLM works well when the alternatives are dissimilar. Care in specifying the model to involve distinct outcomes that are not substitutes for one another seems to be reasonable, albeit unfortunately ambiguous, advice.

### 6.5 Measures of fit

As with the binary and ordinal models, scalar measures of fit for the MNLM model can be computed with the SPost command fitstat. The same caveats against overstating the importance of these scalar measures apply here as to the other models we consider (see also Chapter 3). To examine the fit of individual observations, you can estimate the series of binary logits implied by the multinomial logit model and use the established methods of examining the fit of observations to binary logit estimates. This is the same approach that was recommended in Chapter 5 for ordinal models.

### 6.6 Interpretation

While the MNLM is a mathematically simple extension of the binary model, interpretation is made difficult by the large number of possible comparisons. Even in our simple example with five outcomes, we have many possible comparisons: $M|P, B| P, C|P, W| P, M|W, B| W, C|W, M| C$, $B \mid C$, and $M \mid B$. It is tedious to write all of the comparisons, let alone to interpret each of them for each of the independent variables. Thus, the key to interpretation is to avoid being overwhelmed by the many comparisons. Most of the methods we propose are very similar to those for ordinal outcomes, and accordingly these are treated very briefly. However, methods of plotting discrete changes and factor changes are new, so these are considered in greater detail.

### 6.6.1 Predicted probabilities

Predicted probabilities can be computed with the formula

$$
\widehat{\operatorname{Pr}}(y=m \mid \mathbf{x})=\frac{\exp \left(\mathbf{x} \widehat{\beta}_{m \mid J}\right)}{\sum_{j=1}^{J} \exp \left(\mathbf{x} \widehat{\beta}_{j \mid J}\right)}
$$

where x can contain values from individuals in the sample or hypothetical values. The most basic command for computing probabilities is predict, but we also illustrate a series of SPost commands that compute predicted probabilities in useful ways.

[^38]
### 6.6.2 Predicted probabilities with predict

After estimating the model with mlogit, the predicted probabilities within the sample can be calculated with the command

```
predict newvarl [newvar2...[newvarJ]] [if exp] [in range]
```

where you must provide one new variable name for each of the $J$ categories of the dependent variable, ordered from the lowest to highest numerical values. For example,

```
. mlogit occ white ed exper, basecategory(5) nolog
    (output omitted)
. predict ProbM ProbB ProbC ProbW ProbP
(option p assumed; predicted probabilities)
```

The variables created by predict are

| variable name |  | storage type | display <br> format | value <br> label | variable label |
| :---: | :---: | :---: | :---: | :---: | :---: |
| ProbM |  | float | \%9.0g |  | $\operatorname{Pr}(0 \subset c==1)$ |
| ProbB |  | float | $\% 9.0 \mathrm{~g}$ |  | $\operatorname{Pr}(\mathrm{occ}==2)$ |
| ProbC |  | float | $\% 9.0 \mathrm{~g}$ |  | $\operatorname{Pr}(\mathrm{occ}==3)$ |
| ProbW |  | float | $\% 9.0 \mathrm{~g}$ |  | $\operatorname{Pr}(\mathrm{occ}==4)$ |
| ProbP |  | float | \%9.0g |  | $\operatorname{Pr}(\mathrm{occ}==5)$ |
| . sum Prob* |  |  |  |  |  |
| Variable | Obs | Mean | Std. Dev. | Min | Max |
| ProbM | 337 | . 0919881 | . 059396 | . 0010737 | . 3281906 |
| ProbB | 337 | . 2047478 | . 1450568 | . 0012066 | . 6974148 |
| ProbC | 337 | . 2492582 | . 1161309 | . 0079713 | . 551609 |
| ProbW | 337 | . 1216617 | . 0452844 | . 0083857 | . 2300058 |
| ProbP | 337 | . 3323442 | . 2870992 | . 0001935 | . 9597512 |

## Using predict to compare mlogit and ologit

An interesting way to illustrate how predictions can be plotted is to compare predictions from ordered logit and multinomial logit when the models are applied to the same data. Recall from Chapter 5 that the range of the predicted probabilities for middle categories abruptly ended, while predictions for the end categories had a more gradual distribution. To illustrate this point, the example in Chapter 5 is estimated using ologit and mlogit, with predicted probabilities computed for each case:

```
. use ordwarm2,clear
(77 & 89 General Social Survey)
. ologit warm yr89 male white age ed prst, nolog
    (output omitted)
. predict SDologit Dologit Aologit SAologit
(option p assumed; predicted probabilities)
. label var Dologit "ologit-D"
```

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```
. mlogit warm yr89 male white age ed prst, nolog
    (output omitted)
. predict SDmlogit Dmlogit Amlogit SAmlogit
(option p assumed; predicted probabilities)
. label var Dmlogit "mlogit-D"
```

We can plot the predicted probabilities of disagreeing in the two models with the command dotplot Dologit Dmlogit, ylabel ( $0, .25, .5, .75$ ), which leads to

```
.75
.5-
```

While the two sets of predictions have a correlation of .92 (computed by the command corr Dologit Dmlogit), the abrupt truncation of the distribution for the ordered logit model strikes us as substantively unrealistic.

### 6.6.3 Individual predicted probabilities with prvalue

Predicted probabilities for individuals with specified characteristics can be computed with prvalue. For example, we might compute the probabilities of each occupational category to compare nonwhites and whites who are average on education and experience:

```
. mlogit occ white ed exper, basecategory(5) nolog
    (output omitted)
. quietly prvalue, x(white 0) rest(mean) save
. prvalue, x(white 1) rest(mean) dif
mlogit: Change in Predictions for occ
Predicted probabilities for each category:
\begin{tabular}{|c|c|c|c|}
\hline & Current & Saved & Difference \\
\hline \(\operatorname{Pr}(\mathrm{y}=\) Menial|x) : & 0.0860 & 0.2168 & -0.1309 \\
\hline \(\operatorname{Pr}(\mathrm{y}=\) BlueCol|x) : & 0.1862 & 0.1363 & 0.0498 \\
\hline \(\operatorname{Pr}(\mathrm{y}=\operatorname{Craft|x)}\) : & 0.2790 & 0.4387 & -0.1597 \\
\hline \(\operatorname{Pr}(\mathrm{y}=\) WhiteCol|x) : & 0.1674 & 0.0877 & 0.0797 \\
\hline \(\operatorname{Pr}(\mathrm{y}=\operatorname{Prof} \mid \mathrm{x})\) : & 0.2814 & 0.1204 & 0.1611 \\
\hline white & ed & exper & \\
\hline Current= 1 & 13.094955 & 20.501484 & \\
\hline Saved= 0 & 13.094955 & 20.501484 & \\
\hline Diff= & 0 & 0 & \\
\hline
\end{tabular}
```

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This example also illustrates how to use prvalue to compute differences between two sets of probabilities. Our first call of prvalue is done quietly, but we save the results. The second call uses the dif option, and the output compares the results for the first and second set of values computed.

### 6.6.4 Tables of predicted probabilities with prtab

If you want predicted probabilities for all combinations of a set of categorical independent variables, prtab is useful. For example, we might want to know how white and nonwhite respondents differ in their probability of having a menial job by years of education:

```
. label def lwhite 0 NonWhite 1 White
. label val white lwhite
. prtab ed white, novarlbl outcome(1)
mlogit: Predicted probabilities of outcome 1 (Menial) for occ
\begin{tabular}{r|cc}
\hline & \multicolumn{2}{|c}{\begin{tabular}{r} 
white \\
ed
\end{tabular}} \\
& NonWhite & White \\
\hline 3 & 0.2847 & 0.1216 \\
6 & 0.2987 & 0.1384 \\
7 & 0.2988 & 0.1417 \\
8 & 0.2963 & 0.1431 \\
9 & 0.2906 & 0.1417 \\
10 & 0.2814 & 0.1366 \\
11 & 0.2675 & 0.1265 \\
12 & 0.2476 & 0.1104 \\
13 & 0.2199 & 0.0883 \\
14 & 0.1832 & 0.0632 \\
15 & 0.1393 & 0.0401 \\
16 & 0.0944 & 0.0228 \\
17 & 0.0569 & 0.0120 \\
18 & 0.0310 & 0.0060 \\
19 & 0.0158 & 0.0029 \\
20 & 0.0077 & 0.0014 \\
\hline
\end{tabular}
\begin{tabular}{lrr} 
& white & ed
\end{tabular} exper
```

Tip: outcome () option In this example, we use the outcome() option to restrict the output to a single outcome category. Without this option, prtab will produce a separate table for each outcome category.

The table produced by prtab shows the substantial differences between whites and nonwhites in the probabilities of having menial jobs and how these probabilities are affected by years of education. However, given the number of categories for ed, plotting these predicted probabilities with prgen is probably a more useful way to examine the results.

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### 6.6.5 Graphing predicted probabilities with prgen

Predicted probabilities can be plotted using the same methods considered for the ordinal regression model. After estimating the model, we use prgen to compute the predicted probabilities for whites with average working experience as education increases from 6 years to 20 years:

```
. prgen ed, x(white=1) from(6) to(20) generate(wht) ncases(15)
mlogit: Predicted values as ed varies from 6 to 20.
\begin{tabular}{lrrr} 
& white & ed & exper \\
\(x=\) & 1 & 13.094955 & 20.501484
\end{tabular}
```

Here is what the options specify:
x (white=1) sets white to 1 . Since the rest () option is not included, all other variables are set to their means by default.
from (6) and to (20) set the minimum and maximum values over which ed is to vary. The default is to use the variable's minimum and maximum values.
ncases (15) indicates that 15 evenly spaced values of ed between 6 and 20 are to be generated. We chose 15 for the number of values from 6 to 20 inclusive.
gen (wht) specifies the root name for the new variables generated by prgen. For example, the variable whtx contains values of ed; the p-variables (e.g., whtp2) contain the predicted probabilities for each outcome; and the s-variables contain the summed probabilities. A list of these variables should make this clear:
. describe wht*

| variable name | storage type | display <br> format | value <br> label | variable label |
| :---: | :---: | :---: | :---: | :---: |
| whtx | float | $\% 9.0 \mathrm{~g}$ |  | Changing value of ed |
| whtp1 | float | $\% 9.0 \mathrm{~g}$ |  | pr (Menial) [1] |
| whts1 | float | \%9.0g |  | $\operatorname{pr}(\mathrm{y}<=1$ ) |
| whtp2 | float | \%9.0g |  | pr (BlueCol) [2] |
| whts2 | float | $\% 9.0 \mathrm{~g}$ |  | $\mathrm{pr}(\mathrm{y}<=2)$ |
| whtp3 | float | \%9.0g |  | pr (Craft) [3] |
| whts3 | float | $\% 9.0 \mathrm{~g}$ |  | $\operatorname{pr}(\mathrm{y}<=3)$ |
| whtp4 | float | $\% 9.0 \mathrm{~g}$ |  | pr (WhiteCol) [4] |
| whts4 | float | $\% 9.0 \mathrm{~g}$ |  | $\operatorname{pr}(\mathrm{y}<=4)$ |
| whtp5 | float | $\% 9.0 \mathrm{~g}$ |  | pr (Prof) [5] |
| whts5 | float | \%9.0g |  | $\operatorname{pr}(\mathrm{y}<=5)$ |

The same thing can be done to compute predicted probabilities for nonwhites:

```
. prgen ed, x(white=0) from(6) to(20) generate(nwht) ncases(15)
mlogit: Predicted values as ed varies from 6 to 20.
\begin{tabular}{lrrr} 
& white & ed & exper \\
\(x=\) & 0 & 13.094955 & 20.501484
\end{tabular}
```

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## Plotting probabilities for one outcome and two groups

The variables nwhtp1 and whtp1 contain the predicted probabilities of having menial jobs for nonwhites and whites. Plotting these provides clearer information than the results of prtab given above:
. label var whtp1 "Whites"
. label var nwhtp1 "Nonwhites"
. set textsize 125
. graph whtp1 nwhtp1 nwhtx, b2("Years of Education") l1("Pr(Menial Job)") gap(3) /*
$>* /$ ylabel $(0, .25, .50)$ yscale $(0, .5)$ xscale $(6,20) \mathrm{s}(0 S)$ connect (ss) border
These commands produce the following graph:


## Graphing probabilities for all outcomes for one group

Even though nominal outcomes are not ordered, plotting the summed probabilities can be a useful way to show predicted probabilities for all outcome categories. To illustrate this, we construct a graph to show how education affects the probability of each occupation for whites (a similar graph could be plotted for nonwhites). This is done using the roots\# variables created by prgen, which provide the probability of being in an outcome less than or equal to some value. For example, the label for whts 3 is $\operatorname{pr}(\mathrm{y}<=3)$, which indicates that all nominal categories coded as 3 or less are added together. To plot these probabilities, the first thing we do is change the variable labels to the name of the highest category in the sum, which makes the graph clearer (as you will see below):
. label var whts1 "Menial"
. label var whts2 "Blue Collar"
. label var whts3 "Craft"
. label var whts4 "White Collar"
To create the summed plot, we use the following commands:

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```
. set textsize 125
. graph whts1 whts2 whts3 whts4 whtx, b2("Whites: Years of Education") gap(3) /*
*/ l1("Summed Probability") xlabel(6 8 to 20) ylabel(0,.25,.50,.75,1) /*
*/ yscale(0,1) xscale(6,20) s(OdST) connect(ssss) border
```

which produce


The graph plots the four summed probabilities against whtx, where standard options for graph are used. This graph is not ideal, but before revising it, let's make sure we understand what is being plotted. The lowest line with circles, labeled "Menial" in the key, plots the probability of having a menial job for a given year of education. This is the same information as plotted in our prior graph for whites. The next line with small diamonds, labeled "Blue Collar" in the key, plots the sum of the probability of having a menial job or a blue collar job. Thus, the area between the line with circles and the line with diamonds is the probability of having a blue collar job, and so on.

Unfortunately, this graph is not as effective as we would like, and we cannot improve it with Stata 7 (although this is about to change with new graphics commands that are under development by StataCorp; check www. stata.com for the latest news). Accordingly, we illustrate a useful approach for customizing a graph. First, we saved it as a Windows Metafile using the command translate @Graph 06prsum.wmf, replace (see Chapter 2 for details). Next, we loaded it into a graphics package that can read and edit these files (e.g., Adobe's Illustrator, Microsoft PowerPoint). Using the graphic editing commands, we create a much clearer graph:

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### 6.6.6 Changes in predicted probabilities

Marginal and discrete change can be used in the same way as in models for ordinal outcomes. As before, both can be computed using prchange.

Marginal change is defined as

$$
\frac{\partial \operatorname{Pr}(y=m \mid \mathbf{x})}{\partial x_{k}}=\operatorname{Pr}(y=m \mid \mathbf{x})\left[\beta_{k, m \mid J}-\sum_{j=1}^{J} \beta_{k, j \mid J} \operatorname{Pr}(y=j \mid \mathbf{x})\right]
$$

Since this equation combines all of the $\beta_{k, j \mid J}$ 's, the value of the marginal change depends on the levels of all variables in the model. Further, as the value of $x_{k}$ changes, the sign of the marginal can change. For example, at one point the marginal effect of education on having a craft occupation could be positive, while at another point the marginal effect could be negative.

Discrete change is defined as

$$
\frac{\Delta \operatorname{Pr}(y=m \mid \mathbf{x})}{\Delta x_{k}}=\operatorname{Pr}\left(y=m \mid \mathbf{x}, x_{k}=x_{E}\right)-\operatorname{Pr}\left(y=m \mid \mathbf{x}, x_{k}=x_{S}\right)
$$

where the magnitude of the change depends on the levels of all variables and the size of the change that is being made. The $J$ discrete change coefficients for a variable (one for each outcome category) can be summarized by computing the average of the absolute values of the changes across all of the outcome categories,

$$
\bar{\Delta}=\frac{1}{J} \sum_{j=1}^{J}\left|\frac{\Delta \operatorname{Pr}(y=j \mid \overline{\mathbf{x}})}{\Delta x_{k}}\right|
$$

where the absolute value is taken because the sum of the changes without taking the absolute value is necessarily zero.

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## Computing marginal and discrete change with prchange

Discrete and marginal changes are computed with prchange (the full syntax for which is provided in Chapter 3). For example,


The first thing to notice is the output labeled $\operatorname{Pr}(\mathrm{y} \mid \mathrm{x})$, which is the predicted probabilities at the values set by x() and rest (). Marginal change is listed in the rows MargEfct. For variables that are not binary, discrete change is reported over the entire range of the variable (reported as Min $->$ Max), for changes of one unit centered around the base values (reported as $-+1 / 2$ ), and for changes of one standard deviation centered around the base values (reported as $-+s d / 2$ ). If the uncentered option is used, the changes begin at the value specified by x() or rest () and increase one unit or one standard deviation from there. For binary variables, the discrete change from 0 to 1 is the only appropriate quantity and is the only quantity that is presented. Looking at the results for white above, we can see that for someone who is average in education and experience, the predicted probability of having a professional job is .16 higher for whites than nonwhites. The average change

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is listed in the column Avg $\mid$ Chg $\mid$. For example, for white, $\bar{\Delta}=0.12$; the average absolute change in the probability of various occupational categories for being white as opposed to nonwhite is .12 .

## Marginal change with mfx compute

The marginal change can also be computed using mfx compute, where the at () option is used to set values of the independent variables. Like prchange, mfx compute sets all values of the independent variables to their means by default. As noted in Chapter 5, mfx compute does not allow you to compute effects only for a subset of variables in the model. Also, we must estimate the marginal effects for one outcome at a time, using the predict (outcome(\#)) option to specify the outcome for which we want marginal effects:
. mfx compute, predict(outcome(1))
Marginal effects after mlogit
$\mathrm{y}=\operatorname{Pr}(\mathrm{occ}==1)$ (predict, outcome(1))
$=.09426806$

| variable | dy/dx | Std. Err. | $z$ | P>\|z| | [ $95 \%$ | C.I. | $]$ | $X$ |
| :---: | ---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| white* | -.1308552 | .08915 | -1.47 | 0.142 | -.305592 | .043882 | .916914 |  |
| ed | -.025791 | .00688 | -3.75 | 0.000 | -.039269 | -.012312 | 13.0950 |  |
| exper | -.00227 | .00126 | -1.80 | 0.071 | -.004737 | .000197 | 20.5015 |  |

(*) dy/dx is for discrete change of dummy variable from 0 to 1
These results are for the Menial category ( $o c c==1$ ). Estimates for exper and ed match the results in the MargEfct rows of the prchange output above. Meanwhile, for the binary variable white, the discrete change from 0 to 1 is presented, which also matches the corresponding result from prchange. An advantage of mfx compute is that standard errors for the effects are also provided; a disadvantage is that mfx compute can take a long time to produce results after mlogit, especially if the number of observations and independent variables is large.

### 6.6.7 Plotting discrete changes with prchange and mlogview

One difficulty with nominal outcomes is the large number of coefficients that need to be considered: one for each variable times the number of outcome categories minus one. To help you sort out all of this information, discrete change coefficients can be plotted using our program mlogview. After estimating the model with mlogit and computing discrete changes with prchange, executing mlogview opens the dialog box: ${ }^{2}$
(Continued on next page)

[^39]

Dialog boxes are easier to use than to explain. So, as we describe various features, the best advice is to generate the dialog box shown above and experiment.

Selecting variables If you click and hold on a button, you can select a variable to be plotted. The same variable can be plotted more than once, for example, showing the effects of different amounts of change.

Selecting the amount of change The radio buttons allow you to select the type of discrete change coefficient to plot for each selected variable: +1 selects coefficients for a change of one unit; + SD selects coefficients for a change of one standard deviation; $0 / 1$ selects changes from 0 to 1 ; and Don't Plot is self explanatory.

Making a plot Even though there are more options to explain, you should try plotting your selections by clicking on DC Plot, which produces a graph. The command mlogview works by generating the syntax for the command mlogplot, which actually draws the plot. In the Results Window, you will see the mlogplot command that was used to generate your graph (full details on mlogplot are given in Section 6.6.9). If there is an error in the options you select, the error message will appear in the Results Window.

Assuming everything has worked, we generate the following graph:

| white-0/1 | C | M |  |  |  | B | W |  | P |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| ed |  |  | B C | M | W |  |  |  |  |
| exper | * |  |  |  |  |  |  |  |  |
|  | - 16 | - 12 | -. 08 | - 04 | 0 | . 04 | . 08 | . 12 | . 16 |
|  | Chang | in Predicte | Proabaily | for occ |  |  |  |  |  |

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The graph immediately shows how a unit increase in each variable affects the probability of each outcome. While it appears that the effects of being white are the largest, changes of one unit in education and (especially) experience are often too small to be as informative. It would make more sense to look at the effects of a standard deviation change in these variables. To do this, we return to the dialog box and click on the radio button + SD. Before we see what this does, let's consider several other options that can be used.

Adding labels The box Note allows you to enter text that will be placed at the top of the graph. Clicking the box for Use variable labels replaces the names of the variables on the left axis with the variable labels associated with each variable. When you do this, you may find that the labels are too long. If so, you can use the label variable command to change them.

Tic marks The values for the tic marks are determined by specifying the minimum and maximum values to plot and the number of tic marks. For example, we could specify a plot from -.2 to .4 with 7 tick marks. This will lead to labels every .1 units.

Using some of the features discussed above, our dialog box looks like this:


Clicking on DC Plot produces the following graph:

| White Worker-0/1 | C M |  |  | B W |  | P |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Yrs of Education-std | B | C | M | W |  |  |  |  |
| Yrs of Experience-std | BM |  |  | QNP |  |  |  |  |
|  |  |  |  |  |  |  |  |  |

In this figure, you can see that the effects of education are largest, and that those of experience are smallest. Or, each coefficient can be interpreted individually, such as

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The effects of a standard deviation change in education are largest, with an increase of over .35 in the probability of having a professional occupation.
The effects of race are also substantial, with average blacks being less likely to enter blue collar, white collar, or professional jobs than average whites.
Expected changes due to a standard deviation change in experience are much smaller and show that experience increases the probabilities of more highly skilled occupations.

In using these graphs, keep in mind that different values for discrete change are obtained at different levels of the variables, which are specified with the x() and rest () options for prchange.

Value labels with mlogview The value labels for the different categories of the dependent variables must begin with different letters since the plots generated with mlogview use the first letter of the value label.

### 6.6.8 Odds ratios using listcoef and mlogview

Discrete change does little to illuminate the dynamics among the outcomes. For example, a decrease in education increases the probability of both blue collar and craft jobs, but how does it affect the odds of a person choosing a craft job relative to a blue collar job? To deal with these issues, odds ratios (also referred to as factor change coefficients) can be used. Holding other variables constant, the factor change in the odds of outcome $m$ versus outcome $n$ as $x_{k}$ increases by $\delta$ equals

$$
\frac{\Omega_{m \mid n}\left(\mathbf{x}, x_{k}+\delta\right)}{\Omega_{m \mid n}\left(\mathbf{x}, x_{k}\right)}=e^{\beta_{k, m \mid n} \delta}
$$

If the amount of change is $\delta=1$, then the odds ratio can be interpreted as

For a unit change in $x_{k}$, the odds of $m$ versus $n$ are expected to change by a factor of $\exp \left(\beta_{k, m \mid n}\right)$, holding all other variables constant.

If the amount of change is $\delta=s_{x_{k}}$, then the odds ratio can be interpreted as
For a standard deviation change in $x_{k}$, the odds of $m$ versus $n$ are expected to change by a factor of $\exp \left(\beta_{k, m \mid n} \times s_{k}\right)$, holding all other variables constant.

## Listing odds ratios with listcoef

The difficulty in interpreting odds ratios for the MNLM is that to understand the effect of a variable, you need to examine the coefficients for comparisons among all pairs of outcomes. The standard

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output from mlogit includes only $J-1$ comparisons with the base category. While you could estimate coefficients for all possible comparisons by re-running mlogit with different base categories (e.g., mlogit occ white ed exper, basecategory (3)), using listcoef is much simpler. For example, to examine the effects of race, type

| mlogit ( $\mathrm{N}=337$ ) : Factor Change in the Odds of occ |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Variable: white (sd= .276423) |  |  |  |  |  |
| Odds comparing Group 1 vs Group 2 | b | z | $P>\|z\|$ | $e^{\wedge} \mathrm{b}$ | e^bStdX |
| Menial -BlueCol | -1.23650 | -1.707 | 0.088 | 0.2904 | 0.7105 |
| Menial -Craft | -0.47234 | -0.782 | 0.434 | 0.6235 | 0.8776 |
| Menial -WhiteCol | -1.57139 | -1.741 | 0.082 | 0.2078 | 0.6477 |
| Menial -Prof | -1.77431 | -2.350 | 0.019 | 0.1696 | 0.6123 |
| BlueCol -Menial | 1.23650 | 1.707 | 0.088 | 3.4436 | 1.4075 |
| BlueCol -Craft | 0.76416 | 1.208 | 0.227 | 2.1472 | 1.2352 |
| BlueCol -WhiteCol | -0.33488 | -0.359 | 0.720 | 0.7154 | 0.9116 |
| BlueCol -Prof | -0.53780 | -0.673 | 0.501 | 0.5840 | 0.8619 |
| Craft -Menial | 0.47234 | 0.782 | 0.434 | 1.6037 | 1.1395 |
| Craft -BlueCol | -0.76416 | -1.208 | 0.227 | 0.4657 | 0.8096 |
| Craft -WhiteCol | -1.09904 | -1.343 | 0.179 | 0.3332 | 0.7380 |
| Craft -Prof | -1.30196 | -2.011 | 0.044 | 0.2720 | 0.6978 |
| WhiteCol-Menial | 1.57139 | 1.741 | 0.082 | 4.8133 | 1.5440 |
| WhiteCol-BlueCol | 0.33488 | 0.359 | 0.720 | 1.3978 | 1.0970 |
| WhiteCol-Craft | 1.09904 | 1.343 | 0.179 | 3.0013 | 1.3550 |
| WhiteCol-Prof | -0.20292 | -0.233 | 0.815 | 0.8163 | 0.9455 |
| Prof -Menial | 1.77431 | 2.350 | 0.019 | 5.8962 | 1.6331 |
| Prof -BlueCol | 0.53780 | 0.673 | 0.501 | 1.7122 | 1.1603 |
| Prof -Craft | 1.30196 | 2.011 | 0.044 | 3.6765 | 1.4332 |
| Prof -WhiteCol | 0.20292 | 0.233 | 0.815 | 1.2250 | 1.0577 |
| $\mathrm{b}=$ raw coefficient |  |  |  |  |  |
| $z=z$-score for test of b=0 |  |  |  |  |  |
| $\mathrm{P}>\|z\|=\mathrm{p}$-value for z -test |  |  |  |  |  |
| $e^{\wedge} \mathrm{b}=\exp (\mathrm{b})=$ factor change in odds for unit increase in X |  |  |  |  |  |
| $e^{\wedge} \mathrm{bStdX}=\exp (\mathrm{b} * S \mathrm{SD}$ of X$)=$ change in odds for SD increase in X |  |  |  |  |  |

The odds ratios of interest are in the column labeled $e^{\wedge} b$. For example, the odds ratio for the effect of race on having a professional versus a menial job is 5.90, which can be interpreted as

The odds of having a professional occupation relative to a menial occupation are 5.90 times greater for whites than for blacks, holding education and experience constant.

## Plotting odds ratios

However, examining all of the coefficients for even a single variable with only five dependent categories is complicated. An odds ratio plot makes it easy to quickly see patterns in results for even a complex MNLM (see Long 1997, Chapter 6 for full details). To explain how to interpret an odds ratio plot, we begin with some hypothetical output from a MNLM with three outcomes and three independent variables:

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|  |  | Logit Coefficient for |  |  |
| :--- | :---: | ---: | ---: | ---: |
| Comparison | $x_{1}$ | $x_{2}$ | $x_{3}$ |  |
| $B \mid A$ | $\beta_{B \mid A}$ | -0.693 | 0.693 | 0.347 |
|  | $\exp \left(\beta_{B \mid A}\right)$ | 0.500 | 2.000 | 1.414 |
|  | $p$ | 0.04 | 0.01 | 0.42 |
| $C \mid A$ | $\beta_{C \mid A}$ | 0.347 | -0.347 | 0.693 |
|  | $\exp \left(\beta_{C \mid A}\right)$ | 1.414 | 0.707 | 2.000 |
|  | $p$ | 0.21 | 0.04 | 0.37 |
| $C \mid B$ | $\beta_{C \mid B}$ | 1.040 | -1.040 | 0.346 |
|  | $\exp \left(\beta_{C \mid B}\right)$ | 2.828 | 0.354 | 1.414 |
|  | $p$ | 0.02 | 0.03 | 0.21 |

These coefficients were constructed to have some fixed relationships among categories and variables:

- The effects of $x_{1}$ and $x_{2}$ on $B \mid A$ (which you can read as $B$ versus $A$ ) are equal but of opposite size. The effect of $x_{3}$ is half as large.
- The effects of $x_{1}$ and $x_{2}$ on $C \mid A$ are half as large (and in opposite directions) as the effects on $B \mid A$, while the effect of $x_{3}$ is in the same direction but twice as large.

In the odds ratio plot, the independent variables are each represented on a separate row, and the horizontal axis indicates the relative magnitude of the $\beta$ coefficients associated with each outcome. Here is the plot, where the letters correspond to the outcome categories:

| x1 | Factor Change Scale Relative to Category A |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | . 5 | . 63 | . 79 | 1 | 1.26 |  | 1.59 | 2 |
|  | B |  |  | A |  | C |  |  |
| x2 |  |  |  | A |  |  |  | B |
| x3 |  |  |  | A |  | B |  | C |
|  | -. 69 | -. 46 | - ${ }^{-23}$ | 0 | . 23 |  | . 46 | . 69 |
|  | Logit Coefficient Scale Relative to Category A |  |  |  |  |  |  |  |

The plot reveals a great deal of information, which we now summarize.

Sign of coefficients If a letter is to the right of another letter, increases in the independent variable make the outcome to the right more likely. Thus, relative to outcome $A$, an increase in $x_{1}$ makes it more likely that we will observe outcome $C$ and less likely that will observe outcome $B$. This corresponds to the positive sign of the $\beta_{1, C \mid A}$ coefficient and the negative sign of the $\beta_{1, B \mid A}$ coefficient. The signs of these coefficients are reversed for $x_{2}$ and, accordingly, the odds ratio plot for $x_{2}$ is a mirror image of that for $x_{1}$.

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Magnitude of effects The distance between a pair of letters indicates the magnitude of the effect. For both $x_{1}$ and $x_{2}$, the distance between $A$ and $B$ is twice the distance between $A$ and $C$, which reflects that $\beta_{B \mid A}$ is twice as large as $\beta_{C \mid A}$ for both variables. For $x_{3}$, the distance between $A$ and $B$ is half the distance between $A$ and $C$, reflecting that $\beta_{3, C \mid A}$ is twice as large as $\beta_{3, B \mid A}$.

The additive relationship The additive relationships among coefficients shown in Equation 6.1 are also fully reflected in this graph. For any of the independent variables, $\beta_{C \mid A}=\beta_{B \mid A}+\beta_{C \mid B}$. Accordingly, the distance from $A$ to $C$ is the sum of the distances from $A$ to $B$ and $B$ to $C$.

The base category The additive scale on the bottom axis measures the value of the $\beta_{k, m \mid n}$ 's. The multiplicative scale on the top axis measures the $\exp \left(\beta_{k, m \mid n}\right)$ 's. The reason why the $A$ 's are stacked on top of one another is that the plot uses $A$ as its base category for graphing the coefficients. The choice of base category is arbitrary. We could have used outcome $B$ instead. If we had, the rows of the graph would be shifted to the left or right so that the $B$ 's lined up. Doing this leads to the following graph:

|  | Factor Change Scale Relative to Category B |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | . 35 | . 5 | . 71 | 1 | 1.41 | 2 | 2.83 |
| x1 |  |  |  | B |  | A | C |
| X | C | A |  | B |  |  |  |
| x3 |  |  | A | B | C |  |  |
|  | -1.04 | -. 69 | -. 35 | 0 | . 35 | . 69 | 1.04 |
|  | Logit Coefficient Scale Relative to Category B |  |  |  |  |  |  |

Creating odds ratio plots These graphs can be created using mlogview after running mlogit. Using our example and after changing a few options, we obtain the dialog box:


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Clicking on OR Plot leads to


Several things are immediately apparent. The effect of experience is the smallest, although increases in experience make it more likely that one will be in a craft, white collar, or professional occupation relative to a menial or blue collar one. We also see that education has the largest effects; as expected, increases in education increase the odds of having a professional job relative to any other type.

Adding significance levels The current graph does not reflect statistical significance. This is added by drawing a line between categories for which there is not a significant coefficient. The lack of statistical significance is shown by a connecting line, suggesting that those two outcomes are "tied together." You can add the significance level to the plot with the Connect if box on the dialog box. For example, if we enter . 1 in this box and uncheck the "pack odds ratio plot" box, we obtain


In order to make the connecting lines clear, vertical spacing is added to the graph. This vertical spacing has no meaning and is only used to make the lines clearer. The graph shows that race orders occupations from menial to craft to blue collar to white collar to professional, but the connecting lines show that none of the adjacent categories are significantly differentiated by race. Being white increases the odds of being a craft worker relative to having a menial job, but the effect is not significant. However, being white significantly increases the odds of being a blue collar worker, a white collar worker, or a professional relative to having a menial job. The effects of ed and exper can be interpreted similarly.

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Adding discrete change In Chapter 4, we emphasized that while the factor change in the odds is constant across the levels of all variables, the discrete change gets larger or smaller at different values of the variables. For example, if the odds increase by a factor of 10 but the current odds are 1 in 10,000 , then the substantive impact is small. But, if the current odds were 1 in 5 , the impact is large. Information on the discrete change in probability can be incorporated in the odds ratio graph by making the size of the letter proportional to the discrete change in the odds (specifically, the area of the letter is proportional to the size of the discrete change). This can be added to our graph very simply. First, after estimating the MNLM, run prchange at the levels of the variables that you want. Then, enter mlogview to open the dialog box. Set any of the options, and then click the OR+DC Plot button:


With a little practice, you can create and interpret these graphs very quickly.

### 6.6.9 Using mlogplot*

The dialog box mlogview does not actually draw the plots, but only sends the options you select to mlogplot which creates the graph. Once you click a plot button in mlogview, the necessary mlogplot command, including options, appears in the Results Window. This is done because mlogview invokes a dialog box and so cannot be used effectively in a do-file. But, once you create a plot using the dialog box, you can copy the generated mlogplot command from the Results Window and paste it into a do-file. This should be clear by looking at the following screenshot:
(Continued on next page)

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The dialog box with selected options appears in the upper left. After clicking on the OR Plot button, the graph in the upper right appeared along with the following command in the Results Window:

```
. mlogplot white ed exper, std(0ss) p(.1) min(-2.75) max(.55) or ntics(7)
```

If you enter this command from the Command Window or run it from a do-file, the same graph would be generated. The full syntax for mlogplot is described in Appendix A.

### 6.6.10 Plotting estimates from matrices with mlogplot*

You can also use mlogplot to construct odds ratio plots (but not discrete change plots) using coefficients that are contained in matrices. For example, you can plot coefficients from published papers or generate examples like those we used above. To do this, you must construct matrices containing the information to be plotted and add the option matrix to the command. The easiest way to see how this is done is with an example, followed by details on each matrix. The commands

```
. matrix mnlbeta = (-.693, .693, . 347 \ . 347, -. 347, .693 )
. matrix mnlsd = (1, 2, 4)
. global mnlname = "x1 x2 x3"
. global mnlcatnm = "B C A"
. global mnldepnm "depvar"
. mlogplot, matrix std(uuu) vars(x1 x2 x3) packed
```

create the following plot:
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| x1 | Factor Change Scale Relative to Category A |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | . 5 | . 63 | . 79 | 1 | 1.26 |  | 1.59 | 2 |
|  | B |  |  | A |  | C |  |  |
| x2 |  |  |  | A |  |  |  | B |
| x3 |  |  |  | A |  | B |  | C |
|  | -69 | -. 46 | -23 | 0 | . 23 |  | . 46 | 69 |

## Options for using matrices with mlogplot

matrix indicates that the coefficients to be plotted are contained in matrices.
vars (variable-list) contains the names of the variables to be plotted. This list must contain names from mnlname, described next, but does not need to be in the same order as in mnlname. The list can contain the same name more than once and can select a subset of the names from mnlname.

## Global macros and matrices used by mlogplot

mnlname is a string containing the names of the variables corresponding to the columns of the matrix mnlbeta. For example, global mnlname $=$ "x1 x2 x3".
mnlbeta is a matrix with the $\beta$ 's, where element $(i, j)$ is the coefficient $\beta_{j, i \mid b}$. That is, rows $i$ are for different contrasts; columns $j$ are for variables. For example, matrix mnlbeta $=$ ( -.693 , $.693, .347 \backslash .347,-.347, .693$ ). Since constant terms are not plotted, they are not included in mnlbeta.
mnlsd is a vector with the standard deviations for the variables listed in mnlname. For example, matrix mnlsd $=(1,2,4)$. If you do not want to view standardized coefficients, this matrix can be made all 1 s .
mnlcatnm is a string with labels for the outcome categories with each label separated by a space. For example, global mnlcatnm $=$ "B C A". The first label corresponds to the first row of mnlbeta, the second to the second, and so on. The label for the base category is last.

## Example

Suppose that you want to compare the logit coefficients estimated from two groups, such as whites and nonwhites from the example used in this chapter. We begin by estimating the logit coefficients for whites:

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6.6 Interpretation

| (19se nomocc2, clear(1982 General Social Survey) |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Multinomial regression |  |  |  | Number of obs <br> LR chi2(8) <br> Prob > chi2 <br> Pseudo R2 |  | 309 |
|  |  |  |  | 154.60 |
|  |  |  |  | 0.0000 |
| Log likelihood $=-388.21313$ |  |  |  |  |  | 0.1660 |
| occ | Coef. | Std. Err. | z |  |  | $\mathrm{P}>\|\mathrm{z}\|$ | [95\% Con | Interval] |
| Menial |  |  |  |  |  |  |
| ed | -. 8307514 | . 1297238 | -6.40 |  |  | 0.000 | -1.085005 | -. 5764973 |
| exper | -. 0338038 | . 0192045 | -1.76 | 0.078 | -. 071444 | . 0038364 |
| _cons | 10.34842 | 1.779603 | 5.82 | 0.000 | 6.860465 | 13.83638 |
| BlueCol |  |  |  |  |  |  |
| ed | -. 9225522 | . 1085452 | -8.50 | 0.000 | -1.135297 | -. 7098075 |
| exper | -. 031449 | . 0150766 | -2.09 | 0.037 | -. 0609987 | -. 0018994 |
| _cons | 12.27337 | 1.507683 | 8.14 | 0.000 | 9.318368 | 15.22838 |
| Craft |  |  |  |  |  |  |
| ed | -. 6876114 | . 0952882 | -7.22 | 0.000 | -. 8743729 | -. 50085 |
| exper | -. 0002589 | . 0131021 | -0.02 | 0.984 | -. 0259385 | . 0254207 |
| _cons | 9.017976 | 1.36333 | 6.61 | 0.000 | 6.345897 | 11.69005 |
| WhiteCol |  |  |  |  |  |  |
| ed | -. 4196403 | . 0956209 | -4.39 | 0.000 | -. 6070539 | -. 2322268 |
| exper | . 0008478 | . 0147558 | 0.06 | 0.954 | -. 0280731 | . 0297687 |
| _cons | 4.972973 | 1.421146 | 3.50 | 0.000 | 2.187578 | 7.758368 |

(Outcome occ==Prof is the comparison group)
Next, we compute coefficients for nonwhites:

| Multinomial regression |  |  |  | Number of obs LR chi2(8) <br> Prob > chi2 <br> Pseudo R2 |  | $\begin{array}{r} 28 \\ 17.79 \\ 0.0228 \\ 0.2135 \end{array}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| occ | Coef. | Std. Err. | z | $P>\|z\|$ | [95\% Con | Interval] |
| Menial |  |  |  |  |  |  |
| ed | -. 7012628 | . 3331146 | -2.11 | 0.035 | -1.354155 | -. 0483701 |
| exper | -. 1108415 | . 0741488 | -1.49 | 0.135 | -. 2561705 | . 0344876 |
| _cons | 12.32779 | 6.053743 | 2.04 | 0.042 | . 4626714 | 24.19291 |
| BlueCol |  |  |  |  |  |  |
| ed | -. 560695 | . 3283292 | -1.71 | 0.088 | -1.204208 | . 0828185 |
| exper | -. 0261099 | . 0682348 | -0.38 | 0.702 | -. 1598477 | . 1076279 |
| _cons | 8.063397 | 6.008358 | 1.34 | 0.180 | -3.712768 | 19.83956 |
| Craft |  |  |  |  |  |  |
| ed | -. 882502 | . 3359805 | -2.63 | 0.009 | -1.541012 | -. 2239924 |
| exper | -. 1597929 | . 0744172 | -2.15 | 0.032 | -. 305648 | -. 0139378 |
| _cons | 16.21925 | 6.059753 | 2.68 | 0.007 | 4.342356 | 28.09615 |
| WhiteCol |  |  |  |  |  |  |
| ed | -. 5311514 | . 369815 | -1.44 | 0.151 | -1.255976 | . 1936728 |
| exper | -. 0520881 | . 0838967 | -0.62 | 0.535 | -. 2165227 | . 1123464 |
| _cons | 7.821371 | 6.805367 | 1.15 | 0.250 | -5.516904 | 21.15965 |

(Outcome occ==Prof is the comparison group)
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The two sets of coefficients for ed are placed in mnlbeta:

```
. matrix mnlbeta = (-.8307514, -.9225522, -.6876114, -.4196403 \ /*
>*/ -.7012628, -. 560695 , -. 882502 , -.5311514)
```

Notice that the rows of the matrix correspond to the variables (i.e., ed for whites and ed for nonwhites) since this was the easiest way to enter the coefficients. For mlogplot, the columns need to correspond to variables, so we transpose the matrix:
. matrix mnlbeta = mnlbeta-
We assign names to the columns using mnlname and to the rows using mnlcatnm (where the last element is the name of the reference category):

```
. global mnlname = "White NonWhite"
. global mnlcatnm = "Menial BlueCol Craft WhiteCol Prof"
```

We named the coefficients for ed for whites, White, and the coefficients for ed for nonwhites, NonWhite, since this will make the plot clearer. Next, we compute the standard deviation of ed:
. sum ed

| Variable | Obs | Mean | Std. Dev. | Min | Max |
| ---: | ---: | ---: | ---: | ---: | ---: |
| ed | 337 | 13.09496 | 2.946427 | 3 | 20 |

and enter the information into mnlsd :

```
matrix mnlsd = (2.946427,2.946427)
```

The same value is entered twice since we want to use the overall standard deviation in education for both groups. To create the plot, we use the command

```
. mlogplot, vars(White NonWhite) packed /*
> */ or matrix std(ss) note("Racial Differences in Effects of Education")
```

which leads to

Racial Differences in Effects of Education


Given the limitations of our dataset (e.g., there were only 28 cases in the logit for nonwhites) and our simple model, these results do not represent serious research on racial differences in occupational outcomes, but they do illustrate the flexibility of the mlogplot command.

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### 6.7 The conditional logit model

In the multinomial logit model, we estimate how individual-specific variables affect the likelihood of observing a given outcome. For example, we considered how individual characteristics such as education and experience affect a person's occupation. In the conditional logit model (CLM), alternative-specific variables that vary by outcome and individual are used to predict the outcome that is chosen. Consider the following examples:

- The dependent variable is the mode of transportation that an individual uses to get to work: car, bus (of any color), or train (e.g., Hensher 1986). We are interested in the effect of time: we think that how long it would take a person to get to work for a given alternative might affect her probability of selecting the alternative. We want to estimate the effect of time on the respondent's choice, but the amount of time for a given mode of travel is different for each respondent.
- The dependent variable is the type of car an individual purchases: European, American, or Japanese (see [R] clogit). We are interested in the effect of the number of dealerships in the buyer's city: we think that the more dealerships that sell cars of a given type, the more likely it is that buyers will purchase cars of that type. We want to estimate the effect of the number of dealerships, but the number of dealerships of each type in the buyer's city varies for different buyers (since they live in different cities).
- The dependent variable is which candidate a respondent votes for in a multiparty election (see, e.g., Alvarez and Nagler 1998). For example, in 1992, the major candidates were Clinton, Bush, and Perot. We are interested in the effect of the distance between the respondent and the candidate on particular issues (e.g., taxation, defense, gun control). We want to estimate how distance on different issues affects vote choice, but the distance from each candidate to the respondent varies for each respondent.

The conditional logit model (CLM) allows us to estimate how nominal outcomes are affected by characteristics of the outcomes that vary across individuals. In the CLM, the predicted probability of observing outcome $m$ is given by

$$
\begin{equation*}
\operatorname{Pr}\left(y_{i}=m \mid \mathbf{z}_{i}\right)=\frac{\exp \left(\mathbf{z}_{i m} \gamma\right)}{\sum_{j=1}^{J} \exp \left(\mathbf{z}_{i j} \gamma\right)} \text { for } m=1 \text { to } J \tag{6.2}
\end{equation*}
$$

where $\mathbf{z}_{i m}$ contains values of the independent variables for outcome $m$ for individual $i$. In the example of the CLM that we use, there are three choices for transportation: train, bus, and car. Suppose that we consider a single independent variable, where $z_{i m}$ is the amount of time it would take respondent $i$ to travel using mode of transportation $m$. Then, $\gamma$ is a single parameter indicating the effect of time on the probability of choosing one mode over another. In general, for each variable $z_{k}$, there are $J$ values of the variable for each individual, but only the single parameter $\gamma_{k}$.

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### 6.7.1 Data arrangement for conditional logit

Estimating the CLM in Stata requires that the data be arranged differently than for the other models we consider in this book, which we illustrate with an example from Greene and Hensher (1997). We have data on 152 groups of people traveling for their vacation, choosing between three modes of travel: train, bus or car. The group is indicated by the variable id. For each group of travelers, there are three rows of data corresponding to the three choices faced by each group. Accordingly, we have $N \times J=152 \times 3=456$ observations. For each group, the first observation is for the option of taking a train; the second for taking a bus; and the third for taking a car. Two dummy variables are used to indicate the mode of travel corresponding to a given row of data. Variable train is 1 if the observation contains information about taking the train, else train is 0 . bus is 1 if the observation contains information about taking a bus, else 0 . If both train and bus are 0 , the observation has information about driving a car. The actual choice made for a group is indicated with the dummy variable choice equal to 1 if the person took the mode of travel corresponding to a specific observation. For example, let's look at the first two groups (i.e., six records):
. use travel2.dta, clear
(Greene \& Hensher 1997 data on travel mode choice)
. list id mode train bus time invc choice in $1 / 6$, nodisplay

1. 

id
2.

Both groups traveled by car, as indicated by choice, which equals 1 in the third observation for each group. The variable time indicates how long a group thinks it will take them to travel using a given mode of transportation. Thus, time is an alternative-specific variable. For the first group, we can see that their trip would take 406 minutes by train, 452 minutes by bus, and 180 minutes by car. We might expect that the longer the time required, the less likely a person is to choose a particular mode of transportation. Similarly, the variable invc contains the in-vehicle cost of the trip: we might expect that the higher the cost of traveling by some mode, the less likely a person is to choose that mode. While many datasets with alternative-specific variables are already arranged in this way, later we talk about commands for setting up your data.

### 6.7.2 Estimating the conditional logit model

The syntax for clogit is

```
clogit depvar [indepvars] [weight] [if exp] [in range], group(varname)
    [level(\#) or ]
```

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## Options

group (varname) is required and specifies the variable that identifies the different groups of observations in the dataset. In our example, the group variable is id, which identifies the different respondents.
level(\#) specifies the level, in percent, for confidence intervals. The default is 95 percent. or requests that odds ratios $\exp \left(\widehat{\gamma}_{k}\right)$ be reported instead of $\widehat{\gamma}_{k}$.

## Example of the clogit model

For our transportation example, the dependent variable is choice, a binary variable indicating which mode of transportation was actually chosen. The independent variables include the $J-1$ dummy variables train and bus that identify each alternative mode of transportation and the alternativespecific variables time and invc. To estimate the model, we use the option group (id) to specify that the id variable identifies the groups in the sample:
. clogit choice train bus time invc, group(id) nolog
Conditional (fixed-effects) logistic regression
Log likelihood $=-80.961135$

### 6.7.3 Interpreting results from clogit

## Using odds ratios

In the results that we just obtained, the coefficients for time and invc are negative. This indicates that the longer it takes to travel by a given mode, the less likely that mode is to be chosen. Similarly, the more it costs, the less likely a mode is to be chosen. More specific interpretations are possible by using listcoef to transform the estimates into odds ratios:
. listcoef
clogit ( $\mathrm{N}=456$ ) : Factor Change in Odds
Odds of: 1 vs 0

| choice | b | z | $\mathrm{P}\rangle\|\mathrm{z}\|$ | $\mathrm{e}^{\wedge} \mathrm{b}$ |
| ---: | ---: | ---: | ---: | ---: |
| train | 2.67124 | 5.895 | 0.000 | 14.4579 |
| bus | 1.47233 | 3.674 | 0.000 | 4.3594 |
| time | -0.01915 | -7.812 | 0.000 | 0.9810 |
| invc | -0.04817 | -4.030 | 0.000 | 0.9530 |

[^40]For the alternative-specific variables, time and invc, the odds ratios are the multiplicative effect of a unit change in a given independent variable on the odds of any given mode of travel. For example,

Increasing the time of travel by one minute for a given mode of transportation decreases the odds of using that mode of travel by a factor of $.98(2 \%)$, holding the values for the other alternatives constant.

That is, if the time it take to travel by car increases by one minute while the time it takes to travel by train and bus remain constant, the odds of traveling by car decrease by 2 percent.

The odds ratios for the alternative-specific constants bus and train indicate the relative likelihood of selecting these alternatives versus travelling by car (the omitted category), assuming that cost and time are the same for all modes. For example,

If cost and time were equal, individuals would be 4.36 times more likely to travel by bus than by car, and they would be 14.46 times more likely to travel by train than by car.

## Using predicted probabilities

While the SPost commands prvalue, prtab, prcounts, and prgen do not work with clogit, you can use Stata's predict to compute predicted probabilities for each alternative for each group in the sample, where the predicted probabilities sum to 1 for each group. For example,

```
. predict prob
(option pc1 assumed; conditional probability for single outcome within group)
```

The message in parentheses indicates that by default conditional probabilities are being computed. To see what was done, let's list the variables in the model along with the predicted probabilities for the first group:
. list train bus time invc choice prob in $1 / 3$, nodisplay

| train | bus | time | invc | choice | prob |
| ---: | ---: | ---: | ---: | ---: | ---: |
| 1 | 0 | 406 | 31 | 0 | .0642477 |
| 0 | 1 | 452 | 25 | 0 | .0107205 |
| 0 | 0 | 180 | 10 | 1 | .9250318 |

The predicted probability of traveling by car (the option chosen) is .93 , while the predicted probability of traveling by train is only .06 . In this case, the choice corresponds to choosing the cheapest and quickest mode of transportation. If we consider another observation where train was chosen,

|  | train | bus | time | invc | choice | prob |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 16. | 1 | 0 | 385 | 20 | 1 | . 5493771 |
| 17. | 0 | 1 | 452 | 13 | 0 | . 0643481 |
| 18. | 0 | 0 | 284 | 12 | 0 | . 3862748 |

we see that the probability of choosing train was estimated to be .55 , while the probability of driving was .39. In this case, the respondent chose to travel by train even though it was neither cheapest nor fastest.

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### 6.7.4 Estimating the multinomial logit model using clogit*

Any multinomial logit model can be estimated using clogit by expanding the dataset (explained below) and respecifying the independent variables as a set of interactions. This is of more than academic interest for two reasons. First, it opens up the possibility of mixed models that include both individual-specific and alternative-specific variables (see Section 6.7.5). Second, it is possible to impose constraints on parameters in clogit that are not possible with mlogit (see mclgen and mclest by Hendrickx (2000) for further details).

## Setting up the data

To illustrate how this is done, we show how to use clogit to estimate the model of occupational attainment that we used to illustrate mlogit earlier in the chapter. The first step in rearranging the data is to create one record for each outcome:

```
. use nomocc2, clear
(1982 General Social Survey)
. gen id = _n
. expand 5
(1348 observations created)
```

The command gen id $=\_n$ creates an id number that is equal to the observation's row number. The expand $n$ command creates $n$ duplicate observations for each current observation. We need 5 observations per individual since there are 5 alternatives. Next, we sort the data so that observations with the same id value are adjacent. This is necessary so that we can use the mod (modulo) function to generate variable alt with values 1 through 5 corresponding to the codings for the different values of occ (our dependent variable):
. sort id
. gen alt $=\bmod \left(\_n, 5\right)$
. replace alt $=5$ if alt $==0$
(337 real changes made)
The values of alt are then used to create the four dummy variables for the different occupational types, leaving professional as the reference category (alt==5).

```
. gen menial = (alt==1)
. gen bluecol = (alt==2)
. gen craft = (alt==3)
. gen whitecol = (alt==4)
```

Finally, we generate a new variable choice that equals 1 if choice==alt and equals 0 otherwise. That is, choice indicates the occupation attained:

```
. gen choice = (occ==alt)
```

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For the first two individuals (which is 10 observations),

|  | id | menial | bluecol | craft | whitecol | choice |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1. | 1 | 1 | 0 | 0 | 0 | 1 |
| 2. | 1 | 0 | 1 | 0 | 0 | 0 |
| 3. | 1 | 0 | 0 | 1 | 0 | 0 |
| 4. | 1 | 0 | 0 | 0 | 1 | 0 |
| 5. | 1 | 0 | 0 | 0 | 0 | 0 |
| 6. | 2 | 1 | 0 | 0 | 0 | 1 |
| 7. | 2 | 0 | 1 | 0 | 0 | 0 |
| 8. | 2 | 0 | 0 | 1 | 0 | 0 |
| 9. | 2 | 0 | 0 | 0 | 1 | 0 |
| 10. | 2 | 0 | 0 | 0 | 0 | 0 |

## Creating interactions

Next, we create interactions by multiplying each of the four dummy variables by each of the independent variables white, ed, and exper:

```
. gen whiteXm = white*menial
. gen whiteXbc = white*bluecol
. gen whiteXc = white*craft
. gen whiteXwc = white*whitecol
. gen edXm = ed*menial
. gen edXbc = ed*bluecol
. gen edXc = ed*craft
. gen edXwc = ed*whitecol
. gen experXm = exper*menial
. gen experXbc = exper*bluecol
. gen experXc = exper*craft
. gen experXwc = exper*whitecol
```

To see what this does, we list the interactions with ed:

|  | menial | bluecol | craft | whitecol | edXm | edXbc | edXc | edXwc |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1. | 1 | 0 | 0 | 0 | 11 | 0 | 0 | 0 |
| 2. | 0 | 1 | 0 | 0 | 0 | 11 | 0 | 0 |
| 3. | 0 | 0 | 1 | 0 | 0 | 0 | 11 | 0 |
| 4. | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 11 |
| 5. | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

The trick is that the interaction of ed with the indicator variable for a given outcome is only equal to ed in the record corresponding to that outcome (see Long 1997, 181 for details on the mathematics involved).

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## Estimating the model

These interactions are then included as independent variables for clogit, where we order the terms in the same way as the output from mlogit on page 177.


Since the estimated parameters are identical to those produced by mlogit earlier, their interpretation is also the same.

### 6.7.5 Using clogit to estimate mixed models*

The MNLM has individual-specific variables, such as an individual's income. For individual-specific variables, the value of a variable does not differ across outcomes, but we want to estimate $J-1$ parameters for each individual-specific variable. The CLM has alternative-specific variables, such as the time it takes to get to work with a given mode of transportation. For alternative-specific variables, values varied across alternatives, but we estimate a single parameter for the effect of the variable. An interesting possibility is combining the two in a single model, referred to as a mixed model. For example, in explaining the choices people make about mode of transportation, we might want to know if wealthier people are more likely to drive than take the bus.

To create a mixed model, we combine the formulas for the MNLM and the CLM (see Long 1997, 178-182 and Powers and Xie 2000, 242-245):

$$
\begin{equation*}
\operatorname{Pr}\left(y_{i}=m \mid \mathbf{x}_{i}, \mathbf{z}_{i}\right)=\frac{\exp \left(\mathbf{z}_{i m} \gamma+\mathbf{x}_{i} \beta_{m}\right)}{\sum_{j=1}^{J} \exp \left(\mathbf{z}_{i j} \gamma+\mathbf{x}_{i} \beta_{j}\right)} \quad \text { where } \beta_{1}=0 \tag{6.3}
\end{equation*}
$$

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As in the CLM, $\mathbf{z}_{i m}$ contains values of the alternative-specific variables for outcome $m$ and individual $i$, and $\gamma$ contains the effects of the alternative-specific variables. As in the multinomial logit model, $\mathbf{x}_{i}$ contains individual-specific independent variables for individual $i$, and $\beta_{m}$ contains coefficients for the effects on outcome $m$ relative to the base category.

This mixed model can be estimated using clogit. For the alternative-specific variables the data are set up in the same way as for the conditional logit model above. For individual-specific variables, interaction terms are created as illustrated in the last section. To illustrate this approach, we add two individual-specific variables to our model of travel demand: hhinc is household income and psize is the number of people who will be traveling together. First we create the interactions:

```
. use travel2, clear
(Greene & Hensher 1997 data on travel mode choice)
. gen hincXbus = hinc*bus
. gen hincXtrn = hinc*train
. gen sizeXbus = psize*bus
. gen sizeXtrn = psize*train
```

Then we estimate the model with clogit:

| Conditional (fixed-effects) logistic regression | Number of obs | = | 456 |
| :---: | :---: | :---: | :---: |
|  | LR chi2(8) | = | 178.97 |
|  | Prob > chi2 | = | 0.0000 |
| Log likelihood = -77.504846 | Pseudo R2 | = | 0.5359 |


| choice | Coef. | Std. Err. | z | P>\|z| | [95\% Conf. Interval] |  |
| ---: | ---: | ---: | :---: | ---: | ---: | ---: |
| train | 3.499641 | .7579659 | 4.62 | 0.000 | 2.014055 | 4.985227 |
| bus | 2.486465 | .8803643 | 2.82 | 0.005 | .7609827 | 4.211947 |
| time | -.0185035 | .0025035 | -7.39 | 0.000 | -.0234103 | -.0135966 |
| invc | -.0402791 | .0134851 | -2.99 | 0.003 | -.0667095 | -.0138488 |
| hincXbus | -.0080174 | .0200322 | -0.40 | 0.689 | -.0472798 | .031245 |
| hincXtrn | -.0342841 | .0158471 | -2.16 | 0.031 | -.0653438 | -.0032243 |
| sizeXbus | -.5141037 | .4007012 | -1.28 | 0.199 | -1.299464 | .2712563 |
| sizeXtrn | -.0038421 | .3098074 | -0.01 | 0.990 | -.6110533 | .6033692 |

To interpret these results, we can again transform the coefficients into odds ratios using listcoef:
. listcoef
clogit ( $N=456$ ) : Factor Change in Odds
Odds of: 1 vs 0

| choice | b | z | $\mathrm{P}>\|\mathrm{z}\|$ | $\mathrm{e}^{\wedge} \mathrm{b}$ |
| ---: | ---: | ---: | ---: | ---: |
| train | 3.49964 | 4.617 | 0.000 | 33.1036 |
| bus | 2.48647 | 2.824 | 0.005 | 12.0187 |
| time | -0.01850 | -7.391 | 0.000 | 0.9817 |
| invc | -0.04028 | -2.987 | 0.003 | 0.9605 |
| hincXbus | -0.00802 | -0.400 | 0.689 | 0.9920 |
| hincXtrn | -0.03428 | -2.163 | 0.031 | 0.9663 |
| sizeXbus | -0.51410 | -1.283 | 0.199 | 0.5980 |
| sizeXtrn | -0.00384 | -0.012 | 0.990 | 0.9962 |

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The interpretation for the individual-specific variables is the same as the interpretation of odds ratios in the MNLM. For example, a unit increase in income decreases the odds of traveling by train versus traveling by car by a factor of .97 . Similarly, each additional member of the travelling party decreases the odds of traveling by bus versus traveling by car by a factor of .60 .

Note We have only considered the conditional logit model in the context of choices among an unordered set of alternatives. The possible uses of clogit are much broader. The Stata Reference Manual entry for clogit contains additional examples and references.

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# REGRESSION MODELS FOR CATEGORICAL DEPENDENT VARIABLES USING STATA 

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## 7 Models for Count Outcomes

Count variables indicate how many times something has happened. While the use of regression models for counts is relatively recent, even a brief survey of recent applications illustrates how common these outcomes are and the importance of this class of models. Examples include the number of patients, hospitalizations, daily homicides, international conflicts, beverages consumed, industrial injuries, new companies, and arrests by police, to name only a few.

While the linear regression model has often been applied to count outcomes, this can result in inefficient, inconsistent, and biased estimates. Even though there are situations in which the LRM provides reasonable results, it is much safer to use models specifically designed for count outcomes. Four such models are considered in this chapter: Poisson regression (PRM), negative binomial regression (NBRM), and variations of these models for zero-inflated counts (ZIP and ZINB). As with earlier chapters, we begin with a quick review of the statistical model, consider issues of testing and fit, and then discuss methods of interpretation. These discussions are intended as a review for those who are familiar with the models. For further details, see Long (1997) or Cameron and Trivedi (1998, the definitive work in this area). As always, you can obtain sample do-files and data files by downloading the spostst4 package (see Chapter 1 for details).

### 7.1 The Poisson distribution

The univariate Poisson distribution is fundamental to understanding regression models for counts. Accordingly, we start by exploring this distribution. Let $y$ be a random variable indicating the number of times an event has occurred. If $y$ has a Poisson distribution, then

$$
\operatorname{Pr}(y \mid \mu)=\frac{e^{-\mu} \mu^{y}}{y!} \quad \text { for } y=0,1,2, \ldots
$$

where $\mu>0$ is the sole parameter defining the distribution. The easiest way to get a sense of this distribution is to compare the plot of the predicted probability for different values of the rate parameter $\mu$ (labeled as mu in the graph):

[^41]

The plot illustrates four characteristics of the Poisson distribution that are important for understanding regression models for counts:

1. $\mu$ is the mean of the distribution. As $\mu$ increases, the mass of the distribution shifts to the right.
2. $\mu$ is also the variance. Thus, $\operatorname{Var}(y)=\mu$, which is known as equidispersion. In real data, many count variables have a variance greater than the mean, which is called overdispersion.
3. As $\mu$ increases, the probability of a zero count decreases. For many count variables, there are more observed zeros than predicted by the Poisson distribution.
4. As $\mu$ increases, the Poisson distribution approximates a normal distribution. This is shown by the distribution for $\mu=10.5$.

### 7.1.1 Fitting the Poisson distribution with poisson

To illustrate the models in this chapter, we use data from Long (1990) on the number of publications produced by Ph.D. biochemists. The variables considered are

```
. use couart2, clear
(Academic Biochemists / S Long)
```

(Continued on next page)

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| ```Contains data obs: vars: size:``` | $\begin{array}{r} \text { from cou } \\ 915 \\ 6 \\ 14,640 \end{array}$ | uart2.dta <br> (99.7\% of | ory free) | Academic Biochemists / S Long <br> 15 Jan 2001 15:23 <br> (_dta has notes) |
| :---: | :---: | :---: | :---: | :---: |
| variable name | storage type | display <br> format | value <br> label | variable label |
| art | byte | $\% 9.0 \mathrm{~g}$ |  | Articles in last 3 yrs of PhD |
| fem | byte | \%9.0g | sexlbl | Gender: 1=female 0=male |
| mar | byte | $\% 9.0 \mathrm{~g}$ | marlbl | Married: 1=yes 0=no |
| kid5 | byte | \%9.0g |  | Number of children < 6 |
| phd | float | \%9.0g |  | PhD prestige |
| ment | float | \%9.0g |  | Article by mentor in last 3 yrs |

Sorted by: art
. summarize

| Variable | Obs | Mean | Std. Dev. | Min | Max |
| ---: | ---: | ---: | ---: | ---: | ---: |
| art | 915 | 1.692896 | 1.926069 | 0 | 19 |
| fem | 915 | .4601093 | .4986788 | 0 | 1 |
| mar | 915 | .6622951 | .473186 | 0 | 1 |
| kid5 | 915 | .495082 | .76488 | 0 | 3 |
| phd | 915 | 3.103109 | .9842491 | .755 | 4.62 |
| ment | 915 | 8.767212 | 9.483915 | 0 | 76.99998 |

A useful place to begin when analyzing a count outcome is to compare the observed distribution to a Poisson distribution that has the same mean. The command poisson estimates the Poisson regression model that is presented in Section 7.2. Here we use poisson without any independent variables in order to fit a univariate Poisson distribution with a mean equal to that of our outcome variable art. That is, we estimate the model:

$$
\mu=\exp \left(\beta_{0}\right)
$$

The results are:
• poisson art, nolog
Poisson regression
Log likelihood $=-1742.5735$

Since $\widehat{\beta}_{0}=.5264, \widehat{\mu}=\exp (.5264)=1.6929$, which is the same as the estimated mean of art obtained with summarize earlier. To compute the observed probabilities for each count and the predictions from counts drawn from a Poisson distribution with this mean, we use prcounts, which is part of SPost.

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### 7.1.2 Computing predicted probabilities with prcounts

For poisson and other models for count outcomes, prcounts extends the features of predict by computing the predicted rate and predicted probabilities of each count from 0 to the specified maximum for every observation. Optionally, prcounts creates variables with observed probabilities and sample averages of predicted probabilities for each count; these variables can be used to construct plots to assess the fit of count models, as shown in Section 7.5.1.

## Syntax

prcounts name [if exp] [in range] [, max(max) plot]
where name is a prefix to the new variables that are generated. name cannot be the name of an existing variable.

## Options

$\max (\#)$ is the maximum count for which predicted probabilities should be computed. The default is 9. For example, with $\max (2)$ predictions for $\operatorname{Pr}(y=0), \operatorname{Pr}(y=1)$, and $\operatorname{Pr}(y=2)$ are computed.
plot specifies that variables for plotting expected counts should be generated. If this option is not used, only predictions for individual observations are computed.
if and in restrict the sample for which predictions are made. By default, prcounts computes predicted values for all observations in memory. To restrict the computations to the estimation sample, you should add the condition: if e(sample)==1.

## Variables generated

The following variables are generated, where name represents the prefix specified with prcounts. $y$ is the count variable and each prediction is conditional on the independent variables in the regression. If there are no independent variables, as in our current example, the values are unconditional.
namerate predicted rate or count $E(y)$.
nameprk predicted probability $\operatorname{Pr}(y=k)$ for $k=0$ to max. By default, max $=9$.
nameprgt predicted probability $\operatorname{Pr}(y>\max )$.
namecuk predicted cumulative probability $\operatorname{Pr}(y \leq k)$ for $k=0$ to max. By default, $\max =9$.
When the plot option is specified, max +1 observations (for counts 0 through max) are generated for the following variables:
nameval the value $k$ of the count $y$ ranging from 0 to max.
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nameobeq the observed probability $\operatorname{Pr}(y=k)$. These values are the same as the ones you could obtain by running tabulate on the count variable (e.g., tabulate art).
nameoble the observed cumulative probability $\operatorname{Pr}(y \leq k)$.
namepreq the average predicted probability $\operatorname{Pr}(y=k)$.
nameprle the average predicted cumulative probability $\operatorname{Pr}(y \leq k)$.

Which observations are used to compute the averages? By default, prcounts computes averages for all observations in memory, which could include observations that were not used in the estimation. For example, if your model was poisson art if fem==1, then the averages computed by prcounts would be based on all observations, including those where fem is not 1 . To restrict the averages to the sample used in estimation, you need to add the condition if e $($ sample $)==1$. For example, prcounts isfem if e(sample)==1, plot.

### 7.1.3 Comparing observed and predicted counts with prcounts

If the plot option was used with prcounts, it is simple to construct a graph that compares the observed probabilities for each value of the count variable to the predicted probabilities from fitting the Poisson distribution. For example,

| . prcounts psn, plot max(9) |  |  |  |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: |
| . label var psnobeq "Observed Proportion" |  |  |  |  |  |
| . label var psnpreq "Poisson Prediction" |  |  |  |  |  |
| . label var psnval "\# of Articles" |  |  |  |  |  |
| . list psnval psnobeq psnpreq in $1 / 10$ |  |  |  |  |  |
| psnval |  |  |  | psnobeq | psnpreq |
| 1. |  |  |  |  |  |
| 2. |  |  |  |  |  |

The listed values are the observed and predicted probabilities for observing scientists with 0 through 9 publications. These can be plotted with graph:

[^42]This leads to the following graph:


The graph clearly shows that the fitted Poisson distribution (represented by $\triangle$ 's) under-predicts 0 s and over-predicts counts 1,2 , and 3 . This pattern of over- and under-prediction is characteristic of fitting a count model that does not take into account heterogeneity among sample members in their rate $\mu$. Since fitting the univariate Poisson distribution assumes that all scientists have the same rate of productivity, which is clearly unrealistic, our next step is to allow heterogeneity in $\mu$ based on observed characteristics of the scientists.

Advanced: plotting Poisson distributions Earlier we plotted the Poisson distribution for four values of $\mu$. The trick to doing this is to construct artificial data with a given mean rate of productivity. Here are the commands we used to generate the graph on page 224:
. clear
. set obs 25
. gen ya $=.8$
. poisson ya, nolog
. prcounts pya, plot max(20)
. gen $\mathrm{yb}=1.5$

- poisson yb, nolog
. prcounts pyb, plot max(20)
. gen yc $=2.9$
- poisson yc, nolog
. prcounts pyc, plot max(20)

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```
. gen yd = 10.5
. poisson yd, nolog
. prcounts pyd, plot max(20)
. label var pyapreq "mu=0.8"
. label var pybpreq "mu=1.5"
. label var pycpreq "mu=2.9"
. label var pydpreq "mu=10.5"
. label var pyaval "y=# of Events"
. set textsize 125
. graph pyapreq pybpreq pycpreq pydpreq pyaval, c(llll) gap(3) /*
*/ 12("Probability") yscale(0,.5) ylabel(0 .1 to .5) xlabel(0 2 to 20)
```


### 7.2 The Poisson regression model

The Poisson regression model (PRM) extends the Poisson distribution by allowing each observation to have a different value of $\mu$. More formally, the PRM assumes that the observed count for observation $i$ is drawn from a Poisson distribution with mean $\mu_{i}$, where $\mu_{i}$ is estimated from observed characteristics. This is sometimes referred to as incorporating observed heterogeneity, and leads to the structural equation:

$$
\mu_{i}=E\left(y_{i} \mid \mathbf{x}_{i}\right)=\exp \left(\mathbf{x}_{i} \beta\right)
$$

Taking the exponential of $\mathbf{x} \beta$ forces $\mu$ to be positive, which is necessary since counts can only be 0 or positive. To see how this works, consider the PRM with a single independent variable, $\mu=\exp (\alpha+\beta x)$, which can be plotted as ${ }^{1}$


[^43]In this graph the mean $\mu$, shown by the curved line, increases as $x$ increases. For each value of $\mu$, the distribution around the mean is shown by the dots which should be thought of as coming out of the page and which represent the probability of each count. Interpretation of the model involves assessing how changes in the independent variables affect the conditional mean and the probabilities of various counts. Details on interpretation are given after we consider estimation.

### 7.2.1 Estimating the PRM with poisson

The Poisson regression model is estimated with the command:

```
poisson depvar [indepvars] [weight] [if exp] [in range] [, level(#) nolog
    table cluster(varname) irr exposure(varname) robust ]
```

In our experience, poisson converges quickly and difficulties are rarely encountered.

## Variable lists

depvar is the dependent variable. poisson does not require this to be an integer. But, if you have noninteger values, you obtain the warning:

Note: you are responsible for interpretation of non-count dep variable.
indepvars is a list of independent variables. If indepvars is not included, a model with only an intercept is estimated, which corresponds to fitting a univariate Poisson distribution, as shown in the last section.

## Specifying the estimation sample

if and in qualifiers can be used to restrict the estimation sample. For example, if you want to estimate a model for only women, you could specify poisson art mar kid5 phd ment if $\mathrm{fem}==1$.

Listwise Deletion Stata excludes observations in which there are missing values for any of the variables in the model. Accordingly, if two models are estimated using the same data but have different independent variables, it is possible to have different samples. We recommend that you use mark and markout (discussed in Chapter 3) to explicitly remove observations with missing data.

## Weights

poisson can be used with fweights, pweights, and iweights. See Chapter 3 for details.
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## Options

nolog suppresses the iteration history.
level (\#) specifies the level of the confidence interval for estimated parameters. By default, a $95 \%$ interval is used. You can change the default level, say, to a $90 \%$ interval, with the command set level 90.
irr reports estimated coefficients transformed to incidence rate ratios defined as $\exp (\beta)$. These are discussed in Section 7.2.3.
exposure (varname) specifies a variable indicating the amount of time during which an observation was "at risk" of the event occurring. Details are given in an example below.
robust requests that robust variance estimates be used. See Chapter 3 for details.
cluster (varname) specifies that the observations are independent across the groups specified by unique values of varname but not necessarily within the groups. When cluster() is specified, robust standard errors are automatically used. See Chapter 3 for details.

### 7.2.2 Example of estimating the PRM

If scientists who differ in their rates of productivity are combined, the univariate distribution of articles will be overdispersed (i.e., the variance is greater than the mean). Differences among scientists in their rates of productivity could be due to factors such as gender, marital status, number of young children, prestige of the graduate program, and the number of articles written by a scientist's mentor. To account for these differences, we add these variables as independent variables:

| Poisson regression |  |  |  | Number of obs <br> LR chi2(5) <br> Prob > chi2 <br> Pseudo R2 |  | $\begin{array}{r} 915 \\ .83 .03 \\ .0000 \\ .0525 \end{array}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| art | Coef. | Std. Err. | z | $P>\|z\|$ | [95\% | Interval] |
| fem | -. 2245942 | . 0546138 | -4.11 | 0.000 | -. 3316 | -. 1175532 |
| mar | . 1552434 | . 0613747 | 2.53 | 0.011 | . 0349 | . 2755356 |
| kid5 | -. 1848827 | . 0401272 | -4.61 | 0.000 | -. 2635 | -. 1062349 |
| phd | . 0128226 | . 0263972 | 0.49 | 0.627 | -. 038 | . 0645601 |
| ment | . 0255427 | . 0020061 | 12.73 | 0.000 | . 0216 | . 0294746 |
| _cons | . 3046168 | . 1029822 | 2.96 | 0.003 | . 1027 | . 5064581 |

The way in which you interpret a count model depends on whether you are interested in the expected value of the count variable or in the distribution of counts. If interest is in the expected count, several methods can be used to compute the change in the expectation for a change in an independent variable. If interest is in the distribution of counts or perhaps just the probability of a specific count, the probability of a count for a given level of the independent variables can be computed. Each of these methods is now considered.

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### 7.2.3 Interpretation using the rate $\mu$

In the PRM,

$$
\mu=E(y \mid \mathbf{x})=\exp (\mathbf{x} \beta)
$$

Changes in $\mu$ for changes in the independent variable can be interpreted in a variety of ways.

## Factor Change in $E(y \mid x)$

Perhaps the most common method of interpretation is the factor change in the rate. If we define $E\left(y \mid \mathbf{x}, x_{k}\right)$ as the expected count for a given $\mathbf{x}$ where we explicitly note the value of $x_{k}$, and define $E\left(y \mid \mathbf{x}, x_{k}+\delta\right)$ as the expected count after increasing $x_{k}$ by $\delta$ units, then

$$
\begin{equation*}
\frac{E\left(y \mid \mathbf{x}, x_{k}+\delta\right)}{E\left(y \mid \mathbf{x}, x_{k}\right)}=e^{\beta_{k} \delta} \tag{7.1}
\end{equation*}
$$

Therefore, the parameters can be interpreted as

For a change of $\delta$ in $x_{k}$, the expected count increases by a factor of $\exp \left(\beta_{k} \times \delta\right)$, holding all other variables constant.

For example,

Factor change: For a unit change in $x_{k}$, the expected count changes by a factor of $\exp \left(\beta_{k}\right)$, holding all other variables constant.

Standardized factor change: For a standard deviation change in $x_{k}$, the expected count changes by a factor of $\exp \left(\beta_{k} \times s_{k}\right)$, holding all other variables constant.

Incidence Rate Ratio In some discussions of count models, $\mu$ is referred to as the incidence rate and Equation 7.1 for $\delta=1$ is called the incidence rate ratio. These coefficients can be computed by adding the option irr to the estimation command. Alternatively, they are computed with our listcoef, which is illustrated below.

## Percent change in $E(\mathbf{y} \mid \mathbf{x})$

Alternatively, the percentage change in the expected count for a $\delta$ unit change in $x_{k}$, holding other variables constant, can be computed as

$$
100 \times \frac{E\left(y \mid \mathbf{x}, x_{k}+\delta\right)-E\left(y \mid \mathbf{x}, x_{k}\right)}{E\left(y \mid \mathbf{x}, x_{k}\right)}=100 \times\left[\exp \left(\beta_{k} \times \delta\right)-1\right]
$$

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### 7.2 The Poisson regression model

## Example of factor and percent change

Factor change coefficients can be computed using listcoef:
. poisson art fem mar kid5 phd ment, nolog
(output omitted)

- listcoef fem ment, help
poisson (N=915): Factor Change in Expected Count
Observed SD: 1.926069

| art | $b$ | $z$ | $P>\|z\|$ | $e^{\wedge} b$ | $e^{\wedge} b S t d X$ | SDofX |
| ---: | ---: | :---: | :---: | :---: | :---: | ---: |
| fem | -0.22459 | -4.112 | 0.000 | 0.7988 | 0.8940 | 0.4987 |
| ment | 0.02554 | 12.733 | 0.000 | 1.0259 | 1.2741 | 9.4839 |

b = raw coefficient
$z=z$-score for test of $b=0$
$\mathrm{P}>|z|=\mathrm{p}$-value for z -test
$e^{\wedge} b=\exp (b)=$ factor change in expected count for unit increase in $X$
$e^{\wedge} b S t d X=\exp (b * S D$ of $X)=$ change in expected count for $S D$ increase in $X$
SDof $X=$ standard deviation of $X$
For example, the coefficients for fem and ment can be interpreted as
Being a female scientist decreases the expected number of articles by a factor of .80 , holding all other variables constant.
For a standard deviation increase in the mentor's productivity, roughly 9.5 articles, a scientist's mean productivity increases by a factor of 1.27 , holding other variables constant.

To compute percent change, we add the option percent:

```
. listcoef fem ment, percent help
poisson (N=915): Percentage Change in Expected Count
    Observed SD: 1.926069
```

| art | b | z | $\mathrm{P}>\|\mathrm{z}\|$ | $\%$ | \%StdX | SDofX |
| ---: | :---: | :---: | :---: | :---: | ---: | ---: |
| fem | -0.22459 | -4.112 | 0.000 | -20.1 | -10.6 | 0.4987 |
| ment | 0.02554 | 12.733 | 0.000 | 2.6 | 27.4 | 9.4839 |

        b = raw coefficient
        \(z=z\)-score for test of \(b=0\)
    \(\mathrm{P}>|\mathrm{z}|=\mathrm{p}\)-value for z -test
        \(\%=\) percent change in expected count for unit increase in X
    \(\% S t d X=\) percent change in expected count for \(S D\) increase in \(X\)
    SDofX = standard deviation of \(X\)
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For example, the percent change coefficients for fem and ment can be interpreted as

Being a female scientist decreases the expected number of articles by 20 percent, holding all other variables constant.

For every additional article by the mentor, a scientist's predicted mean productivity increases by 2.6 percent, holding other variables constant.

The standardized percent change coefficient can be interpreted as

For a standard deviation increase in the mentor's productivity, a scientist's mean productivity increases by 27 percent, holding all other variables constant.

## Marginal change in $E(\mathbf{y} \mid \mathbf{x})$

Another method of interpretation is the marginal change in $E(y \mid \mathbf{x})$ :

$$
\frac{\partial E(y \mid \mathbf{x})}{\partial x_{k}}=E(y \mid \mathbf{x}) \beta_{k}
$$

For $\beta_{k}>0$, the larger the current value of $E(y \mid \mathbf{x})$, the larger the rate of change; for $\beta_{k}<0$, the smaller the rate of change. The marginal with respect of $x_{k}$ depends on both $\beta_{k}$ and $E(y \mid \mathbf{x})$. Thus, the value of the marginal depends on the levels of all variables in the model. In practice, this measure is often computed with all variables held at their means.

## Example of marginal change using prchange

Since the marginal is not appropriate for binary independent variables, we only request the change for the continuous variables phd and ment. The marginal effects are in the column that is labeled MargEfct:
. prchange phd ment, rest(mean)
poisson: Changes in Predicted Rate for art

|  | n->max | $0->1$ | -+1/2 | -+sd/2 | MargEfct |
| :---: | :---: | :---: | :---: | :---: | :---: |
| phd | 0.0794 | 0.0200 | 0.0206 | 0.0203 | 0.0206 |
| ment | 7.9124 | 0.0333 | 0.0411 | 0.3910 | 0.0411 |
| $\exp (\mathrm{xb})$ | 1.6101 |  |  |  |  |
|  | fem | mar | kid5 | phd | ment |
| $\mathrm{x}=$ | . 460109 | . 662295 | . 495082 | 3.10311 | 8.76721 |
| $s d(x)=$ | . 498679 | . 473186 | . 76488 | . 984249 | 9.48392 |

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## Example of marginal change using mfx compute

By default, mfx compute computes the marginal change with variables held at their means:

```
. mfx compute
```

Marginal effects after poisson
$\mathrm{y}=$ predicted number of events (predict)
$=1.6100936$

| variable | dy/dx | Std. Err. | z | $\mathrm{P}>\|\mathrm{z}\|$ | $[$ | $95 \%$ | C.I. $]$ | X |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | :---: |
| fem* | -.3591461 | .08648 | -4.15 | 0.000 | -.528643 | -.189649 | .460109 |  |
| mar* | .2439822 | .09404 | 2.59 | 0.009 | .059671 | .428293 | .662295 |  |
| kid5 | -.2976785 | .06414 | -4.64 | 0.000 | -.423393 | -.171964 | .495082 |  |
| phd | .0206455 | .04249 | 0.49 | 0.627 | -.062635 | .103926 | 3.10311 |  |
| ment | .0411262 | .00317 | 12.97 | 0.000 | .034912 | .04734 | 8.76721 |  |

(*) $d y / d x$ is for discrete change of dummy variable from 0 to 1
The estimated marginals for phd and ment match those given above. For dummy variables, mfx computes the discrete change as the variable changes from 0 to 1 , a topic we now consider.

## Discrete change in $E(\mathbf{y} \mid \mathbf{x})$

It is also possible to compute the discrete change in the expected count for a change in $x_{k}$ from $x_{S}$ to $x_{E}$,

$$
\frac{\Delta E(y \mid \mathbf{x})}{\Delta x_{k}}=E\left(y \mid \mathbf{x}, x_{k}=x_{E}\right)-E\left(y \mid \mathbf{x}, x_{k}=x_{S}\right)
$$

which can be interpreted as

For a change in $x_{k}$ from $x_{S}$ to $x_{E}$, the expected count changes by $\Delta E(y \mid \mathbf{x}) / \Delta x_{k}$, holding all other variables at the specified values.

As was the case in earlier chapters, the discrete change can be computed in a variety of ways depending on your purpose:

1. The total possible effect of $x_{k}$ is found by letting $x_{k}$ change from its minimum to its maximum.
2. The effect of a binary variable $x_{k}$ is computed by letting $x_{k}$ change from 0 to 1 . This is the quantity computed by mfx compute for binary variables.
3. The uncentered effect of a unit change in $x_{k}$ at the mean is computed by changing from $\bar{x}_{k}$ to $\bar{x}_{k}+1$. The centered discrete change is computed by changing from $\left(\bar{x}_{k}-1 / 2\right)$ to $\left(\bar{x}_{k}+1 / 2\right)$.
4. The uncentered effect of a standard deviation change in $x_{k}$ at the mean is computed by changing from $\bar{x}_{k}$ to $\bar{x}_{k}+s_{k}$. The centered change is computed by changing from $\left(\bar{x}_{k}-s_{k} / 2\right)$ to $\left(\bar{x}_{k}+s_{k} / 2\right)$.

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5. The uncentered effect of a change of $\delta$ units in $x_{k}$ from $\bar{x}_{k}$ to $\bar{x}_{k}+\delta$. The centered change is computed by changing from $\left(\bar{x}_{k}-\delta / 2\right)$ to $\left(\bar{x}_{k}+\delta / 2\right)$.

Discrete changes are computed with prchange. By default changes are computed centered around the values specified with x() and rest (). To compute changes that begin at the specified values, such as a change from $\bar{x}_{k}$ to $\bar{x}_{k}+1$, you must specify the uncentered option. By default, prchange computes results for changes in the independent variables of 1 unit and a standard deviation. With the delta(\#) option, you can request changes of \# units. When using discrete change, remember that the magnitude of the change in the expected count depends on the levels of all variables in the model.

## Example of discrete change using prchange

In this example, we set all variables to their mean:

```
. prchange fem ment, rest(mean)
poisson: Changes in Predicted Rate for art
```

|  | min->max | $0->1$ | $-+1 / 2$ | -+ sd/2 | MargEfct |
| ---: | ---: | ---: | ---: | ---: | ---: |
| fem | -0.3591 | -0.3591 | -0.3624 | -0.1804 | -0.3616 |
| ment | 7.9124 | 0.0333 | 0.0411 | 0.3910 | 0.0411 |

$\exp (x b): \quad 1.6101$

|  | fem | mar | kid5 | phd | ment |
| ---: | ---: | ---: | ---: | ---: | ---: |
| $x=$ | .460109 | .662295 | .495082 | 3.10311 | 8.76721 |
| $\operatorname{sd}(\mathrm{x})=$ | .498679 | .473186 | .76488 | .984249 | 9.48392 |

Examples of interpretation are

Being a female scientist decreases the expected productivity by .36 articles, holding all other variables at their means.
A standard deviation increase in the mentor's articles increases the scientist's rate of productivity by .39 , holding all other variables at their mean.

To illustrate the use of the uncentered option, suppose that we want to know the effect of a change from 1 to 2 young children:

```
. prchange kid5, uncentered x(kid5=1)
poisson: Changes in Predicted Rate for art
\begin{tabular}{lrrrrr} 
& min->max & \(0->1\) & +1 & + sd & MargEfct \\
kid5 & -0.7512 & -0.2978 & -0.2476 & -0.1934 & -0.2711
\end{tabular}
exp(xb): 1.4666
\begin{tabular}{rrrrrr} 
& fem & mar & kid5 & phd & ment \\
\(\mathrm{x}=\) & .460109 & .662295 & 1 & 3.10311 & 8.76721 \\
\(\mathrm{sd}(\mathrm{x})=\) & .498679 & .473186 & .76488 & .984249 & 9.48392
\end{tabular}
```

The rate of productivity decreases by .25 as the number of young children increases from 1 to 2 . To examine the effect of a change from 1 to 3 children, we add the delta() option:

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```
. prchange kid5, uncentered x(kid5=1) delta(2)
poisson: Changes in Predicted Rate for art
(Note: delta = 2)
\begin{tabular}{lrrrrr} 
& min->max & \(0->1\) & +delta & +sd & MargEfct \\
kid5 & -0.7512 & -0.2978 & -0.4533 & -0.1934 & -0.2711
\end{tabular}
exp(xb): 1.4666
\begin{tabular}{rrrrrr} 
& fem & mar & kid5 & phd & ment \\
\(\mathrm{x}=\) & .460109 & .662295 & 1 & 3.10311 & 8.76721 \\
\(\operatorname{sd}(\mathrm{x})=\) & .498679 & .473186 & .76488 & .984249 & 9.48392
\end{tabular}
```

The results shows a decrease of .45 in the expected number of articles as the number of young children increases from 1 to 3 .

### 7.2.4 Interpretation using predicted probabilities

The estimated parameters can also be used to compute predicted probabilities using the following formula:

$$
\widehat{\operatorname{Pr}}(y=m \mid \mathbf{x})=\frac{e^{-\mathbf{x} \widehat{\beta}}(\mathbf{x} \widehat{\beta})^{m}}{m!}
$$

Predicted probabilities at specified values can be computed using prvalue. Predictions at the observed values for all observations can be made using prcounts, or prgen can be used to compute predictions that can be plotted. These commands are now illustrated.

## Example of predicted probabilities using prvalue

prvalue computes predicted probabilities for values of the independent variables specified with x () and rest (). For example, to compare the predicted probabilities for married and unmarried women without young children, we first compute the predicted counts for single women without children by specifying $x$ (mar= 0 fem=1 kid5=0) and rest (mean). We suppress the output with quietly but save the results for later use:

```
. * single women without children
. quietly prvalue, x(mar=0 fem=1 kid5=0) rest(mean) save
```

Next, we compute the predictions for married women without children and use the dif option to compare these results to those we just saved:

```
. * compared to married women without children
. prvalue, x(mar=1 fem=1 kid5=0) rest(mean) dif
poisson: Change in Predictions for art
Predicted rate: 1.6471 95% CI [1.4895 , 1.8213]
        Saved: 1.4102
    Difference: . }2368
```

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Predicted probabilities:

|  |  | Current | Saved | Difference |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\operatorname{Pr}(\mathrm{y}=0 \mid \mathrm{x})$ : |  | 0.1926 | 0.2441 | -0.0515 |  |
| $\operatorname{Pr}(\mathrm{y}=1 \mid \mathrm{x})$ : |  | 0.3172 | 0.3442 | -0.0270 |  |
| $\operatorname{Pr}(\mathrm{y}=2 \mid \mathrm{x})$ : |  | 0.2613 | 0.2427 | 0.0186 |  |
| $\operatorname{Pr}(\mathrm{y}=3 \mid \mathrm{x})$ : |  | 0.1434 | 0.1141 | 0.0293 |  |
| $\operatorname{Pr}(\mathrm{y}=4 \mid \mathrm{x})$ : |  | 0.0591 | 0.0402 | 0.0188 |  |
| $\operatorname{Pr}(\mathrm{y}=5 \mid \mathrm{x})$ : |  | 0.0195 | 0.0113 | 0.0081 |  |
| $\operatorname{Pr}(\mathrm{y}=6 \mid \mathrm{x})$ : |  | 0.0053 | 0.0027 | 0.0027 |  |
| $\operatorname{Pr}(\mathrm{y}=7 \mid \mathrm{x})$ : |  | 0.0013 | 0.0005 | 0.0007 |  |
| $\operatorname{Pr}(\mathrm{y}=8 \mid \mathrm{x})$ : |  | 0.0003 | 0.0001 | 0.0002 |  |
| $\operatorname{Pr}(\mathrm{y}=9 \mid \mathrm{x})$ : |  | 0.0000 | 0.0000 | 0.0000 |  |
|  | fem | mar | kid5 | phd | ment |
| Current= | 1 | 1 | 0 | 3.1031093 | 8.7672131 |
| Saved= | 1 | 0 | 0 | 3.1031093 | 8.7672131 |
| Diff= | 0 | 1 | 0 | 0 | 0 |

The results show that married women are less likely to have 0 or 1 publications, and more likely to have higher counts. Overall, their rate of productivity is .24 higher.

To examine the effects of the number of young children, we can use a series of calls to prvalue, where the brief option limits the amount of output:

```
. prvalue, x(mar=1 fem=1 kid5=0) rest(mean) brief
Predicted probabilities:
    Pr}(y=0|x): 0.1926 Pr (y=1|x): 0.3172
    Pr}(y=2|x): 0.2613 Pr (y=3|x): 0.1434
    Pr}(y=4|x): 0.0591 Pr (y=5|x): 0.0195
    Pr}(y=6|x):\quad0.0053 Pr Pr y=7|x): 0.0013
    Pr}(y=8|x):0.0003 Pr (y=9|x): 0.000
. prvalue, x(mar=1 fem=1 kid5=1) rest(mean) brief
Predicted probabilities:
    Pr}(y=0|x): 0.2544 Pr (y=1|x): 0.3482
    Pr}(y=2|x): 0.2384 Pr (y=3|x): 0.108
    Pr}(y=4|x): 0.0372 Pr (y=5|x): 0.0102
    Pr}(y=6|x):0.0023 Pr Pr y=7 |x): 0.0005
    Pr}(y=8|x): 0.0001 Pr (y=9|x): 0.000
. prvalue, x(mar=1 fem=1 kid5=2) rest(mean) brief
Predicted probabilities:
```



```
    Pr}(y=2|x):0.2075 Pr (y=3|x): 0.078
    Pr}(y=4|x): 0.0224 Pr (y=5|x): 0.005
    Pr}(y=6|x): 0.0010 Pr Pr y=7|x): 0.0002
    Pr}(y=8|x): 0.0000 Pr (y=9|x): 0.000
. prvalue, x(mar=1 fem=1 kid5=3) rest(mean) brief
Predicted probabilities:
\begin{tabular}{llll}
\(\operatorname{Pr}(\mathrm{y}=0 \mid \mathrm{x}):\) & 0.3883 & \(\operatorname{Pr}(\mathrm{y}=1 \mid \mathrm{x}):\) & 0.3673 \\
\(\operatorname{Pr}(\mathrm{y}=2 \mid \mathrm{x}):\) & 0.1737 & \(\operatorname{Pr}(\mathrm{y}=3 \mid \mathrm{x}):\) & 0.0548 \\
\(\operatorname{Pr}(\mathrm{y}=4 \mid \mathrm{x}):\) & 0.0130 & \(\operatorname{Pr}(\mathrm{y}=5 \mid \mathrm{x}):\) & 0.0025 \\
\(\operatorname{Pr}(\mathrm{y}=6 \mid \mathrm{x}):\) & 0.0004 & \(\operatorname{Pr}(\mathrm{y}=7 \mid \mathrm{x}):\) & 0.0001 \\
\(\operatorname{Pr}(\mathrm{y}=8 \mid \mathrm{x}):\) & 0.0000 & \(\operatorname{Pr}(\mathrm{y}=9 \mid \mathrm{x}):\) & 0.0000
\end{tabular}
```

These values could be presented in a table or plotted, but overall it is clear that the probabilities of a zero count increase as the number of young children increases.

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7.2 The Poisson regression model

## Example of predicted probabilities using prgen

The command prgen computes a series of predictions by holding all variables but one constant and allowing that variable to vary. The resulting predictions can then be plotted. In this example we plot the predicted probability of not publishing for married men and married women with different numbers of children. First we compute the predictions for women using the prefix fprm to indicate female predictions from the PRM:

```
. prgen kid5, x(fem=1 mar=1) rest(mean) from(0) to(3) gen(fprm) n(4)
poisson: Predicted values as kid5 varies from 0 to 3.
\begin{tabular}{lrrrrr} 
& fem & mar & kid5 & phd & ment \\
\(x=\) & 1 & 1 & .49508197 & 3.1031093 & 8.7672125
\end{tabular}
```

Next, we compute predictions for men, using the prefix mprm:

```
. prgen kid5, x(fem=0 mar=1) rest(mean) from(0) to(3) gen(mprm) n(4)
poisson: Predicted values as kid5 varies from 0 to 3.
\begin{tabular}{lrrrrr} 
& fem & mar & kid5 & phd & ment
\end{tabular}
```

In both calls of prgen we requested four values with the $n(4)$ option. This creates predictions for $0,1,2$, and 3 children. To plot these predictions, we begin by adding value labels to the newly generated variables. Then, we use the now familiar graph command:
. label var fprmp0 "Married Women"
. label var mprmp0 "Married Men"
. label var mprmx "Number of Children"
. graph fprmp0 mprmp0 mprmx, c(ll) s(OS) ylab(0,.1 to .4) yline(.1,.2,.3) /*
> */ xlab( $0,1,2,3$ ) gap(3) 12 (Probability of No Articles)
This leads to the following graph, where the points marked with $\square$ 's and $\bigcirc$ 's are placed at the tic marks for the number of children:


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If you compare the values plotted for women to those computed with prvalue in the prior section, you will see that they are exactly the same, just computed in a different way.

## Example of predicted probabilities using prcounts

prcounts computes predictions for all observations in the dataset. In addition, the predictions are averaged across observations:

$$
\overline{\operatorname{Pr}}(y=m)=\frac{1}{N} \sum_{i=1}^{N} \widehat{\operatorname{Pr}}\left(y_{i}=m \mid \mathbf{x}_{i}\right)
$$

To illustrate how this command can be used to compare predictions from different models, we begin by fitting a univariate Poisson distribution and computing predictions with prcounts:

```
. poisson art, nolog
\begin{tabular}{lll} 
Poisson regression & Number of obs & \(=\) \\
& LR chi2 \((0)\) & 915 \\
& \(=\) & -0.00 \\
Log likelihood \(=-1742.5735\) & Prob \(>\) chi2 & \(=\) \\
\hline
\end{tabular}
\begin{tabular}{r|rccccr}
\hline art & Coef. & Std. Err. & z & P>|z| & [95\% Conf. Interval] \\
\hline _cons & .5264408 & .0254082 & 20.72 & 0.000 & .4766416 & .57624 \\
\hline
\end{tabular}
- prcounts psn, plot max (9)
. label var psnpreq "Univariate Poisson Dist."
```

Since we specified the plot option and the prefix psn, the command prcounts created a new variable called psnpreq that contains the average predicted probabilities of counts 0 through 9 from a univariate Poisson distribution. We then estimate the PRM with independent variables and again compute predictions with prcounts:


- prcounts prm, plot max (9)
. label var prmpreq "PRM"
. label var prmobeq "Observed"
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In addition to the new variable prmpreq, prcounts also generates prmobeq, which contains the observed probability of counts 0 through 9 . Another new variable, prmval, contains the value of the count. We now plot the values of psnpreq, prmpreq, and prmobeq with prmval on the $x$-axis:
. graph prmobeq psnpreq prmpreq prmval, /*
$>$ */ c(ll[.]l[-]l) gap(3) 12 ("Probability of Count") s(oxT) /*
$>* / \operatorname{yscale}(0, .4)$ ylabel $(0, .1, .2, .3, .4) \operatorname{xlabel}(0,1,2,3,4,5,6,7,8,9)$
This produces the following graph:


This graph shows that even though many of the independent variables have significant effects on the number of articles published, there is only a modest improvement in the predictions made by the PRM over the univariate Poisson distribution, with somewhat more 0 s predicted and slightly fewer 2 s and 3 s . While this suggests the need for an alternative model, we will first discuss how different periods of exposure can be incorporated into count models.

### 7.2.5 Exposure time*

So far we have implicitly assumed that each observation was "at risk" of an event occurring for the same amount of time. In terms of our example, this means that for each person in the sample we counted their articles over the same period of time. Often when collecting data, however, different observations have different exposure times. For example, the sample of scientists might have received their degrees in different years and our outcome might have been total publications from Ph.D. to the date of the survey. Clearly the amount of time in the career affects the total number of publications.

Different exposure times can be incorporated quite simply into count models. Let $t_{i}$ be the amount of time that observation $i$ is at risk. If the rate (i.e., the expected number of observations for a single unit of time) for that case is $\mu_{i}$, then we would expect $t_{i} \mu_{i}$ to be the expected count

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over a period of length $t_{i}$. Then, assuming only two independent variables for simplicity, our count equation becomes

$$
\mu_{i} t_{i}=\left[\exp \left(\beta_{0}+\beta_{1} x_{1}+\beta_{2} x_{2}\right)\right] \times t_{i}
$$

Since $t=\exp (\ln t)$, the equation can be rewritten as

$$
\mu_{i} t_{i}=\exp \left(\beta_{0}+\beta_{1} x_{1}+\beta_{2} x_{2}+\ln t_{i}\right)
$$

This shows that the effect of different exposure times can be included as the log of the exposure time with a regression coefficient constrained to equal 1 . While we do not have data with different exposure times, we have artificially constructed three variables to illustrate this issue. profage is a scientist's professional age, which corresponds to the time a scientist has been "exposed" to the possibility of publishing; lnage is the natural log of profage; and totalarts is the total number of articles during the career (to see how these were created, you can examine the sample file st4ch7.do). To estimate the model including exposure time, we use the exposure() option:

| Poisson regression |  |  |  | Number of obs <br> LR chi2(5) <br> Prob > chi2 <br> Pseudo R2 |  | $\begin{array}{r} 915 \\ 791.07 \\ 0.0000 \\ 0.0606 \end{array}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| totalarts | Coef. | Std. Err. | z | $\mathrm{P}>\|z\|$ | [95\% | Interval] |
| fem | -. 2109383 | . 022453 | -9.39 | 0.000 | -. 2549 | -. 1669313 |
| mar | . 1051588 | . 0253274 | 4.15 | 0.000 | . 0555 | . 1547996 |
| kid5 | -. 1507171 | . 0161878 | -9.31 | 0.000 | -. 1824 | -. 1189897 |
| phd | . 0542277 | . 0108399 | 5.00 | 0.000 | . 0329 | . 0754736 |
| ment | . 0205018 | . 0008338 | 24.59 | 0.000 | . 0188 | . 0221361 |
| $\begin{array}{r} \text { _cons } \\ \text { profage } \end{array}$ | $\begin{array}{r} .2351063 \\ \text { (exposure) } \end{array}$ | . 0426229 | 5.52 | 0.000 | . 151 | . 3186457 |

The results can be interpreted using the same methods discussed above.
To show you what the exposure () option is doing, we can obtain the same results by adding lnage as an independent variable and constraining the coefficient for lnage to 1 :


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7.3 The negative binomial regression model

You can also obtain the same result with offset() instead of exposure(), except that with offset () you specify a variable that is equal to the $\log$ of the exposure time. For example,
. poisson totalarts fem mar kid5 phd ment, nolog offset(lnage)
While the exposure() and offset () are not considered further in this chapter, they can be used with the other models we discuss.

### 7.3 The negative binomial regression model

The PRM accounts for observed heterogeneity (i.e., observed differences among sample members) by specifying the rate $\mu_{i}$ as a function of observed $x_{k}$ 's. In practice the PRM rarely fits due to overdispersion. That is, the model underestimates the amount of dispersion in the outcome. The negative binomial regression model (NBRM) addresses the failure of the PRM by adding a parameter $\alpha$ that reflects unobserved heterogeneity among observations. ${ }^{2}$ For example, with three independent variables, the PRM is

$$
\mu_{i}=\exp \left(\beta_{0}+\beta_{1} x_{i 1}+\beta_{2} x_{i 2}+\beta_{3} x_{i 3}\right)
$$

The NBRM adds an error $\varepsilon$ that is assumed to be uncorrelated with the $x$ 's,

$$
\begin{aligned}
\widetilde{\mu}_{i} & =\exp \left(\beta_{0}+\beta_{1} x_{i 1}+\beta_{2} x_{i 2}+\beta_{3} x_{i 3}+\varepsilon_{i}\right) \\
& =\exp \left(\beta_{0}+\beta_{1} x_{i 1}+\beta_{2} x_{i 2}+\beta_{3} x_{i 3}\right) \exp \left(\varepsilon_{i}\right) \\
& =\exp \left(\beta_{0}+\beta_{1} x_{i 1}+\beta_{2} x_{i 2}+\beta_{3} x_{i 3}\right) \delta_{i}
\end{aligned}
$$

where the second step follows by basic algebra, and the last step simply defines $\delta \equiv \exp (\varepsilon)$. To identify the model, we assume that

$$
E(\delta)=1
$$

which corresponds to the assumption $E(\varepsilon)=0$ in the LRM. With this assumption, it is easy to show that

$$
E(\widetilde{\mu})=\mu E(\delta)=\mu
$$

Thus, the PRM and the NBRM have the same mean structure. That is, if the assumptions of the NBRM are correct, the expected rate for a given level of the independent variables will be the same in both models. However, the standard errors in the PRM will be biased downward, resulting in spuriously large $z$-values and spuriously small $p$-values (Cameron and Trivedi 1986, 31).

The distribution of observations given both the values of the $x$ 's and $\delta$ is still Poisson in the NBRM. That is,

$$
\operatorname{Pr}\left(y_{i} \mid \mathbf{x}_{i}, \delta_{i}\right)=\frac{e^{-\widetilde{\mu}_{i}} \widetilde{\mu}_{i}^{y_{i}}}{y_{i}!}
$$

Since $\delta$ is unknown, we cannot compute $\operatorname{Pr}(y \mid \mathbf{x})$. This is resolved by assuming that $\delta$ is drawn from a gamma distribution (see Long 1997, 231-232 or Cameron and Trivedi 1998, 70-79 for details). Then we can compute $\operatorname{Pr}(y \mid \mathbf{x})$ as a weighted combination of $\operatorname{Pr}(y \mid \mathbf{x}, \delta)$ for all values

[^44]of $\delta$, where the weights are determined by $\operatorname{Pr}(\delta)$. The mathematics for this mixing of values of $\operatorname{Pr}(y \mid \mathbf{x}, \delta)$ is complex (and not particularly helpful for understanding the interpretation of the model), but lead to the negative binomial distribution
$$
\operatorname{Pr}(y \mid \mathbf{x})=\frac{\Gamma\left(y+\alpha^{-1}\right)}{y!\Gamma\left(\alpha^{-1}\right)}\left(\frac{\alpha^{-1}}{\alpha^{-1}+\mu}\right)^{\alpha^{-1}}\left(\frac{\mu}{\alpha^{-1}+\mu}\right)^{y}
$$
where $\Gamma$ is the gamma function.
In the negative binomial distribution, the parameter $\alpha$ determines the degree of dispersion in the predictions, as illustrated by the following figure:


In both panels the dispersion of predicted counts for a given value of $x$ is larger than in the PRM. In particular, note the greater probability of a 0 count. Further, the larger value of $\alpha$ in Panel B results in greater spread in the data. Indeed, if $\alpha=0$, the NBRM reduces to the PRM, which turns out to the be the key to testing for overdispersion. This is discussed in Section 7.3.3.

### 7.3.1 Estimating the NBRM with nbreg

The NBRM is estimated with the following command:

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```
nbreg depvar [indepvars] [weight] [if exp] [in range] [, level(\#) nolog
    table cluster(varname) irr exposure(varname) robust ]
```

where the options are the same as those for poisson. Because of differences in how poisson and nbreg are implemented in Stata, models estimated with nbreg take substantially longer to converge.

### 7.3.2 Example of estimating the NBRM

Here we use the same example as for the PRM above:

| Negative binomial regression |  |  |  | Number of obs LR chi2(5) <br> Prob > chi2 <br> Pseudo R2 |  | $=$ $=$ $=$ $=$ | $\begin{array}{r} 915 \\ 97.96 \\ 0.0000 \\ 0.0304 \end{array}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| art | Coef. | Std. Err. | z | $\mathrm{P}>\|\mathrm{z}\|$ | [95\% |  | Interval] |
| fem | -. 2164184 | . 0726724 | -2.98 | 0.003 | -. 3588 |  | -. 0739832 |
| mar | . 1504895 | . 0821063 | 1.83 | 0.067 | -. 0104 |  | . 3114148 |
| kid5 | -. 1764152 | . 0530598 | -3.32 | 0.001 | -. 2804 |  | -. 07242 |
| phd | . 0152712 | . 0360396 | 0.42 | 0.672 | -. 0553 |  | . 0859075 |
| ment | . 0290823 | . 0034701 | 8.38 | 0.000 | . 0222 |  | . 0358836 |
| _cons | . 256144 | . 1385604 | 1.85 | 0.065 | -. 0154 |  | . 5277174 |
| /lnalpha | -. 8173044 | . 1199372 |  |  | -1.052 |  | -. 5822318 |
| alpha | . 4416205 | . 0529667 |  |  | . 3491 |  | . 5586502 |

Likelihood ratio test of alpha=0: chibar2(01) = 180.20 Prob>=chibar2 $=0.000$
The output is similar to that of poisson, with the exception of the results at the bottom of the output, which initially can be confusing. While the model was defined in terms of the parameter $\alpha$, nbreg estimates $\ln (\alpha)$ with the estimate given in the line /lnalpha. This is done because estimating $\ln (\alpha)$ forces the estimated $\alpha$ to be positive. The value of $\widehat{\alpha}$ is given on the next line. $z$-values are not given since they require special treatment, as discussed in Section 7.3.3.

## Comparing the PRM and NBRM using outreg

We can use outreg to combine the results from poisson and nbreg:

```
. poisson art fem mar kid5 phd ment, nolog
(output omitted)
. outreg using 07prmnbrm, replace
. nbreg art fem mar kid5 phd ment, nolog
(output omitted)
. outreg using 07prmnbrm, append xstats
```

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The option xstats requests that auxiliary parameters be included in the table, which in the case of nbreg leads to the estimate of $\alpha$ being included. After some modifications to the output of outreg, we obtain

|  | PRM | NBRM |
| :---: | :---: | :---: |
| Gender: 1=female 0=male | -0.225 | -0.216 |
|  | (4.11)** | (2.98) ** |
| Married: 1=yes 0=no | 0.155 | 0.150 |
|  | (2.53)* | (1.83) |
| Number of children < 6 | -0.185 | -0.176 |
|  | (4.61) ** | (3.32) ** |
| PhD prestige | 0.013 | 0.015 |
|  | (0.49) | (0.42) |
| Article by mentor in last 3 yrs | 0.026 | 0.029 |
|  | (12.73) ** | (8.38) ** |
| Constant | 0.305 | 0.256 |
|  | (2.96) ** | (1.85) |
| lnalpha |  | -0.817 |
|  |  | (6.81)** |
| Observations | 915 | 915 |
| Absolute value of $z$-statistics in parentheses |  |  |
| * significant at 5\% level | ** signif | at 1\% leve |

The estimates of the corresponding parameters from the PRM and the NBRM are close, but the $z$-values for the NBRM are consistently smaller than those for the PRM. This is the expected consequence of overdispersion. outreg includes the estimate of lnalpha along with a $z$-value. But, as the next section shows, you should not use this to test for overdispersion.

### 7.3.3 Testing for overdispersion

If there is overdispersion, estimates from the PRM are inefficient with standard errors that are biased downward, even if the model includes the correct variables. Accordingly, it is important to test for overdispersion. Since the NBRM reduces to the PRM when $\alpha=0$, we can test for overdispersion by testing $H_{0}: \alpha=0$. There are two points to keep in mind in making this test. First, nbreg estimates $\ln (\alpha)$ rather than $\alpha$. A test of $H_{0}: \ln (\alpha)=0$ corresponds to testing $H_{0}: \alpha=1$, which is not the test we want. Second, since $\alpha$ must be greater than or equal to 0 , the asymptotic distribution of $\widehat{\alpha}$ when $\alpha=0$ is only half of a normal distribution. That is, all values less than 0 have a probability of 0 . This requires an adjustment to the usual significance level of the test.

To test the hypothesis $H_{0}: \alpha=0$, Stata provides a LR test that is listed after the estimates of the parameters.

```
Likelihood ratio test of alpha=0: chibar2(01) = 180.20 Prob > =chibar2 = 0.000
```

Since this output is different than that from lrtest, it is worth clarifying what it means. The test statistic chibar2 (01) is computed by the same formula shown in Chapter 3:

$$
\begin{aligned}
G^{2} & =2\left(\ln L_{\mathrm{NBRM}}-\ln L_{\mathrm{PRM}}\right) \\
& =2(-1560.96--1651.06)=180.2
\end{aligned}
$$

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The significance level of the test is adjusted to account for the truncated sampling distribution of $\widehat{\alpha}$. For details, you can click on chibar2(01), which will be listed in blue in the Results Window (recall that blue means that you can click for further information). In our example, the results are very significant and provide strong evidence of overdispersion. You can summarize this by saying that

Since there is significant evidence of overdispersion ( $G^{2}=180.2, p<.01$ ), the negative binomial regression model is preferred to the Poisson regression model.

### 7.3.4 Interpretation using the rate $\mu$

Since the mean structure for the NBRM is identical to that for the PRM, the same methods of interpretation based on $E(y \mid \mathbf{x})$ can be used based on the equation

$$
\frac{E\left(y \mid \mathbf{x}, x_{k}+\delta\right)}{E\left(y \mid \mathbf{x}, x_{k}\right)}=e^{\beta_{k} \delta}
$$

This leads to the interpretation that

For a change of $\delta$ in $x_{k}$, the expected count increases by a factor of $\exp \left(\beta_{k} \times \delta\right)$, holding all other variables constant.

Factor and percent change coefficients can be obtained using listcoef. For example,


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| art | b | z | $P>\|z\|$ | \% | \%StdX | SDofX |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| fem | -0.21642 | -2.978 | 0.003 | -19.5 | -10.2 | 0.4987 |
| ment | 0.02908 | 8.381 | 0.000 | 3.0 | 31.8 | 9.4839 |
| ln alpha <br> alpha | $\begin{array}{r} -0.81730 \\ 0.44162 \end{array}$ | SE(alph | $)=0.0$ |  |  |  |
| LR test of alpha=0: 180.20 Prob>=LRX2 $=0.000$ |  |  |  |  |  |  |
| b = raw coefficient |  |  |  |  |  |  |
| $z=z$-score for test of b=0 |  |  |  |  |  |  |
| $\mathrm{P}>\|z\|=\mathrm{p}$-value for z-test |  |  |  |  |  |  |
| $\% S t d X=$ percent change in expected count for SD increase in X |  |  |  |  |  |  |

These coefficients can be interpreted as

Being a female scientist decreases the expected number of articles by a factor of .81, holding all other variables constant. Equivalently, being a female scientist decreases the expected number of articles by 19.5 percent, holding all other variables constant.
For every additional article by the mentor, a scientist's expected mean productivity increases by 3.0 percent, holding other variables constant.
For a standard deviation increase in the mentor's productivity, a scientist's expected mean productivity increases by 32 percent, holding all other variables constant.

Interpretations for marginal and discrete change can be computed and interpreted using the methods discussed for the PRM.

### 7.3.5 Interpretation using predicted probabilities

The methods from the PRM can also be used for interpreting predicted probabilities. The only difference is that the predicted probabilities are computed with the formula

$$
\widehat{\operatorname{Pr}}(y \mid \mathbf{x})=\frac{\Gamma\left(y+\alpha^{-1}\right)}{y!\Gamma\left(\alpha^{-1}\right)}\left(\frac{\widehat{\alpha}^{-1}}{\widehat{\alpha}^{-1}+\widehat{\mu}}\right)^{\widehat{\alpha}^{-1}}\left(\frac{\widehat{\mu}}{\widehat{\alpha}^{-1}+\widehat{\mu}}\right)^{y}
$$

where $\widehat{\mu}=\exp (\mathbf{x} \widehat{\beta})$. As before, predicted probabilities can be computed using prchange, prgen, prcounts, and prvalue. Since there is nothing new in how to use these commands, we provide only two examples that are designed to illustrate key differences and similarities between the PRM and the NBRM. First, we use prvalue to compute predicated values for an "average" respondent. For the PRM,

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| poisson: Predictions for art |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Predicted rate: 1.6101 <br> Predicted probabilities: |  |  | 95\% CI [1.5286 , 1.6959] |  |  |  |
|  |  |  |  |  |  |  |
|  | $\operatorname{Pr}(\mathrm{y}=0 \mid \mathrm{x})$ : | 0.1999 | $\operatorname{Pr}(\mathrm{y}=1 \mid \mathrm{x})$ : | 0.3218 |  |  |
|  | $\operatorname{Pr}(\mathrm{y}=2 \mid \mathrm{x})$ : | 0.2591 | $\operatorname{Pr}(\mathrm{y}=3 \mid \mathrm{x})$ : | 0.1390 |  |  |
|  | $\operatorname{Pr}(\mathrm{y}=4 \mid \mathrm{x})$ : | 0.0560 | $\operatorname{Pr}(\mathrm{y}=5 \mid \mathrm{x})$ : | 0.0180 |  |  |
|  | $\operatorname{Pr}(\mathrm{y}=6 \mid \mathrm{x}):$ | 0.0048 | $\operatorname{Pr}(\mathrm{y}=7 \mid \mathrm{x})$ : | 0.0011 |  |  |
|  | $\operatorname{Pr}(\mathrm{y}=8 \mid \mathrm{x})$ : | 0.0002 | $\operatorname{Pr}(\mathrm{y}=9 \mid \mathrm{x})$ : | 0.0000 |  |  |
|  | fem | mar | kid5 | phd |  | ment |
| $\mathrm{x}=$ | . 46010929 | . 66229508 | . 49508197 | 3.1031093 | 8 | 72125 |

and for the NBRM,
. quietly nbreg art fem mar kid5 phd ment

- prvalue
nbreg: Predictions for art
Predicted rate: 1.602
Predicted probabilities:

| $\operatorname{Pr}(\mathrm{y}=0 \mid \mathrm{x}):$ | 0.2978 | $\operatorname{Pr}(\mathrm{y}=1 \mid \mathrm{x}):$ | 0.2794 |  |
| ---: | ---: | ---: | ---: | ---: |
| $\operatorname{Pr}(\mathrm{y}=2 \mid \mathrm{x}):$ | 0.1889 | $\operatorname{Pr}(\mathrm{y}=3 \mid \mathrm{x}):$ | 0.1113 |  |
| $\operatorname{Pr}(\mathrm{y}=4 \mid \mathrm{x}):$ | 0.0607 | $\operatorname{Pr}(\mathrm{y}=5 \mid \mathrm{x}):$ | 0.0315 |  |
| $\operatorname{Pr}(\mathrm{y}=6 \mid \mathrm{x}):$ | 0.0158 | $\operatorname{Pr}(\mathrm{y}=7 \mid \mathrm{x}):$ | 0.0077 |  |
| $\operatorname{Pr}(\mathrm{y}=8 \mid \mathrm{x}):$ | 0.0037 | $\operatorname{Pr}(\mathrm{y}=9 \mid \mathrm{x}):$ | 0.0018 |  |
| fem | mar | kid5 | phd | ment |
| $\mathrm{x}=$ | .46010929 | .66229508 | .49508197 | 3.1031093 |

The first thing to notice is that the predicted rate is nearly identical for both models: 1.610 versus 1.602. This illustrates that even with overdispersion (which there is in this example), the estimates from the PRM are consistent. But, substantial differences emerge when we examine predicted probabilities: $\widehat{\operatorname{Pr}}_{\mathrm{PRM}}(y=0 \mid \overline{\mathbf{x}})=0.200$ compared to $\widehat{\operatorname{Pr}}_{\mathrm{NBRM}}(y=0 \mid \overline{\mathbf{x}})=0.298$. We also find higher probabilities in the NBRM for larger counts. For example, $\widehat{\operatorname{Pr}}_{\mathrm{NBRM}}(y=5 \mid \overline{\mathbf{x}})=0.0315$ compared to $\widehat{\operatorname{Pr}}_{\text {PRM }}(y=5 \mid \overline{\mathbf{x}})=0.0180$. These probabilities reflect the greater dispersion in the NBRM compared to the PRM.

Another way to see the greater probability for 0 counts in the NBRM is to plot the probability of 0s as values of an independent variable change. This is done with prgen:
. quietly nbreg art fem mar kid5 phd ment, nolog

- prgen ment, rest(mean) $f(0) t(50)$ gen(nb) $n(20)$
nbreg: Predicted values as ment varies from 0 to 50.

|  | fem | mar | kid5 | phd | ment |
| ---: | ---: | ---: | ---: | ---: | ---: |

. quietly poisson art fem mar kid5 phd ment

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```
. prgen ment, rest(mean) f(0) t(50) gen(psn) n(20)
poisson: Predicted values as ment varies from 0 to 50.
\begin{tabular}{rrrrrr} 
& fem & mar & kid5 & phd & ment \\
\(x=\) & .46010929 & .66229508 & .49508197 & 3.1031093 & 8.7672125
\end{tabular}
. label var psnp0 "Pr(0) for PRM"
. label var nbpO "Pr(0) for NBRM"
. graph psnp0 nbp0 nbx, c(ll) ylab(0,.1,.2,.3,.4) yline(.1,.2,.3) /*
> */ gap(3) l1("Probability of a Zero Count")
```

which leads to the following graph:


The probability of having zero publications is computed when each variable except the mentor's number of articles is held at its mean. For both models, the probability of a zero decreases as the mentor's articles increase. But, the proportion of predicted zeros is significantly higher for the NBRM. Since both models have the same expected number of publications, the higher proportion of predicted zeros for the NBRM is offset by the higher proportion of larger counts that are also predicted by this model.

### 7.4 Zero-inflated count models

The NBRM improves upon the underprediction of zeros in the PRM by increasing the conditional variance without changing the conditional mean, which was illustrated by the output from prvalue in the prior section. Zero-inflated count models, introduced by Lambert (1992), respond to the failure of the PRM model to account for dispersion and excess zeros by changing the mean structure to allow zeros to be generated by two distinct processes. To make this clearer, consider our example of scientific productivity. The PRM and NBRM assume that every scientist has a positive probability of publishing any given number of papers. The probability differs across individuals according

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to their characteristics, but all scientists have some probability of publishing. Substantively, this is unrealistic because some scientists are not potential publishers. For example, they could hold positions, perhaps in industry, where publishing is not allowed. Zero-inflated models allow for this possibility and, in the process, they increase the conditional variance and the probability of zero counts.

The zero-inflated model assumes that there are two latent (i.e., unobserved) groups. An individual in the Always-0 Group (Group A) has an outcome of 0 with a probability of 1 , while an individual in the Not Always-0 Group (Group ~A) might have a zero count, but there is a nonzero probability that she has a positive count. This process is developed in three steps: Step 1) Model membership into the latent groups; Step 2) Model counts for those in Group ~A; and Step 3) Compute observed probabilities as a mixture of the probabilities for the two groups.

Step 1: Membership in Group A Let $A=1$ if someone is in Group A, else $A=0$. Group membership is a binary outcome that can be modeled using the logit or probit model of Chapter 4,

$$
\begin{equation*}
\psi_{i}=\operatorname{Pr}\left(A_{i}=1 \mid \mathbf{z}_{i}\right)=F\left(\mathbf{z}_{i} \gamma\right) \tag{7.2}
\end{equation*}
$$

where $\psi_{i}$ is the probability of being in Group A for individual $i$. The $z$-variables are referred to as inflation variables since they serve to inflate the number of 0 s as shown below. To illustrate Equation 7.2, assume that two variables affect the probability of an individual being in Group A and that we model this with a logit equation:

$$
\psi_{i}=\frac{\exp \left(\gamma_{0}+\gamma_{1} z_{1}+\gamma_{2} z_{2}\right)}{1+\exp \left(\gamma_{0}+\gamma_{1} z_{1}+\gamma_{2} z_{2}\right)}
$$

If we had an observed variable indicating group membership, this would be a standard, binary regression model. But, since group membership is a latent variable, we do not know whether an individual is in Group A or Group ~A.

Step 2: Counts for those in Group ${ }^{\sim} \mathbf{A}$ Among those who are not always zero, the probability of each count (including zeros) is determined by either a Poisson or a negative binomial regression. Notice that in the equations that follow we are conditioning both on the $x_{k}$ 's and on $A=0$. Also note that the $x_{k}$ 's are not necessarily the same as the inflation variables $z_{k}$ in the first step (although the two sets of variables can be the same). For the zero-inflated Poisson (ZIP) model, we have

$$
\operatorname{Pr}\left(y_{i} \mid \mathbf{x}_{i}, A_{i}=0\right)=\frac{e^{-\mu_{i}} \mu_{i}^{y_{i}}}{y_{i}!}
$$

or, for the zero inflated negative binomial (ZINB) model,

$$
\operatorname{Pr}\left(y_{i} \mid \mathbf{x}_{i}, A_{i}=0\right)=\frac{\Gamma\left(y_{i}+\alpha^{-1}\right)}{y_{i}!\Gamma\left(\alpha^{-1}\right)}\left(\frac{\alpha^{-1}}{\alpha^{-1}+\mu_{i}}\right)^{\alpha^{-1}}\left(\frac{\mu_{i}}{\alpha^{-1}+\mu_{i}}\right)^{y_{i}}
$$

In both equations, $\mu_{i}=\exp \left(\mathbf{x}_{i} \beta\right)$. If we knew which observations were in Group ${ }^{\sim} \mathrm{A}$, these equations would define the PRM and the NBRM. But, here the equations only apply to those observations in Group ${ }^{\text {A }}$, and we do not have an observed variable indicating group membership.

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Step 3: Mixing Groups $\mathbf{A}$ and ${ }^{\sim} \mathbf{A}$ The simplest way to understand the mixing is to start with an example. Suppose that retirement status is indicated by $r=1$ for retired folks and $r=0$ for those not retired, where

$$
\begin{aligned}
& \operatorname{Pr}(r=1)=.2 \\
& \operatorname{Pr}(r=0)=1-.2=.8
\end{aligned}
$$

Let $y$ indicate living in a warm climate, with $y=1$ for yes and $y=0$ for no. Suppose that the conditional probabilities are

$$
\begin{aligned}
& \operatorname{Pr}(y=1 \mid r=1)=.5 \\
& \operatorname{Pr}(y=1 \mid r=0)=.3
\end{aligned}
$$

so that people are more likely to live in a warm climate if they are retired. What is the probability of living in a warm climate for the population as a whole? The answer is a mixture of the probabilities for the two groups weighted by the proportion in each group:

$$
\begin{aligned}
\operatorname{Pr}(y=1)= & {[\operatorname{Pr}(r=1) \times \operatorname{Pr}(y=1 \mid r=1)] } \\
& \quad+[\operatorname{Pr}(r=0) \times \operatorname{Pr}(y=1 \mid r=0)] \\
= & {[.2 \times .5]+[.8 \times .3]=.34 }
\end{aligned}
$$

In other words, the two groups are mixed according to their proportions in the population to determine the overall rate. The same thing is done for the zero-inflated models.

The proportion in each group is defined by

$$
\begin{aligned}
& \operatorname{Pr}\left(A_{i}=1\right)=\psi_{i} \\
& \operatorname{Pr}\left(A_{i}=0\right)=1-\psi_{i}
\end{aligned}
$$

and the probabilities of a zero within each group are

$$
\begin{aligned}
& \operatorname{Pr}\left(y_{i}=0 \mid A_{i}=1, \mathbf{x}_{i}, \mathbf{z}_{i}\right)=1 \quad \text { by definition of the } A \text { Group } \\
& \operatorname{Pr}\left(y_{i}=0 \mid A_{i}=0, \mathbf{x}_{i}, \mathbf{z}_{i}\right)=\text { outcome of PRM or NBRM. }
\end{aligned}
$$

Then, the overall probability of a 0 count is

$$
\begin{aligned}
\operatorname{Pr}\left(y_{i}=0 \mid \mathbf{x}_{i}, \mathbf{z}_{i}\right) & =\left[\psi_{i} \times 1\right]+\left[\left(1-\psi_{i}\right) \times \operatorname{Pr}\left(y_{i}=0 \mid \mathbf{x}_{i}, A_{i}=0\right)\right] \\
& =\psi_{i}+\left[\left(1-\psi_{i}\right) \times \operatorname{Pr}\left(y_{i}=0 \mid \mathbf{x}_{i}, A_{i}=0\right)\right]
\end{aligned}
$$

For outcomes other than 0 ,

$$
\begin{aligned}
\operatorname{Pr}\left(y_{i}=k \mid \mathbf{x}_{i}, \mathbf{z}_{i}\right) & =\left[\psi_{i} \times 0\right]+\left[\left(1-\psi_{i}\right) \times \operatorname{Pr}\left(y_{i}=k \mid \mathbf{x}_{i}, A_{i}=0\right)\right] \\
& =\left(1-\psi_{i}\right) \times \operatorname{Pr}\left(y_{i}=k \mid \mathbf{x}_{i}, A_{i}=0\right)
\end{aligned}
$$

where we use the assumption that the probability of a positive count in Group $A$ is 0 .
Expected counts are computed in a similar fashion:

$$
\begin{aligned}
E(y \mid \mathbf{x}, \mathbf{z}) & =[0 \times \psi]+[\mu \times(1-\psi)] \\
& =\mu(1-\psi)
\end{aligned}
$$

Since $0 \leq \psi \leq 1$, the expected value will be smaller than $\mu$, which shows that the mean structure in zero-inflated models differs from that in the PRM or NBRM.

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7.4 Zero-inflated count models

### 7.4. Estimation of zero-inflated models with zinb and zip

The ZIP and ZINB models are estimated with the zip and zinb commands. The syntax is

```
zip depvar [indepvars] [weight] [if exp] [in range] [, inflate(indepvars2)
    level(#) probit vuong nolog irr robust cluster(varname)
    exposure(varname)]
zinb depvar [indepvars] [weight] [if exp] [in range] [, inflate(indepvars2)
    level(#) probit vuong nolog irr robust cluster(varname)
    exposure(varname)]
```


## Variable lists

depvar is the dependent variable, which must be a count variable.
indepvars is a list of independent variables that determine counts among those who are not always zeros. If indepvars is not included, a model with only an intercept is estimated.
indepvars 2 is a list of inflation variables that determine if you are in the Always-0 Group or the Not Always-0 Group.
indepvars and indepvars 2 can be the same variables, but they do not have to be.

## Options

Here we only consider options that differ from those in earlier models for this chapter.
probit specifies that the model determining the probability of being in the Always-0 Group versus the Not Always-0 Group is to be a binary probit model. By default, a binary logit model is used. vuong requests a Vuong (1989) test of the ZIP model versus the PRM, or of the ZINB versus the NBRM. Details are given in Section 7.5.2.

### 7.4.2 Example of estimating the ZIP and ZINB models

The output from zip and zinb are very similar, so here we show only the output for zinb:

| Zero-inflated negative binomial regression | Number of obs | = | 915 |
| :---: | :---: | :---: | :---: |
|  | Nonzero obs | = | 640 |
|  | Zero obs | = | 275 |
| Inflation model $=$ logit | LR chi2(5) | = | 67.97 |
| Log likelihood = -1549.991 | Prob > chi2 | = | 0.0000 |

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| art | Coef. | Std. Err. | z | $P>\|z\|$ | [95\% Con | Interval] |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| art |  |  |  |  |  |  |
| fem | -. 1955068 | . 0755926 | -2.59 | 0.010 | -. 3436655 | -. 0473481 |
| mar | . 0975826 | . 084452 | 1.16 | 0.248 | -. 0679402 | . 2631054 |
| kid5 | -. 1517325 | . 054206 | -2.80 | 0.005 | -. 2579744 | -. 0454906 |
| phd | -. 0007001 | . 0362696 | -0.02 | 0.985 | -. 0717872 | . 0703869 |
| ment | . 0247862 | . 0034924 | 7.10 | 0.000 | . 0179412 | . 0316312 |
| _cons | . 4167466 | . 1435962 | 2.90 | 0.004 | . 1353032 | . 69819 |
| inflate |  |  |  |  |  |  |
| fem | . 6359327 | . 8489175 | 0.75 | 0.454 | -1.027915 | 2.299781 |
| mar | -1.499469 | . 93867 | -1.60 | 0.110 | -3.339228 | . 3402907 |
| kid5 | . 6284274 | . 4427825 | 1.42 | 0.156 | -. 2394104 | 1.496265 |
| phd | -. 0377153 | . 3080086 | -0.12 | 0.903 | -. 641401 | . 5659705 |
| ment | -. 8822932 | . 3162277 | -2.79 | 0.005 | -1.502088 | -. 2624984 |
| _cons | -. 1916864 | 1.322821 | -0.14 | 0.885 | -2.784368 | 2.400995 |
| /lnalpha | -. 9763565 | . 1354679 | -7.21 | 0.000 | -1.241869 | -. 7108443 |
| alpha | . 376681 | . 0510282 |  |  | . 288844 | . 4912293 |

The top set of coefficients, labeled art at the left margin, correspond to the NBRM for those in the Not Always-0 Group. The lower set of coefficients, labeled inflate, correspond to the binary model predicting group membership.

### 7.4.3 Interpretation of coefficients

When interpreting zero inflated models, it is easy to be confused by the direction of the coefficients. listcoef makes interpretation simpler. For example, consider the results for the ZINB:

```
. zinb art fem mar kid5 phd ment, inf(fem mar kid5 phd ment) nolog
    (output omitted)
. listcoef, help
zinb (N=915): Factor Change in Expected Count
    Observed SD: 1.926069
Count Equation: Factor Change in Expected Count for Those Not Always 0
```

| art | b | z | $\mathrm{P}>\|\mathrm{z}\|$ | $\mathrm{e}^{\wedge} \mathrm{b}$ | $\mathrm{e}^{\wedge} \mathrm{bStdX}$ | SDofX |
| ---: | ---: | ---: | :---: | :---: | ---: | ---: |
| fem | -0.19551 | -2.586 | 0.010 | 0.8224 | 0.9071 | 0.4987 |
| mar | 0.09758 | 1.155 | 0.248 | 1.1025 | 1.0473 | 0.4732 |
| kid5 | -0.15173 | -2.799 | 0.005 | 0.8592 | 0.8904 | 0.7649 |
| phd | -0.00070 | -0.019 | 0.985 | 0.9993 | 0.9993 | 0.9842 |
| ment | 0.02479 | 7.097 | 0.000 | 1.0251 | 1.2650 | 9.4839 |
| ln alpha | -0.97636 |  |  |  |  |  |
| alpha | 0.37668 | SE(alpha) $=0.05103$ |  |  |  |  |

        b = raw coefficient
        \(z=z\)-score for test of \(b=0\)
    \(p>|z|=p\)-value for \(z\)-test
    \(e^{\wedge} b=\exp (b)=\) factor change in expected count for unit increase in \(X\)
    $e^{\wedge} b S t d X=\exp (b * S D$ of $X)=$ change in expected count for $S D$ increase in $X$
SDof $X=$ standard deviation of $X$

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Binary Equation: Factor Change in Odds of Always 0

| Always0 | b | z | $\mathrm{P}>\|\mathrm{z}\|$ | $\mathrm{e}^{\wedge} \mathrm{b}$ | $\mathrm{e}^{\wedge} \mathrm{bStdX}$ | SDofX |
| ---: | :---: | :---: | :---: | :---: | :---: | ---: |
| fem | 0.63593 | 0.749 | 0.454 | 1.8888 | 1.3732 | 0.4987 |
| mar | -1.49947 | -1.597 | 0.110 | 0.2232 | 0.4919 | 0.4732 |
| kid5 | 0.62843 | 1.419 | 0.156 | 1.8747 | 1.6172 | 0.7649 |
| phd | -0.03772 | -0.122 | 0.903 | 0.9630 | 0.9636 | 0.9842 |
| ment | -0.88229 | -2.790 | 0.005 | 0.4138 | 0.0002 | 9.4839 |

```
            b = raw coefficient
            z = z-score for test of b=0
    P>|z| = p-value for z-test
        e^b = exp(b) = factor change in odds for unit increase in X
e^bStdX = exp(b*SD of X) = change in odds for SD increase in X
    SDofX = standard deviation of X
```

The top half of the output, labeled Count Equation, contains coefficients for the factor change in the expected count for those in the Not Always-0 Group. This group comprises those scientists who have the opportunity to publish. The coefficients can be interpreted in the same way as coefficients from the PRM or the NBRM. For example,

Among those who have the opportunity to publish, being a woman decreases the expected rate of publication by a factor of .91 , holding all other factors constant.

The bottom half, labeled Binary Equation, contains coefficients for the factor change in the odds of being in the Always-0 Group compared to the Not Always-0 Group. These can be interpreted just as the coefficients for a binary logit model. For example,

Being a woman increases the odds of not having the opportunity to publish by a factor of 1.89 , holding all other variables constant.

As we found in this example, when the same variables are included in both equations, the signs of the corresponding coefficients from the binary equation are often in the opposite direction of the coefficients for the count equation. This often makes substantive sense since the binary process is predicting membership in the group that always has zero counts, so a positive coefficient implies lower productivity. The count process predicts number of publications so that a negative coefficient would indicate lower productivity.

### 7.4.4 Interpretation of predicted probabilities

For the ZIP model,

$$
\widehat{\operatorname{Pr}}(y=0 \mid \mathbf{x}, \mathbf{z})=\widehat{\psi}+(1-\widehat{\psi}) e^{-\widehat{\mu}}
$$

where $\widehat{\mu}=\exp (\mathbf{x} \widehat{\beta})$ and $\widehat{\psi}=F(\mathbf{z} \widehat{\gamma})$. The predicted probability of a positive count applies only to the $1-\widehat{\psi}$ observations in the Not Always-0 Group:

$$
\widehat{\operatorname{Pr}}(y \mid \mathbf{x})=(1-\widehat{\psi}) \frac{e^{-\widehat{\mu}_{i} \widehat{\mu}^{y}}}{y!}
$$

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Similarly, for the ZINB model,

$$
\widehat{\operatorname{Pr}}(y=0 \mid \mathbf{x}, \mathbf{z})=\widehat{\psi}+(1-\widehat{\psi})\left(\frac{\widehat{\alpha}^{-1}}{\widehat{\alpha}^{-1}+\widehat{\mu}_{i}}\right)^{\widehat{\alpha}^{-1}}
$$

And the predicted probability for a positive count is

$$
\widehat{\operatorname{Pr}}(y \mid \mathbf{x})=(1-\widehat{\psi}) \frac{\Gamma\left(y+\widehat{\alpha}^{-1}\right)}{y!\Gamma\left(\widehat{\alpha}^{-1}\right)}\left(\frac{\widehat{\alpha}^{-1}}{\widehat{\alpha}^{-1}+\widehat{\mu}}\right)^{\widehat{\alpha}^{-1}}\left(\frac{\widehat{\mu}}{\widehat{\alpha}^{-1}+\widehat{\mu}}\right)^{y}
$$

The probabilities can be computed with prvalue, prcounts, and prgen.

## Predicted probabilities with prvalue

prvalue works in the same way for zip and zinb as it did for earlier count models, although the output is slightly different. Suppose we want to compare the predicted probabilities for a married female scientist with young children who came from a weak graduate program to those for a married male from a strong department with a productive mentor:

```
. quietly prvalue, x(fem=0 mar=1 kid5=3 phd=3 ment=10) save
. prvalue, x(fem=1 mar=1 kid5=3 phd=1 ment=0) dif
zinb: Change in Predictions for art
Predicted rate: . }2717
    Saved: 1.3563
        Difference: -1.0845
Predicted probabilities:
```

|  | Current | Saved | Dif |
| :---: | ---: | ---: | ---: |
| $\operatorname{Pr}(\mathrm{y}=0 \mid \mathrm{x}, \mathrm{z}):$ | 0.9290 | 0.3344 | 0.5945 |
| $\operatorname{Pr}(\mathrm{y}=1 \mid \mathrm{x}):$ | 0.0593 | 0.3001 | -0.2408 |
| $\operatorname{Pr}(\mathrm{y}=2 \mid \mathrm{x}):$ | 0.0101 | 0.1854 | -0.1754 |
| $\operatorname{Pr}(\mathrm{y}=3 \mid \mathrm{x}):$ | 0.0015 | 0.0973 | -0.0958 |
| $\operatorname{Pr}(\mathrm{y}=4 \mid \mathrm{x}):$ | 0.0002 | 0.0465 | -0.0463 |
| $\operatorname{Pr}(\mathrm{y}=5 \mid \mathrm{x}):$ | 0.0000 | 0.0209 | -0.0209 |
| $\operatorname{Pr}(\mathrm{y}=6 \mid \mathrm{x}):$ | 0.0000 | 0.0090 | -0.0090 |
| $\operatorname{Pr}(\mathrm{y}=7 \mid \mathrm{x}):$ | 0.0000 | 0.0038 | -0.0038 |
| $\operatorname{Pr}(\mathrm{y}=8 \mid \mathrm{x}):$ | 0.0000 | 0.0015 | -0.0015 |
| $\operatorname{Pr}(\mathrm{y}=9 \mid \mathrm{x}):$ | 0.0000 | 0.0006 | -0.0006 |
| $\operatorname{Pr}($ Always $0 \mid z):$ | 0.6883 | 0.0002 | 0.6882 |

x values for count equation

|  | fem | mar | kid5 | phd | ment |
| ---: | ---: | ---: | ---: | ---: | ---: |
| Current $=$ | 1 | 1 | 3 | 1 | 0 |
| Saved $=$ | 0 | 1 | 3 | 3 | 10 |
| Diff $=$ | 1 | 0 | 0 | -2 | -10 |

z values for binary equation

|  | fem | mar | kid5 | phd | ment |
| ---: | ---: | ---: | ---: | ---: | ---: |
| Current $=$ | 1 | 1 | 3 | 1 | 0 |
| Saved $=$ | 0 | 1 | 3 | 3 | 10 |
| Diff $=$ | 1 | 0 | 0 | -2 | -10 |

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There are two major differences in the output of prvalue for zip and zinb compared to other count models. First, levels of both the $x$ variables from the count equation and the $z$ variables from the binary equation are listed. In this example, they are the same variables, but they could be different. Second, there are two probabilities of 0 counts. For example, for our female scientists, prvalue lists $\operatorname{Pr}(\mathrm{y}=0 \mid \mathrm{x}, \mathrm{z}): 0.9290$, which is the probability of having no publications, either because a scientist does not have the opportunity to publish or because a scientist is a potential publisher who by chance did not publish. The quantity $\operatorname{Pr}(A l w a y s 0 \mid z): 0.6883$ is the probability of not having the opportunity to publish. Thus, most of the 0 s for women are due to being in the group that never publishes. The remaining probabilities listed are the probabilities of observing each count of publications for the specified set of characteristics.

## Predicted probabilities with prgen

prgen is used to plot predictions. In this case, we examine the two sources of 0s. First, we call prgen to compute the predicted values to be plotted:

```
. prgen ment, rest(mean) f(0) t(20) gen(zinb) n(21)
zinb: Predicted values as ment varies from 0 to 20.
base x values for count equation:
```

|  | fem | mar | kid5 | phd | ment |
| :--- | ---: | ---: | ---: | ---: | ---: |
| $\mathrm{x}=$ | .46010929 | .66229508 | .49508197 | 3.1031093 | 8.7672125 |
| base z values for binary | equation: |  |  |  |  |
|  | fem | mar | kid5 | phd | ment |
| $z=$ | .46010929 | .66229508 | .49508197 | 3.1031093 | 8.7672125 |

prgen created two probabilities for 0 counts: zinbp0 contains the probability of a 0 count from both the count and the binary equation. zinball0 is the probability due to observations being in the Always-0 group. We use generate zinbnb0 = zinbp0 - zinball0 to compute the probability of 0 s from the count portion of the model:

```
. gen zinbnb0 = zinbp0 - zinball0
(894 missing values generated)
. label var zinbp0 "Os from Both Equations"
. label var zinball0 "Os from Binary Equation"
. label var zinbnbO "Os from Count Equation"
. label var zinbx "Mentor`s Publications"
```

These are plotted with the command

```
. graph zinball0 zinbnb0 zinbp0 zinbx, s(0TS) c(sss) gap(3) xlabel(0,5 to 20) /*
> */ l2(Probability of Zero) ylabel(0,.05 to .65)
```

which produces the following graph:

## (Continued on next page)

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The curve marked with O's is a probability curve just like those shown in Chapter 4 for binary models. The curve marked with $\triangle$ 's shows the probability of 0 s from a series of negative binomial distributions each with different rate parameters $\mu$ determined by the level of mentor's publications. The overall probability of a zero count is the sum of the two curves, which is shown by the line with $\square$ 's.

### 7.5 Comparisons among count models

There are two methods that can be used to compare the results of the PRM, NBRM, ZIP, and ZINB models.

### 7.5.1 Comparing mean probabilities

One way to compare these models is to compare predicted probabilities across models. First, we compute the mean predicted probability. For example, in the PRM,

$$
\overline{\operatorname{Pr}}_{\mathrm{PRM}}(y=m)=\frac{1}{N} \sum_{i=1}^{N} \widehat{\operatorname{Pr}}_{\mathrm{PRM}}\left(y_{i}=m \mid \mathbf{x}_{i}\right)
$$

This is simply the average across all observations of the probability of each count. The difference between the observed probabilities and the mean prediction can be computed as

$$
\Delta \overline{\operatorname{Pr}}_{\mathrm{PRM}}(y=m)=\widehat{\operatorname{Pr}}_{\text {Observed }}(y=m)-\overline{\operatorname{Pr}}_{\mathrm{PRM}}(y=m)
$$

This can be done for each model and then plotted. The commands are

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7.5 Comparisons among count models
. quietly poisson art fem mar kid5 phd ment, nolog
. quietly prcounts prm, plot max(9)
. label var prmpreq "Predicted: PRM"
. label var prmobeq "Observed"
. quietly nbreg art fem mar kid5 phd ment, nolog
. quietly prcounts nbrm, plot max (9)
. label var nbrmpreq "Predicted: NBRM"
. quietly zip art fem mar kid5 phd ment, /*
> */ inf(fem mar kid5 phd ment) vuong nolog
. quietly prcounts zip, plot max (9)
. label var zippreq "Predicted: ZIP"
. quietly zinb art fem mar kid5 phd ment, /*
> */ inf(fem mar kid5 phd ment) vuong nolog
. quietly prcounts zinb, plot max (9)

- label var zinbpreq "Predicted: ZINB"
. * create deviations
- gen obs = prmobeq
( 905 missing values generated)
- gen dprm = obs - prmpreq
(905 missing values generated)
. label var dprm "PRM"
- gen dnbrm = obs - nbrmpreq
( 905 missing values generated)
. label var dnbrm "NBRM"
. gen dzip = obs - zippreq
(905 missing values generated)
. label var dzip "ZIP"
- gen dzinb $=$ obs - zinbpreq
(905 missing values generated)
. label var dzinb "ZINB"
. graph dprm dnbrm dzip dzinb prmval, s(OSTx) c(llll) gap(3) /*
$>$ */ 12(Observed-Predicted) ylabel(-.10,-. 05 to .10) /*
> */ xlabel (0 1 to 9) yline(0)
which leads to the following graph:
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Points above 0 on the $y$-axis indicate more observed counts than predicted; those below 0 indicate more predicted counts than observed. The graph shows that only the PRM has a problem predicting the average number of 0s. Among the other models, the ZIP does less well, predicting too many 1 s and too few 2 s and 3 s . The NBRM and ZINB do about equally well. Based on these results, we might prefer the NBRM because it is simpler.

### 7.5.2 Tests to compare count models

Plotting predictions is only an informal method of assessing the fit of a count model. More formal testing can be done with an LR test of overdispersion and a Vuong test to compare two models.

## LR tests of $\alpha$

Since the NBRM reduces to the PRM when $\alpha=0$, the PRM and NBRM can be compared by testing $H_{0}: \alpha=0$. As shown in Section 7.3.3, we find that

```
Likelihood ratio test of alpha=0: chibar2(01) = 180.20 Prob > =chibar2 = 0.000
```

which provides strong evidence for preferring the NBRM over the PRM.
Since the ZIP and ZINB models are also nested, the same LR test can be applied to compare them. While Stata does not compute this for you, it is simple to do. First we estimate the ZIP model:
. quietly zip art fem mar kid5 phd ment, inf(fem mar kid5 phd ment) vuong nolog
. scalar llzip = e(ll)
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The command scalar llzip $=e(l l)$ saves the log likelihood that was left in memory by zip. Next we do the same thing for zinb and compute the difference between the two log likelihoods:

```
. quietly zinb art fem mar kid5 phd ment, inf(fem mar kid5 phd ment) nolog
. scalar llzinb = e(ll)
. scalar lr = -2*(llzip-llzinb)
```

The following commands can be used to compute the $p$-value. Note that if you do this with your own model, you need to substitute the value of lnalpha which is listed as part of the output for zinb:
. scalar pvalue $=\operatorname{chiprob}(1, l r) / 2$
. scalar lnalpha = -. 9763565
. if (lnalpha < -20) \{ scalar pvalue= 1 \}

- di as text "Likelihood ratio test comparing ZIP to ZINB: " as res \%8.3f lr /*
> */ as text " Prob>=" as res \% 5.3 f pvalue
The first line is the standard way to compute the $p$-value for a chi-squared test with one degree of freedom, except that we divide by 2 . This is because $\alpha$ cannot be negative, as we discussed earlier with regard to comparing the poisson and nbreg models (Gutierrez, Carter, and Drukker 2001). The next line assigns the estimated value of $\ln \alpha$ to a scalar. If this value is very close to 0 , we conclude that the $p$-value is 1 . The last line simply prints the result:

Likelihood ratio test comparing ZIP to ZINB: 109.564 Prob>=0.000
We conclude that the ZINB significantly improves the fit over the ZIP model.

## Vuong test of non-nested models

Greene (1994) points out that PRM and ZIP are not nested. For the ZIP model to reduce to the PRM, it is necessary for $\psi$ to equal zero. This does not occur when $\gamma=\mathbf{0}$ since $\psi=F(\mathbf{z 0})=.5$. Similarly, the NBRM and the ZINB are not nested. Consequently, Greene proposes using a test by Vuong $(1989,319)$ for non-nested models. This test considers two models, where $\widehat{\operatorname{Pr}}{ }_{1}\left(y_{i} \mid \mathbf{x}_{i}\right)$ is the predicted probability of observing $y$ in the first model and $\widehat{\operatorname{Pr}}_{2}\left(y_{i} \mid \mathbf{x}_{i}\right)$ is the predicted probability for the second model. Defining

$$
m_{i}=\ln \left[\frac{\widehat{\operatorname{Pr}}_{1}\left(y_{i} \mid \mathbf{x}_{i}\right)}{\widehat{\operatorname{Pr}}_{2}\left(y_{i} \mid \mathbf{x}_{i}\right)}\right]
$$

let $\bar{m}$ be the mean and let $s_{m}$ be the standard deviation of $m_{i}$. The Vuong statistic to test the hypothesis that $E(m)=0$ equals

$$
V=\frac{\sqrt{N} \bar{m}}{s_{m}}
$$

$V$ has an asymptotic normal distribution. If $V>1.96$, the first model is favored; if $V<-1.96$, the second model is favored.

For zip, the vuong option computes the Vuong statistic comparing the ZIP model to the PRM; for zinb it compares ZINB to NBRM. For example,

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```
. zip art fem mar kid5 phd ment, inf(fem mar kid5 phd ment) vuong nolog
    (output omitted)
Vuong Test of Zip vs. Poisson: Std. Normal = 4.18 Pr> Z = 0.0000
```

The significant, positive value of $V$ supports the ZIP model over the PRM. If you use listcoef you get more guidance in interpreting the result:

```
. listcoef, help
zip (N=915): Factor Change in Expected Count
    (output omitted)
    Vuong Test = 4.18 ( }\textrm{p}=0.000\mathrm{ ) favoring ZIP over PRM.
```

For the ZINB,

```
. listcoef, help
zinb (N=915): Factor Change in Expected Count
    (output omitted)
    Vuong Test = 2.24 (p=0.012) favoring ZINB over NBRM.
```

While it is possible to compute a Vuong statistic to compare other pairs of models, such as ZIP and NBRM, these are currently not available in Stata.

Overall, these tests provide evidence that the ZINB model fits the data best. However, when fitting a series of models without any theoretical rationale, it is easy to overfit the data. In our example, the most compelling evidence for the ZINB is that it makes substantive sense. Within science, there are some scientists who for structural reasons cannot publish, but for other scientists, the failure to publish in an given period is a matter of chance. This is the basis of the zero inflated models. The negative binomial version of the model seems preferable to the Poisson version, since it is likely that there are unobserved sources of heterogeneity that differentiate the scientists. In sum, the ZINB makes substantive sense and fits the data well.

# REGRESSION MODELS FOR CATEGORICAL DEPENDENT VARIABLES USING STATA 

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## 8 Additional Topics

In this final chapter, we discuss some disparate topics that were not covered in the preceding chapters. We begin by considering complications on the right hand side of the model: nonlinearities, interactions, and nominal or ordinal variables coded as a set of dummy variables. While the same principles of interpretation apply in these cases, several tricks are necessary for computing the appropriate quantities. Next, we discuss briefly what is required if you want to modify SPost to work with other estimation commands. The final section discusses a menagerie of Stata "tricks" that we find useful for working more efficiently in Stata.

### 8.1 Ordinal and nominal independent variables

When an independent variable is categorical, it should be entered into the model as a set of binary, indicator variables. While our example uses an ordinal variable, the discussion applies equally to nominal independent variables, with one exception that is clearly noted.

### 8.1.1 Coding a categorical independent variable as a set of dummy variables

A categorical independent variable with $J$ categories can be included in a regression model as a set of $J-1$ dummy variables. In this section, we use a binary logit model to analyze factors affecting whether a scientist has published. The outcome is a dummy variable hasarts that is equal to 1 if the scientist has one or more publications and equals 0 otherwise. In our analysis in the last chapter, we included the independent variable ment, which we treated as continuous. But, suppose instead that the data were from a survey in which the mentor was asked to indicate whether he or she had 0 articles (none), 1 to 3 articles (few), 4 to 9 (some), 10 to 20 (many), or more than 20 articles (lots). The resulting variable, which we call mentord, has the following frequency distribution: ${ }^{1}$
(Continued on next page)

[^45]| . tab mentord, missing |  |  |  |
| :--- | ---: | ---: | ---: |
| Ordinal <br> measure of <br> mentor's |  |  |  |
| articles |  |  |  |$\quad$ Freq. $\quad$ Percent $\quad$ Cum. |  |  |  |
| ---: | ---: | ---: |
| None | 90 | 9.84 |
| Few | 201 | 21.97 |
| Some | 324 | 35.41 |
| Many | 213 | 23.28 |
| Lots | 87 | 9.51 |
| Total | 915 | 100.00 |

We can convert mentord into a set of dummy variables using a series of generate commands. Since the dummy variables are used to indicate in which category an observation belongs, they are often referred to as indicator variables. First we construct none to indicate that the mentor had no publications:
. gen none $=($ mentord $==0)$ if mentord ${ }^{\sim}=$.
Expressions in Stata equal 1 if true and 0 if false. Accordingly, gen none $=$ (mentord=$=0$ ) creates none equal to 1 for scientists whose mentor had no publications and equal to 0 otherwise. Although we do not have any missing values for mentord, it is a good habit to always add an if condition so that missing values continue to be missing (remember that a missing value is treated by Stata as positive infinity when evaluating expressions). This is done by adding if mentord ${ }^{\sim}=$. to the command. We use tab to verify that none was constructed correctly:
. tab none mentord, missing
none

In the same way, we create indicator variables for the other categories of mentord:

```
. gen few = (mentord == 1) if mentord ~= .
. gen some = (mentord == 2) if mentord ~ = .
. gen many = (mentord == 3) if mentord ~ = .
. gen lots = (mentord == 4) if mentord ~= .
```

Note You can also construct indictor variables using xi or tabulate's gen() option. For further information, type help xi and help tabulate.

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### 8.1.2 Estimation and interpretation with categorical independent variables

Since mentord has $J=5$ categories, we must include $J-1=4$ indicator variables as independent variables in our model. To see why one of the indicators must be dropped, consider our example. If you know that none, few, some, and many are all 0 , it must be the case that lots equals 1 since a person has to be in one of the five categories. Another way to think of this is to note that none $+\mathrm{few}+$ some + many + lots $=1$ so that including all $J$ categories would lead to perfect collinearity. If you include all five indicator variables, Stata automatically drops one of them. For example,
. logit hasarts fem mar kid5 phd none few some many lots, nolog
note: lots dropped due to collinearity

| Logit estimates |
| :--- |
| $\quad$ (output omitted) |$\quad$ Number of obs $\quad=\quad 915$

The category that is excluded is the reference category, since the coefficients for the included indicators are interpreted relative to the excluded category which serves as a point of reference. Which category you exclude is arbitrary, but with an ordinal independent variable it is generally easier to interpret the results when you exclude an extreme category. For nominal categories, it is often useful to exclude the most important category. For example, we estimate a binary logit, excluding the indicator variable none:
. logit hasarts fem mar kid5 phd few some many lots, nolog

| Logit estimates | Number of obs | $=$ |
| :--- | :--- | :--- |
|  | LR chi2 (8) | $=$ |
|  | Prob > chi2 | $=$ |
| Log likelihood $=-522.46467$ | Pseudo R2 | $=$ |
|  |  | 0.0000 |
|  |  |  |


| hasarts | Coef. | Std. Err. | z | $\mathrm{P}>\|\mathrm{z}\|$ | [95\% Conf. Interval] |  |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| fem | -.2579293 | .1601187 | -1.61 | 0.107 | -.5717562 | .0558976 |
| mar | .3300817 | .1822141 | 1.81 | 0.070 | -.0270514 | .6872147 |
| kid5 | -.2795751 | .1118578 | -2.50 | 0.012 | -.4988123 | -.0603379 |
| phd | .0121703 | .0802726 | 0.15 | 0.879 | -.145161 | .1695017 |
| few | .3859147 | .2586461 | 1.49 | 0.136 | -.1210223 | .8928517 |
| some | .9602176 | .2490498 | 3.86 | 0.000 | .4720889 | 1.448346 |
| many | 1.463606 | .2829625 | 5.17 | 0.000 | .9090099 | 2.018203 |
| lots | 2.335227 | .4368715 | 5.35 | 0.000 | 1.478975 | 3.19148 |
| _cons | -.0521187 | .3361977 | -0.16 | 0.877 | -.7110542 | .6068167 |

Logit models can be interpreted in terms of factor changes in the odds, which we compute using listcoef:
(Continued on next page)

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| logit ( $\mathrm{N}=915$ ) : Factor Change in Odds Odds of: Arts vs NoArts |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| hasarts | b | z | $P>\|z\|$ | $e^{\wedge} \mathrm{b}$ | $e^{\wedge} \mathrm{bStdX}$ | SDof X |
| fem | -0.25793 | -1.611 | 0.107 | 0.7726 | 0.8793 | 0.4987 |
| mar | 0.33008 | 1.812 | 0.070 | 1.3911 | 1.1690 | 0.4732 |
| kid5 | -0.27958 | -2.499 | 0.012 | 0.7561 | 0.8075 | 0.7649 |
| phd | 0.01217 | 0.152 | 0.879 | 1.0122 | 1.0121 | 0.9842 |
| few | 0.38591 | 1.492 | 0.136 | 1.4710 | 1.1734 | 0.4143 |
| some | 0.96022 | 3.856 | 0.000 | 2.6123 | 1.5832 | 0.4785 |
| many | 1.46361 | 5.172 | 0.000 | 4.3215 | 1.8568 | 0.4228 |
| lots | 2.33523 | 5.345 | 0.000 | 10.3318 | 1.9845 | 0.2935 |

The effect of an indicator variable can be interpreted in the same way that we interpreted dummy variables in Chapter 4, but with comparisons being relative to the reference category. For example, the odds ratio of 10.33 for lots can be interpreted as

The odds of a scientist publishing are 10.3 times larger if his or her mentor had lots of publications compared to no publications, holding other variables constant.
or, equivalently:
If a scientist's mentor has lots of publications as opposed to no publications, the odds of a scientist publishing are 10.3 times larger, holding other variables constant.

The odds ratios for the other indicators can be interpreted in the same way.

### 8.1.3 Tests with categorical independent variables

The basic ideas and commands for tests that involve categorical independent variables are the same as those used in prior chapters. But, since the tests involve some special considerations, we review them here.

## Testing the effect of membership in one category versus the reference category

When a set of indicator variables are included in a regression, a test of the significance of the coefficient for any indicator variable is a test of whether being in that category compared to being in the reference category affects the outcome. For example, the coefficient for few can be used to test whether having a mentor with few publications compared to having a mentor with no publications significantly affects the scientist's publishing. In our example, $z=1.492$ and $p=0.136$, so we conclude that

The effect of having a mentor with a few publications compared to none is not significant using a two-tailed test $(z=1.492, p=0.14)$.

Often the significance of an indicator variable is reported without mentioning the reference category. For example, the test of many could be reported as

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Having a mentor with a many publications significantly affects a scientist's productivity ( $z=5.17, p<.01$ ).

Here, the comparison is implicitly being made to mentors with no publications. Such interpretations should only be used if you are confident that the implicit comparison will be apparent to the reader.

## Testing the effect of membership in two non-reference categories

What if neither of the categories that we wish to compare is the reference category? A simple solution is to re-estimate the model with a different reference category. For example, to test the effect of having a mentor with some articles compared to a mentor with many publications, we can re-estimate the model using some as the reference category:


The $z$-statistics for the mentor indicator variables are now tests comparing a given category to that of the mentor having some publications.

Advanced: lincom Notice that for the model that excludes some, the estimated coefficient for many equals the difference between the coefficients for many and some in the earlier model that excluded none. This suggests that instead of re-estimating the model, we could have used lincom to estimate $\beta_{\text {many }}-\beta_{\text {some }}$ :


The result is identical to that obtained by re-estimating the model with a different base category.

[^46]
## Testing that a categorical independent variable has no effect

For an omnibus test of a categorical variable, our null hypothesis is that the coefficients for all of the indicator variables are zero. In our model where none is the excluded variable, the hypothesis to test is

$$
H_{0}: \beta_{\text {few }}=\beta_{\text {some }}=\beta_{\text {lots }}=\beta_{\text {many }}=0
$$

This hypothesis can be tested with an LR test by comparing the model with the four indicators to the model that drops the four indicator variables:

```
. logit hasarts fem mar kid5 phd few some many lots, nolog
    (output omitted)
. lrtest, saving(0)
. logit hasarts fem mar kid5 phd, nolog
    (output omitted)
. lrtest, using(0)
Logit: likelihood-ratio test chi2(4) = 58.32
Prob > chi2 = 0.0000
```

We conclude that

The effect of the mentor's productivity is significant at the .01 level $\left(L R X^{2}=58.32\right.$, $d f=4, p<.01)$.

Alternatively, a Wald test can be used, although the LR test is generally preferred:

```
. logit hasarts fem mar kid5 phd few some many lots, nolog
    (output omitted)
. test few some many lots
( 1) few = 0.0
(2) some = 0.0
( 3) many = 0.0
(4) lots = 0.0
    chi2( 4) = 51.60
    Prob > chi2 = 0.0000
```

which leads to the same conclusion as the LR test.
Note that exactly the same results would be obtained for either test if we had used a different reference category and tested, for example,

$$
H_{0}: \beta_{\text {none }}=\beta_{\text {few }}=\beta_{\text {lots }}=\beta_{\text {many }}=0
$$

## Testing whether treating an ordinal variable as interval loses information

Ordinal independent variables are often treated as interval in regression models. For example, rather than include the four indicator variables that were created from mentord, we might simply include only mentord in our model:

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The advantage of this approach is that interpretation is simpler, but to take advantage of this simplicity you must make the strong assumption that successive categories of the ordinal independent variable are equally spaced. For example, it implies that an increase from no publications by the mentor to a few publications involves an increase of the same amount of productivity as an increase from a few to some, from some to many, and from many to lots of publications.

Accordingly, before treating an ordinal independent variable as if it were interval, you should test whether this leads to a loss of information about the association between the independent and dependent variable. A likelihood ratio test can be computed by comparing the model with only mentord to the model that includes both the ordinal variable (mentord) and all but two of the indicator variables. In the example below, we add some, many, and lots, but including any three of the indicators leads to the same results. If the categories of the ordinal variables are equally spaced, then the coefficients of the $J-2$ indicator variables should all be 0 . For example,

```
. logit hasarts fem mar kid5 phd mentord, nolog
    (output omitted)
. lrtest, saving(0)
. logit hasarts fem mar kid5 phd mentord some many lots, nolog
    (output omitted)
. lrtest, saving(1)
. lrtest, model(0) using(1)
Logit: likelihood-ratio test chi2(3) = 1.07
```

We conclude that the indicator variables do not add additional information to the model ( $L R X^{2}=$ $1.07, d f=3, p=.78$ ). If the test was significant, we would have evidence that the categories of mentord are not evenly spaced and so one should not treat mentord as interval. A Wald test can also be computed, leading to the same conclusion:

```
. logit hasarts fem mar kid5 phd mentord some many lots, nolog
(output omitted)
. test some many lots
(1) some = 0.0
(2) many = 0.0
( 3) lots = 0.0
    chi2( 3)}={\begin{array}{ll}{1.03}\\{\mathrm{ Prob > chi2 }=}&{0.7950}
```

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### 8.1.4 Discrete change for categorical independent variables

There are a few tricks that you must be aware of when computing discrete change for categorical independent variables. To show how this is done, we will compute the change in the probability of publishing for those with a mentor with few publications compared to a mentor with no publications. There are two ways to compute this discrete change. The first way is easier, but the second is more flexible.

## Computing discrete change with prchange

The easy way is to use prchange, where we set all of the indicator variables to 0 :

```
. logit hasarts fem mar kid5 phd few some many lots, nolog
    (output omitted)
. prchange few, x(some=0 many=0 lots=0)
logit: Changes in Predicted Probabilities for hasarts
\begin{tabular}{|c|c|c|c|c|c|c|c|c|}
\hline \multicolumn{2}{|r|}{min->max} & 0->1 & -+1/2 & -+sd/2 & \multicolumn{4}{|l|}{MargEfct} \\
\hline few & 0.0957 & 0.0957 & 0.0962 & 0.0399 & 0.096 & & & \\
\hline & NoArts & Arts & & & & & & \\
\hline \(\operatorname{Pr}(\mathrm{y} \mid \mathrm{x})\) & x) 0.4920 & 0.5080 & & & & & & \\
\hline & fem & mar & kid5 & phd & few & some & many & lots \\
\hline \(\mathrm{x}=\) & . 460109 & . 662295 & . 495082 & 3.10311 & . 219672 & 0 & 0 & 0 \\
\hline \(s d(x)=\) & . 498679 & . 473186 & . 76488 & . 984249 & . 414251 & . 478501 & . 422839 & . 293489 \\
\hline
\end{tabular}
```

We conclude that

Having a mentor with a few publications compared to none increases a scientist's probability of publishing by .10 , holding all other variables at their mean.

Even though we say "holding all other variables at their mean", which is clear within the context of reporting substantive results, the key to getting the right answer from prchange is holding all of the indicator variables at 0 , not at their mean. It does not make sense to change few from 0 to 1 when some, many, and lots are at their means.

## Computing discrete change with prvalue

A second approach to computing discrete change is to use a pair of calls to prvalue. The advantage of this approach is that it works in situations where prchange does not. For example, how does the predicted probability change if we compare a mentor with a few publications to a mentor with some publications, holding all other variables constant? This involves computing probabilities as we move from $f e w=1$ and some $=0$, to $f e w=0$ and some $=1$. We cannot compute this with prchange since two variables are changing at the same time. Instead, we use two calls to prvalue: ${ }^{2}$

[^47]8.2 Interactions

| logit: Change in Predictions for hasarts |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | Current | Saved | Difference |  |  |
| $\operatorname{Pr}(\mathrm{y}=\mathrm{Ar}$ | s\|x): | 0.7125 | 0.5825 | 0.1300 |  |  |
| $\operatorname{Pr}(\mathrm{y}=\mathrm{N}$ | rts\|x) : | 0.2875 | 0.4175 | -0.1300 |  |  |
|  | fem | mar | kid5 | phd | few | some |
| Current= | . 46010929 | . 66229508 | . 49508197 | 3.1031093 | 0 | 1 |
| Saved= | . 46010929 | . 66229508 | . 49508197 | 3.1031093 | 1 | 0 |
| Diff= | 0 | 0 | 0 | 0 | -1 | 1 |
|  | many | lots |  |  |  |  |
| Current= | 0 | 0 |  |  |  |  |
| Saved= | 0 | 0 |  |  |  |  |
| Diff= | 0 | 0 |  |  |  |  |

Because we have used the save and dif options, the difference in the predicted probability (i.e., discrete change) is reported. When we use the save and dif options, we usually add quietly to the first prvalue since all of the information is listed by the second prvalue.

### 8.2 Interactions

Interaction terms are commonly included in regression models when the effect of an independent variable is thought to vary depending on the value of another independent variable. To illustrate how interactions are used, we extend the example from Chapter 5, where the dependent variable is a respondent's level of agreement that a working mother can establish as warm a relationship with her children as mothers who do not work.

It is possible that the effect of education on attitudes towards working mothers varies by gender. To allow this possibility, we add the interaction of education (ed) and gender (male) by adding the variable maleXed $=$ male $\times e d$. In estimating this model, we find that


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| _cut1 | -2.107903 | .3043008 | (Ancillary parameters) |
| :--- | ---: | ---: | :--- |
| _cut2 | -.2761098 | .2992857 |  |
| _cut3 | 1.621787 | .3018749 |  |

The interaction is marginally significant $(p=.06)$ for a two-tailed Wald test. Alternatively, we can compute an LR test

```
. ologit warm age prst yr89 white male ed, nolog
    (output omitted)
. lrtest, saving(0)
. ologit warm age prst yr89 white male ed maleXed, nolog
    (output omitted)
. lrtest, saving(1)
. lrtest, model(0) using(1)
Ologit: likelihood-ratio test }\quad\begin{array}{ll}{\mathrm{ chi2(1) }}&{=}\\{\mathrm{ Prob > chi2 }}&{=}
```

which leads to the same conclusion.

### 8.2.1 Computing gender differences in predictions with interactions

What if we want to compute the difference between men and women in the predicted probabilities for the outcome categories? Gender differences are reflected in two ways in the model. First, we want to change male from 0 to 1 to indicate women versus men. If this was the only variable affected by changing the value of male, we could use prchange. But, when the value of male changes, this necessarily changes the value of maleXed (except in the case when ed is 0 ). For women, maleXed $=$ male $\times \mathrm{ed}=0 \times \mathrm{ed}=0$, while for men, maleXed=male $\times \mathrm{ed}=1 \times \mathrm{ed}=\mathrm{ed}$. Accordingly, we must examine the change in the outcome probabilities when two variables change, so prvalue must be used. We start by computing the predicted values for women, which requires fixing male $=0$ and maleXed=0:

```
. prvalue, x(male=0 maleXed=0) rest(mean) save
ologit: Predictions for warm
\begin{tabular}{|c|c|c|c|c|c|}
\hline \(\operatorname{Pr}(\mathrm{y}=\mathrm{SD} \mid \mathrm{x})\) : & \multicolumn{2}{|c|}{0.0816} & & & \\
\hline \(\operatorname{Pr}(\mathrm{y}=\mathrm{D} \mid \mathrm{x})\) : & \multicolumn{2}{|c|}{0.2754} & & & \\
\hline \(\operatorname{Pr}(\mathrm{y}=\mathrm{A} \mid \mathrm{x})\) : & \multicolumn{2}{|c|}{0.4304} & & & \\
\hline \(\operatorname{Pr}(\mathrm{y}=\mathrm{SA} \mid \mathrm{x})\) : & \multicolumn{2}{|c|}{0.2126} & & & \\
\hline age & prst & yr89 & white & male & ed \\
\hline \(\mathrm{x}=44.935456\) & 39.585259 & . 39860445 & . 8765809 & 0 & 12.218055 \\
\hline maleXed & & & & & \\
\hline \(\mathrm{x}=0\) & & & & & \\
\hline
\end{tabular}
```

Next, we compute the predicted probability for men, where male $=1$ and maleXed equals the average value of education (since for men, maleXed=male $\times e d=1 \times e d=e d$ ). The value for maleXed can be obtained by computing the mean of ed:

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. sum ed

| Variable | Obs | Mean | Std. Dev. | Min | Max |
| ---: | ---: | ---: | ---: | ---: | ---: |
| ed | 2293 | 12.21805 | 3.160827 | 0 | 20 |

- global meaned $=r($ mean $)$
summarize returns the mean to $r$ (mean). The command global meaned $=r$ (mean) assigns the mean of ed to the global macro meaned. In the prvalue command, we specify $x$ (male=1 maleXed=\$meaned), where \$meaned tells Stata to substitute the value contained in the global macro:

| ologit: Change in Predictions for warm |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | Current | Saved | Difference |  |  |
| $\operatorname{Pr}(\mathrm{y}=\mathrm{SD}$ | \|x) : | 0.1559 | 0.0816 | 0.0743 |  |  |
| $\operatorname{Pr}(\mathrm{y}=\mathrm{D}$ | x) : | 0.3797 | 0.2754 | 0.1044 |  |  |
| $\operatorname{Pr}(\mathrm{y}=\mathrm{A}$ | x) : | 0.3494 | 0.4304 | -0.0811 |  |  |
| $\operatorname{Pr}(\mathrm{y}=\mathrm{S}$ | \|x) : | 0.1150 | 0.2126 | -0.0976 |  |  |
|  | age | prst | yr89 | white | male | ed |
| Current= | 44.935456 | 39.585259 | . 39860445 | . 8765809 | 1 | 12.218055 |
| Saved= | 44.935456 | 39.585259 | . 39860445 | . 8765809 | 0 | 12.218055 |
| Diff= | 0 | 0 | 0 | 0 | 1 | 0 |

Warning The mean of maleXed does not equal the mean of ed. That is why we could not use the option: x (male=1 maleXed=mean) and instead had to compute the mean with summarize.

While the trick of using maleXed=\$meaned may seem like a lot of trouble to avoid having to type maleXed=12.21805, it can help you avoid errors and in some cases (illustrated below) it saves a lot of time.

Substantively, we conclude that the probability of strongly agreeing that working mothers can be good mothers is .10 higher for woman than men, taking the interaction with education into account and holding other variables constant at their means. The probability of strongly disagreeing is .07 higher for men than women.

### 8.2.2 Computing gender differences in discrete change with interactions

We might also be interested in how the predicted outcomes are affected by a change in education from having a high school diploma (12 years of education) to having a college degree (16 years). Since the interaction term suggests that the effect of education varies by gender, we must look at the discrete change separately for men and women. Again, repeated calls to prvalue using the save and dif options allow us to do this. For women, we hold both male and maleXed to 0 and allow ed to vary. For men, we hold male to 1 and allow both ed and maleXed to vary. For women, we find that

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| ologit: Change in Predictions for warm |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | Current | Saved | Difference |  |  |
| $\operatorname{Pr}(\mathrm{y}=\mathrm{SD}$ | x) : | 0.0579 | 0.0833 | -0.0254 |  |  |
| $\operatorname{Pr}(\mathrm{y}=\mathrm{D}$ |  | 0.2194 | 0.2786 | -0.0592 |  |  |
| $\operatorname{Pr}(\mathrm{y}=\mathrm{A}$ |  | 0.4418 | 0.4291 | 0.0127 |  |  |
| $\operatorname{Pr}(\mathrm{y}=\mathrm{S}$ | x) : | 0.2809 | 0.2090 | 0.0718 |  |  |
|  | age | prst | yr89 | white | male | ed |
| Current= | 44.935456 | 39.585259 | . 39860445 | . 8765809 | 0 | 16 |
| Saved= | 44.935456 | 39.585259 | . 39860445 | . 8765809 | 0 | 12 |
| Diff= | 0 | 0 | 0 | 0 | 0 | 4 |
| maleXed |  |  |  |  |  |  |
| Current= | 0 |  |  |  |  |  |
| Saved= | 0 |  |  |  |  |  |
| Diff= | 0 |  |  |  |  |  |

For men,

| ologit: Change in Predictions for warm |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | Current | Saved | Difference |  |  |
| $\operatorname{Pr}(\mathrm{y}=\mathrm{SD}$ | x) : | 0.1326 | 0.1574 | -0.0248 |  |  |
| $\operatorname{Pr}(\mathrm{y}=\mathrm{D}$ |  | 0.3558 | 0.3810 | -0.0252 |  |  |
| $\operatorname{Pr}(\mathrm{y}=\mathrm{A}$ |  | 0.3759 | 0.3477 | 0.0282 |  |  |
| $\operatorname{Pr}(\mathrm{y}=\mathrm{S}$ | x) : | 0.1357 | 0.1139 | 0.0218 |  |  |
|  | age | prst | yr89 | white | male | ed |
| Current= | 44.935456 | 39.585259 | . 39860445 | . 8765809 | 1 | 16 |
| Saved= | 44.935456 | 39.585259 | . 39860445 | . 8765809 | 1 | 12 |
| Diff= | 0 | 0 | 0 | 0 | 0 | 4 |
| maleXed |  |  |  |  |  |  |
| Current= | 16 |  |  |  |  |  |
| Saved= | 12 |  |  |  |  |  |
| Diff= | 4 |  |  |  |  |  |

The largest difference in the discrete change between the sexes is for the probability of answering "strongly agree." For both men and women, an increase in education from 12 years to 16 years increases the probability of strong agreement, but the increase is .07 for women and only .02 for men.

### 8.3 Nonlinear nonlinear models

The models that we consider in this book are nonlinear models in that the effect of a change in an independent variable on the predicted probability or predicted count depends on the values of all of

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the independent variables. However, the right-hand side of the model includes a linear combination of variables just like the linear regression model. For example,

$$
\begin{array}{lrl}
\text { Linear Regression: } & y=\beta_{0}+\beta_{1} x_{1}+\beta_{2} x_{2}+\varepsilon \\
\qquad \text { Binary Logit: } & \operatorname{Pr}(y=1 \mid \mathbf{x})=\frac{\exp \left(\beta_{0}+\beta_{1} x_{1}+\beta_{2} x_{2}+\varepsilon\right)}{1+\exp \left(\beta_{0}+\beta_{1} x_{1}+\beta_{2} x_{2}+\varepsilon\right)}
\end{array}
$$

In the terminology of the generalized linear model, we would say that both models have the same linear predictor: $\beta_{0}+\beta_{1} x_{1}+\beta_{2} x_{2}$. In the linear regression model, this leads to predictions that are linear surfaces. For example, with one independent variable the predictions are a line, with two a plane, and so on. In the binary logit model, the prediction is a curved surface, as illustrated in Chapter 4.

### 8.3.1 Adding nonlinearities to linear predictors

Nonlinearities in the LRM can be introduced by adding transformations on the right hand side. For example, in the model

$$
y=\alpha+\beta_{1} x+\beta_{2} x^{2}+\varepsilon
$$

we include $x$ and $x^{2}$ to allow predictions that are a quadratic form. For example, if the estimated model is $\widehat{y}=1+-.1 x+.1 x^{2}$, then the plot is far from linear:


In the same fashion, nonlinearities can be added to the right hand side of the models for categorical outcomes that we have been considering. What may seem odd is that adding nonlinearities to a nonlinear model can sometimes make the predictions more linear.

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### 8.3.2 Discrete change in nonlinear nonlinear models

In the model of labor force participation from Chapter 4, we included a woman's age as an independent variable. Often when age is used in a model, terms for both the age and age-squared are included to allow for diminishing (or increasing) effects of an additional year of age. First, we estimate the model without age squared and compute the effect of a change in age from 30 to 50 for an average respondent:

| (Data from 1976 PSID-T Mroz) |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| logit lfp k5 k618 wc hc lwg inc age, nolog (output omitted) |  |  |  |  |  |  |  |
| . prchange age, $x(a g e=30)$ delta(20) uncentered |  |  |  |  |  |  |  |
| logit: Changes in Predicted Probabilities for lfp |  |  |  |  |  |  |  |
| (Note: delta $=20$ ) |  |  |  |  |  |  |  |
| min->max |  | $0->1$ | +delta | +sd MargEfct |  |  |  |
| age -0 | -0.4372 | -0.0030 | -0.2894 | -0.1062 -0.0118 |  |  |  |
| $\operatorname{Pr}(\mathrm{y} \mid \mathrm{x})$ | NotInLF | - inLF |  |  |  |  |  |
|  | x) 0.2494 | 40.7506 |  |  |  |  |  |
|  | k5 | k618 | wc | hc | lwg | inc | age |
| $\mathrm{x}=$ | $=.237716$ | 1.35325 | . 281541 | . 391766 | 1.09711 | 20.129 | 30 |
| $s d(x)=$ | $=.523959$ | 1.31987 | . 450049 | . 488469 | . 587556 | 11.6348 | 8.07257 |

Notice that we have taken advantage of the delta () and uncentered options (see Chapter 3). We find that the predicted probability of a woman working decreases by . 29 as age increases from 30 to 50 , with all other variables at the mean. Now we add age-squared to the model:

| . logit lfp k5 k618 wc hc lwg inc age age2, nolog |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Logit estimates |  |  |  | Number of obs LR chi2(8) |  | 753 |
|  |  |  |  | 125.67 |
|  |  |  |  | Prob > chi2 | 0.0000 |
| Log likelihood $=-452.03836$ |  |  |  |  |  | Pseudo R2 |  | 0.1220 |
| lfp | Coef. | Std. Err. | z | $P>\|z\|$ | [95\% | Interval] |
| k5 | -1.411597 | . 2001829 | -7.05 | 0.000 | -1.803 | -1.019246 |
| k618 | -. 0815087 | . 0696247 | -1.17 | 0.242 | -. 2179 | . 0549531 |
| wc | . 8098626 | . 2299065 | 3.52 | 0.000 | . 3592 | 1. 260471 |
| hc | . 1340998 | . 207023 | 0.65 | 0.517 | -. 2716 | . 5398575 |
| lwg | . 5925741 | . 1507807 | 3.93 | 0.000 | . 2970 | . 8880988 |
| inc | -. 0355964 | . 0083188 | -4.28 | 0.000 | -. 0519 | -. 0192919 |
| age | . 0659135 | . 1188199 | 0.55 | 0.579 | -. 1669 | . 2987962 |
| age2 | -. 0014784 | . 0013584 | -1.09 | 0.276 | -. 0041 | . 001184 |
| _cons | . 511489 | 2.527194 | 0.20 | 0.840 | -4.44 | 5.464698 |

To test for the joint significance of age and age2, we use a likelihood-ratio test:

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```
. quietly logit lfp k5 k618 wc hc lwg inc, nolog
. lrtest, saving(0)
. quietly logit lfp k5 k618 wc hc lwg inc age age2, nolog
. lrtest, saving(2)
. lrtest, model(0) using(2)
Logit: likelihood-ratio test chi2(2) = 26.79
```

We can no longer use prchange to compute the discrete change since we need to change two variables at the same time. Once again we use a pair of prvalue commands, where we change age from 30 to 50 and change age 2 from $30^{2}(=900)$ to $50^{2}(=2500)$. First we compute the prediction with age at 30 :

```
. global age30 = 30
. global age30sq = $age30*$age30
. quietly prvalue, x(age=$age30 age2=$age30sq) rest(mean) save
```

Then, we let age equal 50 and compute the difference:

| - global age50 $=50$ |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| global age50sq $=$ \$age $50 *$ age 50 |  |  |  |  |  |  |
| prvalue, $\mathrm{x}($ age=\$age50 age2=\$age50sq) rest (mean) dif |  |  |  |  |  |  |
| logit: Change in Predictions for lfp |  |  |  |  |  |  |
| $\operatorname{Pr}(\mathrm{y}=$ inLF $\mid x):$ |  | Current | Saved | Difference |  |  |
|  |  | 0.4699 | 0.7164 | -0.2465 |  |  |
| $\operatorname{Pr}(\mathrm{y}=\operatorname{NotInLF} \mid \mathrm{x})$ : |  | 0.5301 | 0.2836 | 0.2465 |  |  |
|  | k5 | k618 | wc | hc | $1 \mathrm{l} g$ | inc |
| Current= | . 2377158 | 1.3532537 | . 2815405 | . 39176627 | 1.0971148 | 20.128965 |
| Saved= | . 2377158 | 1.3532537 | . 2815405 | . 39176627 | 1.0971148 | 20.128965 |
| Diff= | 0 | 0 | 0 | 0 | 0 | 0 |
|  | age | age2 |  |  |  |  |
| Current= | 50 | 2500 |  |  |  |  |
| Saved= | 30 | 900 |  |  |  |  |
| Diff= | 20 | 1600 |  |  |  |  |

We conclude that

An increase in age from 30 to 50 years decreases the probability of being in the labor force by .25 , holding other variables at their mean.

By adding the squared term, we have decreased our estimate of the change. While in this case the difference is not large, the example illustrates the general point of how to add nonlinearities to the model.

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### 8.4 Using praccum and forvalues to plot predictions

In prior chapters, we used prgen to generate predicted probabilities over the range of one variable while holding other variables constant. While prgen is a relatively simple way of generating predictions for graphs, it can be used only when the specification of the right hand side of the model is straightforward. When interactions or polynomials are included in the model, graphing the effects of a change in an independent variable often requires computing changes in the probabilities as more than one of the variables in the model changes (e.g., age and age2). We created praccum to handle such situations. The user calculates each of the points to be plotted through a series of calls to prvalue. Executing praccum immediately after prvalue accumulates these predictions.

The first time praccum is run, the predicted values are saved in a new matrix. Each subsequent call to praccum adds new predictions to this matrix. When all of the calls to prvalue have been completed, the accumulated predictions in the matrix can be added as new variables to the dataset in an arrangement ideal for plotting, just like with prgen. The syntax of praccum is

```
praccum [, xis(value) using(matrixname) saving(matrixname) generate(prefix)]
```

where either using () or saving () is required.

## Options

xis (value) indicates the value of the $x$ variable associated with the predicted values that are accumulated. For example, this could be the value of age if you wish to plot changes in predicted values as age changes. You do not need to include the values of variables created as transformations of this variable. To continue the example, you would not include the value of age squared.
using (matrixname) specifies the name of the matrix where the predictions from the previous call to prvalue should be added. An error is generated if the matrix does not have the correct number of columns. This can happen if you try to append values to a matrix generated from calls to praccum based on a different model. Matrix matrixname will be created if it does not already exist.
saving (matrixname) specifies that a new matrix should be generated to contain the predicted values from the previous call to prvalue. You only use this option when you initially create the matrix. After the matrix is created, you add to it with using(). The difference between saving () and using () is that saving () will overwrite matrixname if it exists, while using () will append results to it.
generate (prefix) indicates that new variables are to be added to the current dataset. These variables begin with prefix and contain the values accumulated in the matrix in prior calls to praccum.
The generality of praccum requires it to be more complicated to use than prgen.

### 8.4.1 Example using age and age-squared

To illustrate the command, we use praccum to plot the effects of age on labor force participation for a model in which both age and age-squared are included. First, we compute the predictions from the

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model without age2:
. quietly logit lfp k5 k618 age wc hc lwg inc
. prgen age, from(20) to(60) gen(prage) ncases(9)
logit: Predicted values as age varies from 20 to 60.

|  | k 5 | k 618 | age | wc | hc |
| :--- | ---: | ---: | ---: | ---: | ---: |
| $\mathrm{x}=$ | .2377158 | 1.3532537 | 42.537849 | .2815405 | .39176627 |
|  | inc |  |  |  | 1.0971148 |
|  |  |  |  |  |  |
| $x=$ | 20.128965 |  |  |  |  |

. label var pragep1 $" \operatorname{Pr}(l p f$ | age)"
This is the same thing we did using prgen in earlier chapters. Next, we estimate the model with age2 added:

```
. logit lfp k5 k618 age age2 wc hc lwg inc
    (output omitted)
```

To compute the predictions from this model, we use a series of calls to prvalue. For these predictions, we let age change by 5 -year increments from 20 to 60 and age 2 increase from $20^{2}(=400)$ to $60^{2}(=3600)$. In the first call of praccum, we use the saving () option to declare that mat_age is the matrix that will hold the results. The xis() option is required since it specifies the value for the $x$-axis of the graph that will plot these probabilities:
. quietly prvalue, $x$ (age=20 age2=400) rest(mean)
. praccum, saving(mat_age) xis(20)
We execute prvalue quietly to suppress the output, since we are only generating these predictions in order to save them with praccum. The next set of calls adds new predictions to mat_age, as indicated by the option using():
. quietly prvalue, $x$ (age=25 age2=625) rest(mean)
. praccum, using(mat_age) xis(25)
. quietly prvalue, $x$ (age=30 age2=900) rest(mean)
. praccum, using(mat_age) xis(30)
(and so on)
. quietly prvalue, $x$ (age=55 age2=3025) rest(mean)
. praccum, using(mat_age) xis(55)
The last call includes not only the using() option, but also gen(), which tells praccum to save the predicted values from the matrix to variables that begin with the specified root, in this case agesq:
. quietly prvalue, $x($ age=60 age2=3600) rest(mean)
. praccum, using(mat_age) xis(60) gen(agesq)
New variables created by praccum:

| Variable | Obs | Mean | Std. Dev. | Min | Max |
| ---: | ---: | ---: | ---: | ---: | ---: |
| agesqx | 9 | 40 | 13.69306 | 20 | 60 |
| agesqp0 | 9 | .4282142 | .1752595 | .2676314 | .7479599 |
| agesqp1 | 9 | .5717858 | .1752595 | .2520402 | .7323686 |

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To understand what has been done, it helps to look at the new variables that were created:
. list agesqx agesqp0 agesqp1 in $1 / 10$

|  | agesqx | agesqp0 | agesqp1 |
| ---: | ---: | ---: | ---: |
| 1. | 20 | .2676314 | .7323686 |
| 2. | 25 | .2682353 | .7317647 |
| 3. | 30 | .2836163 | .7163837 |
| 4. | 35 | .3152536 | .6847464 |
| 5. | 40 | .3656723 | .6343277 |
| 6. | 45 | .4373158 | .5626842 |
| 7. | 50 | .5301194 | .4698806 |
| 8. | 55 | .6381241 | .3618759 |
| 9. | 60 | .7479599 | .2520402 |
| 10. | . | . | . |

The tenth observation is all missing values since we only made nine calls to praccum. Each value of agesqx reproduces the value specified in xis(). The values of agesqp0 and agesqp1 are the probabilities of $y=0$ and $y=1$ that were computed by prvalue. We see that the probability of observing a 1 , that is, being is the labor force, was .73 the first time we executed prvalue with age at 20 ; the probability was .25 the last time we executed prvalue with age at 60 . Now that these predictions have been added to the dataset, we can use graph to show how the predicted probability of being in the labor force changes with age:

```
. label var agesqp1 "Pr(lpf | age,age2)"
. label var agesqx "Age"
. set textsize 120
. graph pragep1 agesqp1 agesqx, s(OS) c(sss) xlabel(20 25 to 60) /*
> */ gap(3) l1("Pr(Being in the Labor Force)") ylabel(0 . 2 to 1)
```

We are also plotting pragep1, which was computed earlier in this section using prgen. The graph command leads to the following plot:


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The graph shows that, as age increases from 20 to 60 , a woman's probability of being in the labor force declines. In the model with only age, the decline is from .85 to .31 , while in the model with age-squared, the decrease is from .73 to .25 . Overall, the changes are smaller during younger years and larger after age 50.

### 8.4.2 Using forvalues with praccum

The use of praccum is often greatly simplified by Stata's forvalues command (which was introduced in Stata 7). The forvalues command allows you repeat a set of commands where the only thing that you vary between successive repetitions is the value of some key number. As a trivial example, we can use forvalues to have Stata count from 0 to 100 by fives. Enter the following three lines either interactively or in a do-file:

```
forvalues count = O(5)100 {
display `count`
}
```

In the forvalues statement, count is the name of a local macro that will contain the successive values of interest (see Chapter 2 if you are unfamiliar with local macros). The combination 0 (5) 100 indicates that Stata should begin by setting the value of count at 0 and should increase its value by 5 with each repetition until it reaches 100 . The $\}$ 's enclose the commands that will be repeated for each value of count. In this case, all we want to do is display the value of count. This is done with the command display 'count'. To indicate that count is a local macro, we use the pair of single quote marks (i.e., `count ${ }^{\text {' }}$ ). The output produced is

0
5
10
(and so on )
95
100
In our earlier example, we graphed the effect of age as it increased from 20 to 60 by 5-year increments. If we specify forvalues count 20 (5)60, Stata will repeatedly execute the code we enclose in brackets with the value of count updated from 20 to 60 by increments of 5 . The following lines reproduce the results we obtained earlier:

```
capture matrix drop mage
forvalues count = 20(5)60 {
    local countsq = `count``2
    prvalue, x(age=`count` age2=`countsq`) rest(mean) brief
    praccum, using(mage) xis(`count`)
}
praccum, using(mage) gen(agsq)
```

The command capture matrix drop mage at the beginning will drop the matrix mage if it exists, but the do-file will not halt with an error if the matrix does not exist. Within the forvalues loop, count is set to the appropriate value of age, and we use the local command to create the local macro countsq that contains the square of count. After the all the predictions have been computed and accumulated to matrix mage, we make a last call to praccum in which we use the generate () option to specify the stem of names of the new variables to be generated.

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### 8.4.3 Using praccum for graphing a transformed variable

praccum can also be used when an independent variable is a transformation of the original variable. For example, you might want to include the natural log of age as independent variable rather than age. Such a model can be easily estimated:

```
. gen ageln = ln(age)
. logit lfp k5 k618 ageln wc hc lwg inc
    (output omitted)
```

As in the last example, we use forvalues to execute a series of calls to prvalue and praccum to generate predictions:

```
capture matrix drop mat_ln
forvalues count = 20(5)60 {
    local countln = ln(`count`)
    prvalue, x(ageln=`countln`) rest(mean) brief
    praccum, using(mat_ln) xis(`count`)
}
praccum, using(mat_ln) gen(ageln)
```

We use a local to compute the $\log$ of age, the value of which is passed to prvalue with the option x (ageln=`countln`). But, in praccum we specify xis(`count`) not xis(`countln`). This is because we want to plot the probability against age in its original units. The saved values can then be plotted:

```
. label var agelnp1 "Pr(lpf | log of age)"
. set textsize 120
. graph pragep1 agesqp1 agelnp1 agesqx, s(OST) c(sss) xlabel(20 25 to 60) /*
> */ gap(3) l1("Pr(Being in the Labor Force)") ylabel(0 . 2 to 1)
```

which leads to


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### 8.4.4 Using praccum to graph interactions

Earlier in this chapter we examined an ordinal regression model of support for working mothers that included an interaction between a respondent's sex and education. Another way to examine the effects of the interaction is to plot the effect of education on the predicted probability of strongly agreeing for men and women separately. First, we estimate the model:

```
. use ordwarm2.dta, clear
(77 & 89 General Social Survey)
. gen maleXed = male*ed
. ologit warm age prst yr89 white male ed maleXed
    (output omitted)
```

Next, we compute the predicted values of strongly agreeing as education increases for women who are average on all other characteristics. This is done using forvalues to make a series of calls to prvalue and praccum. For women, maleXed is always 0 since male is 0 :

```
forvalues count = 8(2)20 {
    quietly prvalue, x(male=0 ed=`count` maleXed=0) rest(mean)
    praccum, using(mat_f) xis(`count`)
}
praccum, using(mat_f) gen(pfem)
```

In the successive calls to prvalue, only the variable ed is changing. Accordingly, we could have used prgen. For the men, however, we must use praccum since both ed and maleXed change together:

| forvalues count $=8(2) 20$ \{ quietly prvalue, $x\left(m a l e=1\right.$ ed=`count \({ }^{\prime}\) maleXed=` count`) rest(mean) praccum, using(mat_m) xis(`count') |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| ```} praccum, using(mat_m) gen(pmal) New variables created by praccum:``` |  |  |  |  |  |
|  |  |  |  |  |  |
| Variable | Obs | Mean | Std. Dev. | Min | Max |
| pmalx | 7 | 14 | 4.320494 | 8 | 20 |
| pmalp1 | 7 | . 1462868 | . 0268927 | . 1111754 | . 1857918 |
| pmalp2 | 7 | . 3669779 | . 0269781 | . 3273872 | . 4018448 |
| pmalp3 | 7 | . 3607055 | . 0301248 | . 317195 | . 40045 |
| pmalp4 | 7 | . 1260299 | . 0237202 | . 0951684 | . 1609874 |
| pmals1 | 7 | . 1462868 | . 0268927 | . 1111754 | . 1857918 |
| pmals2 | 7 | . 5132647 | . 0537622 | . 4385626 | . 5876365 |
| pmals3 | 7 | . 8739701 | . 0237202 | . 8390126 | . 9048315 |
| pmals4 | 7 | 1 | $2.25 \mathrm{e}-08$ | . 9999999 | 1 |

Years of education, as it has been specified with xis(), is stored in pfemx and pmalx. These variables are identical since we used the same levels for both men and women. The probabilities for women are contained in the variables $\mathrm{pfemp} k$, where $k$ is the category value; for models for ordered or count data, the variables pfemsk store the cumulative probabilities $\operatorname{Pr}(y \leq k)$. The corresponding predictions for men are contained in pmalpk and pmalsk. All that remains is to clean up the variable labels and plot the predictions:

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```
. label var pfemp4 "Pr(SA | female)"
. label var pmalp4 "Pr(SA | male)"
. label var pfemx "Education in Years"
. set textsize 120
. graph pfemp4 pmalp4 pfemx, s(OS) c(ss)xlabel(8 10 to 20) /*
>*/ ylabel(0 .1 to .4) gap(3) 11("Pr(Strongly Agreeing)")
```

which produces the following plot:


For all levels of education, women are more likely to strongly agree that working mothers can be good mothers than are men, holding other variables to their mean. This difference between men and women is much larger at higher levels of education than at lower levels.

### 8.5 Extending SPost to other estimation commands

The commands in SPost only work with some of the many estimation commands available in Stata. If you try to use our commands after estimating other types of models, you will be told that the SPost command does not work for the last model estimated. Over the past year as we developed these commands, we have received numerous inquiries about whether we can modify SPost to work with additional estimation commands. While we would like to accommodate such requests, extensions are likely to be made mainly to estimation commands that we are using in our own work. There are two reasons for this. First, our time is limited. Second, we want to be sure that we fully understand the specifics of each model before we incorporate it into SPost. Still, users who know how to program in Stata are welcome to extend our programs to work with other models. Keep in mind, however, that we can only provide limited support. While we have attempted to write each

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command to make it as simple as possible to expand, some of the programs are complex and you will need to be adept at programming in Stata. ${ }^{3}$

Here are some brief points that may be useful for a programmer wishing to modify our commands. First, our commands make use of ancillary programs that we have also written, all of which begin with _pe (e.g., _pebase). As will be apparent as you trace through the logic of one of our adofiles, extending a command to a new model might require modifications to these ancillary programs as well. Since the _pe*. ado files are used by many different commands, be careful that you do not make changes that break other commands. Second, our programs use information returned in e() by the estimation command. Some user-written estimation commands, especially older ones, do not return the appropriate information in e(), and extending programs to work after these estimation commands will be extremely difficult.

### 8.6 Using Stata more efficiently

Our introduction to Stata in Chapter 2 focused on the basics. But, as you use Stata, you will discover various tricks that make your use of Stata more enjoyable and efficient. While what constitutes a "good trick" depends on the needs and skills of the particular users, in this section we describe some things that we have found useful.

### 8.6.1 profile.do

When Stata is launched, it looks for a do-file called profile.do in the directories listed when you type sysdir. ${ }^{4}$ If profile.do is found in one of these directories, Stata runs it. Accordingly, you can customize Stata by including commands in profile.do. While you should consult Getting Started with Stata for full details or enter the command help profile, the following examples show you some things that we find useful. We have added detailed comments within the $/ * * /$ 's. The comments do not need to be included in profile.do.

```
/*
    In Stata all data is kept in memory. If you get memory errors when
    loading a dataset or while estimating a model, you need more memory.
    While you can change the amount of memory from the Command Window,
    we find it easier to set it here. Type -help memory- for details.
*/
set memory 30m
/*
    Many programs in official Stata and many of our commands use matrices.
    Some of our commands, such as -prchange- use a lot of memory. So, we
    suggest setting the amount of space for matrices to the largest value
    allowed. Type -help matsize- for details.
*/
set matsize 800
```

[^48]```
/*
    Starting with Stata 7, output in log files can be written either as text
    (as with earlier versions of Stata), or in SMCL. We find it
    easier to save logs as text since they can be more easily printed, copied
    to a word processor, and so on. Type -help log- for details.
*/
set logtype text
/*
    You can assign commands to function keys F2 through F9. After assigning
    a text string to a key, when you press that key, the string is
    inserted into the Command Window.
*/
global F8 "set trace on"
global F9 "set trace off"
/*
    You can tell Stata what you want your default working directory
    to be.
*/
cd d:\statastart
/*
    You can also add notes to yourself. Here we post a reminder that
    the command -spost- will change the working directory to the directory
    where we have the files for this book.
*/
noisily di "spost == cd d:\spost\examples"
```


### 8.6.2 Changing screen fonts and window preferences

In Windows, the default font for the Results Window works well on a VGA monitor with 640 by 480 resolution. But, with higher resolution monitors, we prefer a larger font. To change the font, click on $\square$ in the upper-left corner of the Results Window. Select the Fonts option and choose a font you like. You do not need to select one of the fonts that are named "Stata ..." since any fixed-width font will work. In Windows, we are fond of Andale Mono, which is freely available from Microsoft. The best way to find it is to use an Internet search engine and search for "Andale mono download". When we wrote this, the font was available at www.microsoft.com/typography/fontpack/default.htm.

You can also change the size and position of the windows using the usual methods of clicking and dragging. After the font is selected and any new placement of windows is done, you can save your new options to be the defaults with the Preference menu and the Save Windowing Preference option.

### 8.6.3 Using ado-files for changing directories

One of the things we like best about Stata is that you can create your own commands using adofiles. These commands work just like the commands that are part of official Stata, and indeed many

[^49]commands in Stata are written as ado-files. If you are like us, at any one time you are working on several different projects. We like to keep each project in a different directory. For example, d: $\backslash$ nas includes research for the National Academy of Sciences, d: $\backslash$ kinsey is a project associated with the Kinsey Institute, and d: \spost \examples is (you guessed it) for this book. While you can change to these directories with the cd command, one of us keeps forgetting the names of directories. So, he writes a simple ado-file

```
program define spost
    cd d:\spost\examples
end
```

and saves this in his PERSONAL directory as spost.ado. Type sysdir to see what directory is assigned as the PERSONAL directory. Then, whenever he types spost, his working directory is immediately changed:
. spost
d:\spost\examples

### 8.6.4 me.hlp file

Help files in Stata are plain text or SMCL files that end with the .hlp extension. When you type help command, Stata searches in the same directories used for ado-files until it finds a file called command.hlp. We have a file called me.hlp that contains information on things we often use but seldom remember. For example,

```
.-
help for "me"
.-
\begin{tabular}{|c|c|}
\hline Reset everything & \begin{tabular}{l}
"clear" \\
-discard^
\end{tabular} \\
\hline List installed packages & - ado dir \({ }^{\text {- }}\) \\
\hline \multirow[t]{4}{*}{Axes options} & \multirow[t]{4}{*}{} \\
\hline & \\
\hline & \\
\hline & \\
\hline \multirow[t]{3}{*}{Connect options "c()^} & \multirow[t]{3}{*}{} \\
\hline & \\
\hline & \\
\hline \multirow[t]{9}{*}{Symbols "s()^} & -0^ large circle \\
\hline & -S' large square \\
\hline & \({ }^{\text {T^ }}\) - large triangle \\
\hline & -0- small circle \\
\hline & -d^ small diamond \\
\hline & "p- small plus \\
\hline & ^x^ x \\
\hline & .^ dot \\
\hline & ^i^ invisible \\
\hline
\end{tabular}
Author: Scott Long
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```

This file is saved in your PERSONAL directory; typing sysdir will tell you what your PERSONAL directory is. Then, whenever we are stuck and want to recall this information, we just need to type help me and it is displayed on our screen.

### 8.6.5 Scrolling in the Results Window in Windows

After you run a command whose output scrolls off the Results Window, you will notice that a scroll bar appears on the right side of the window. You can use the scroll bar to scroll through results that are no longer in the Stata Results Window. While Stata does not allow you to do this with a keystroke, you can use the scroll wheel found on some mice. We find this very convenient.

### 8.7 Conclusions

Our goal in writing this book was to make it routine to carry out the complex calculations necessary for the full interpretation of regression models for categorical outcomes. While we have gone to great lengths to check the accuracy of our commands and to verify that our instructions are correct, it is possible that there are still some "bugs" in our programs. If you have a problem, here is what we suggest:

1. Make sure that you have the most recent version of the Stata executable and ado-file (select Help $\rightarrow$ Official Updates from the menus) and the most recent versions of SPost (while online, type net search spostado). This is the most common solution to problems people send us.
2. Make sure that you do not have another command from someone else with the same name as one of our commands. If you do, one of them will not work and needs to be removed.
3. Check our FAQ (Frequently Asked Questions) Page located at

> www.indiana.edu/~jsl650/spost.htm

You might find the answer there.
4. Make sure that you do not have anything but letters, numbers, and underscores in your value labels. Numerous programs in Stata get hung up when value labels include other symbols or other special characters.
5. Take a look at the sample files in the spostst4 and spostrm4 packages. These can be obtained when you are on-line and in Stata. Type net search spost and follow the directions you receive. It is sometimes easiest to figure out how to use a command by seeing how others use it.

Next, you can contact us with an e-mail to spostsup@indiana.edu. While we cannot guarantee that we can answer every question we get, we will try to help. The best way to have the problem solved is to send us a do-file and sample dataset in which the error occurs. It is very hard to figure out some problems by just seeing the log file. Since you may not want to send your original data due to size or confidentiality, you can construct a smaller dataset with a subset of variables and cases.

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[^0]:    ${ }^{1}$ The use of the LRM with a binary dependent variables leads to the linear probability model (LPM). We do not consider the LPM further, given the advantages of models such as logit and probit. See Long (1997, 35-40) for details.

[^1]:    ${ }^{2}$ If you click on a link and immediately get a beep with an error message saying that Stata is busy, the problem is probably that Stata is waiting for you to press a key. Most often this occurs when you are scrolling output that does not fit on one screen.

[^2]:    This book is for use by faculty, students, staff, and guests of UCLA, and is not to be distributed, either electronically or in printed form, to others.

[^3]:    ${ }^{1}$ Our screen shots and descriptions are based on Stata for Windows. Please refer to the books Getting Started with Stata for Macintosh or Getting Started with Stata for Unix for examples of the screens for those operating systems.

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[^4]:    ${ }^{2}$ For now, we only consider entering one command at a time, but in Section 2.9 we show you how to run a series of commands at once using "do-files".

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[^5]:    ${ }^{3}$ Stata can use virtual memory if you need to allocate memory beyond that physically available on a system, but we find that virtual memory makes Stata unbearably slow. At the time this book was written, StataCorp was considering increasing the dataset limits, so visit www. stata. com for the latest information.

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[^6]:    ${ }^{4}$ Students sometimes find it amusing to use names like dumbproject.do or joanieloveschachi.do. The fun ends when one needs to reconstruct something but can no longer recall which file does what.

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[^7]:    ${ }^{5}$ In Stata 6 and earlier, some official Stata commands changed the sort order of the data, but fortunately this quirk was removed in Stata 7. As of Stata 7, no properly written Stata command should change the sort order of the data, although readers should beware that user-written programs may not always follow proper Stata programming practice.

[^8]:    This book is for use by faculty, students, staff, and guests of UCLA, and is not to be distributed,

[^9]:    ${ }^{6}$ StataCorp also maintains a web site with resources for developing graphs using the new graphics engine that was introduced with Stata 7. See developer.stata.com/graphics/ (with no www in the front) for details.

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[^10]:    This book is for use by faculty, students, staff, and guests of UCLA, and is not to be distributed, either electronically or in printed form, to others.

[^11]:    ${ }^{7}$ If you download this file from our web site, it is called st4ch2tutorial.do.

[^12]:    ${ }^{1}$ While many of the principles and procedures discussed in this book apply to panel models, such as estimated by Stata's xt commands, or the multiple equation systems, such as biprobit or treatreg, these models are not considered here.
    ${ }^{2}$ In many situations there are convincing reasons for using Bayesian or exact methods for the estimation of these models. However, these methods are not generally available and hence are not considered here.

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[^13]:    ${ }^{3}$ mlogit is a multiple-equation estimation command, but the syntax is the same as single-equation commands because the independent variables included in the model are the same for all equations. The zero-inflated count models zip and zinb are the only multiple-equation commands considered in our book where different sets of independent variables can be used in each equation. Details on the syntax for these models are given in Chapter 7.

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[^14]:    ${ }^{4}$ This example uses binlfp2.dta, which does not have any missing data. We have artificially created missing data. Remember that all of our examples are available from www.indiana.edu/~jsl650/spost.htm or can be obtained by with net search spost.

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[^15]:    ${ }^{5}$ These versions can be computed by using the hc2 or hc3 options for regress. Long and Ervin (2000) recommend using hc3.

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[^16]:    b = raw coefficient
    $t=t-s c o r e$ for test of $b=0$
    $\mathrm{P}>|\mathrm{t}|=\mathrm{p}$-value for t -test
    bStdX $=$ x-standardized coefficient
    bStdY = y-standardized coefficient
    bStdXY $=$ fully standardized coefficient
    SDof $X=$ standard deviation of $X$

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[^18]:    This book is for use by faculty, students, staff, and guests of UCLA, and is not to be distributed, either electronically or in printed form, to others.

[^19]:    ${ }^{6}$ In cloglog, the value at iteration 0 is not the log likelihood with only the intercept. For zip and zinb, the "interceptonly" model can be defined in different ways. These commands return as e(ll_0), the value of the log likelihood with the binary portion of the model unrestricted while only the intercept is free for the Poisson or negative binomial portion of the model. fitstat returns the value of the log likelihood from the model with only an intercept in both the binary and count portion of the model.

[^20]:    ${ }^{7}$ The $\alpha, \beta$, and $\delta$ parameters in this equation are unrelated to those in Figure 3.1.
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[^21]:    This book is for use by faculty, students, staff, and guests of UCLA, and is not to be distributed, either electronically or in printed form, to others.

[^22]:    ${ }^{8}$ The estimation sample includes only those cases that were used in estimating a model. Cases that were dropped due to missing values and/or if and in conditions are not part of the estimation sample.

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[^23]:    This book is for use by faculty, students, staff, and guests of UCLA, and is not to be distributed, either electronically or in printed form, to others.

[^24]:    ${ }^{1}$ These data were generously made available by Thomas Mroz.
    ${ }^{2}$ outreg is an user-written command Gallup (2001) that must be added to Stata before it can be used. To install a copy of the command, type net search outreg while on-line and then follow the prompts.

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[^25]:    ${ }^{3}$ Recall that the $>$ in the following command indicates a line wrap. The $/ * * /$ is just a way of executing long lines in do-files. You should not type these characters if you are working from the Command Window.

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[^26]:    ${ }^{4}$ If you attempt to reproduce this example, the observation numbers of the observations with the largest residuals could be slightly different. This is because there are tie values in the data; sort breaks the ties randomly.

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[^27]:    ${ }^{5}$ fitstat, dif computes the difference between all measures even if the models are not nested. As with the Stata command lrtest, it is up to the user to determine if it makes sense to interpret the computed difference.

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[^28]:    ${ }^{6}$ Stata estimation commands create the variable e (sample) indicating whether a case was used when estimating a model. Accordingly, the condition if e (sample) $==1$ selects only cases used in the last estimation.
    ${ }^{7}$ Recall from Chapter 2 that the gap() option controls the distance between the left-side text and the vertical axis.

[^29]:    ${ }^{8}$ mean is the default setting for the rest () option, so rest (mean) does not need to be specified. We include it in many of our examples anyway, because its use emphasizes that the results are contingent on specified values for all of the independent variables.

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[^30]:    ${ }^{9}$ The formula for computing probabilities from odds is $p=\frac{\Omega}{1+\Omega}$.
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[^31]:    This book is for use by faculty, students, staff, and guests of UCLA, and is not to be distributed,

[^32]:    This book is for use by faculty, students, staff, and guests of UCLA, and is not to be distributed,

[^33]:    ${ }^{1}$ This plot illustrates how graph can be used to construct graphs that are not based on real data. The commands for this graph are contained in st4ch5.do, which is part of the package spostst4. See Chapter 1 for details.

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[^34]:    This book is for use by faculty, students, staff, and guests of UCLA, and is not to be distributed, either electronically or in printed form, to others.

[^35]:    ${ }^{2}$ The sterotype model can be set up in several different ways. For example, in some presentations, it is assumed that $\beta_{0}=0$ and fewer constraints are imposed on the $\alpha$ 's. Here we parameterize the model to highlight its links to other models that we consider.

[^36]:    This book is for use by faculty, students, staff, and guests of UCLA, and is not to be distributed,

[^37]:    ${ }^{1}$ Even though mlogtest estimates other models in order to compute various tests, when the command ends it restores the estimates from your original model. Accordingly, other commands that require results from your original mlogit, such as predict and prvalue, will still work correctly.

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[^39]:    ${ }^{2}$ StataCorp recently increased the limits for the number of options that can be contained in a dialog box. Accordingly, future versions of mlogview are likely to have additional options, and the dialog box you get could look different.

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[^41]:    This book is for use by faculty, students, staff, and guests of UCLA, and is not to be distributed, either electronically or in printed form, to others.

[^42]:    . graph psnobeq psnpreq psnval, c(ll) gap(3) l2("Probability") s(OT) /*
    > */ yscale $(0, .4)$ ylabel ( 0.1 to .4) xlabel (0 1 to 9)
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[^43]:    ${ }^{1}$ This and similar graphs cannot be created using Stata 7.
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[^44]:    ${ }^{2}$ The NBRM can also be derived through a process of contagion where the occurrence of an event changes the probability of further events. That approach is not considered further here.

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[^45]:    ${ }^{1}$ Details on creating mentord from the data in couart2. dta are located in st4ch8. do, which is part of the spostrm4 package. For details, when you are in Stata and on-line, type net search spostrm4.

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[^46]:    This book is for use by faculty, students, staff, and guests of UCLA, and is not to be distributed, either electronically or in printed form, to others.

[^47]:    ${ }^{2}$ Alternatively, we could have re-estimated the model adding none and excluding either few or some, and then used prchange.

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[^48]:    ${ }^{3}$ StataCorp offers both introductory and advanced NetCourses in programming; more information on this can be obtained from www.stata.com.
    ${ }^{4}$ The preferred place for the file is in your default data directory (e.g., c: $\backslash$ data).
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